Practice with Definite Integrals

Use antidifferentiation to compute the following definite integrals. Check your work using the geometric definition of the definite integral, graphing and estimation.

a) $\int_0^2 x^2\,dx$

b) $\int_1^e \frac{1}{x}\,dx$

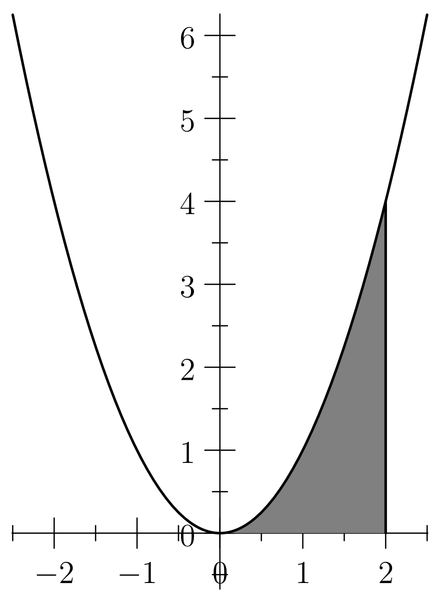
c) $\int_{-\pi/4}^0 \sin x\,dx$

Solution

a) $\int_0^2 x^2\,dx$

The antiderivative of x^2 is $\frac{1}{3}x^3$ (plus a constant), so:

$$\begin{aligned}\int_0^2 x^2\,dx &= \frac{1}{3}x^3\Big|_0^2\\ &= \frac{1}{3}\cdot 2^3 - \frac{1}{3}\cdot 0^3\\ &= \frac{8}{3}.\end{aligned}$$

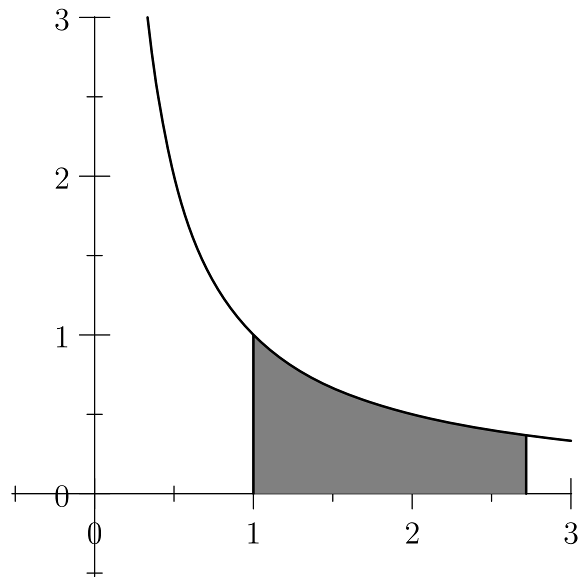


Geometrically, the area under the curve appears to be between 2 and 4 so our answer seems to be correct.

b) $\displaystyle\int_1^e \frac{1}{x}\,dx$

The antiderivative of $\frac{1}{x}$ is $\ln|x|$.

$$\begin{aligned}\int_1^e \frac{1}{x}\,dx &= [\ln|x|]_1^e \\ &= \ln e - \ln 1 \\ &= 1 - 0 \\ &= 1\end{aligned}$$



The area under the curve does appear to be between 1 and 1.5.

c) $\displaystyle\int_{-\pi/4}^{0} \sin x\,dx$

The antiderivative of $\sin x$ is $-\cos x$.

$$\begin{aligned}\int_{-\pi/4}^{0} \sin x\,dx &= -\cos x|_{-\pi/4}^{0} \\ &= -\cos(0) - (-\cos(-\pi/4)) \\ &= -1 + \frac{1}{\sqrt{2}}\end{aligned}$$

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\begin{aligned}
&= \frac{1-\sqrt{2}}{2} \\
&\approx -0.2
\end{aligned}
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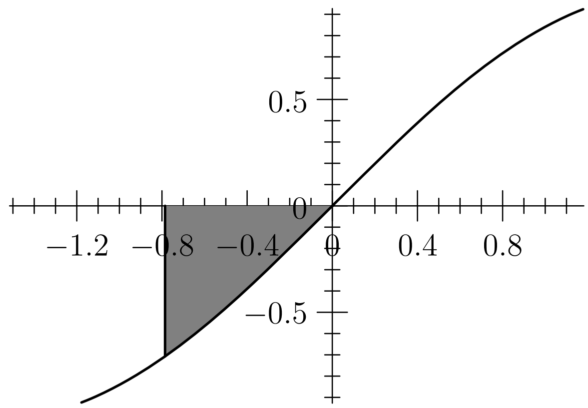

We're computing the area of a region that lies below the x-axis, so we expect the answer to be negative. The estimate -0.2 seems very small but is in fact close to the correct value, as we see when we compare the shaded region to a rectangle with side lengths 0.5 and 0.4.