Antiderivatives are Unique up to a Constant

Theorem: If $F'(x) = f(x)$ and $G'(x) = f(x)$, then $F(x) = G(x) + c$.

In other words, once we've found one antiderivative of a function we know that any other antidervative we might find will only differ from it by some added constant.

Proof: If $F' = G'$ then $(F - G)' = F' - G' = f - f = 0$.

Recall that we proved as a corollary of the Mean Value Theorem that if a function's derivative is zero then it is constant. Hence $G(x) - F(x) = c$ (for some constant c). That is, $G(x) = F(x) + c$.

This is a very important fact. It's the basis for calculus; the reason why it makes sense to do calculus at all. This theorem tells us that if we know the rate of change of a function we can find out everything else about the function except this starting value c.

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