Antiderivatives of $\sec^2 x$ and $\dfrac{1}{\sqrt{1-x^2}}$

Example: $\displaystyle\int \sec^2 x\, dx$

Searching for antiderivatives will help you remember the specific formulas for derivatives. In this case, you need to remember that $\frac{d}{dx}\tan x = \sec^2 x$.

$$\int \sec^2 x\, dx = \tan x + c$$

Example: $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\, dx$

An alternate way to write this integral is $\int \frac{dx}{\sqrt{1-x^2}}$. This is consistent with the idea that dx is an infinitesimal quantity which can be treated like any other number.

We remember $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ and conclude that:

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c.$$

Example: $\displaystyle\int \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + c$$

When looking for antiderivatives, you'll spend a lot of time thinking about derivatives. For a little while you may get the two mixed up and differentiate where you were meant to integrate, or vice-versa. With practice, this problem goes away.

Here is a list of the antiderivatives presented in this lecture:

1. $\displaystyle\int \sin x\, dx = -\cos x + c$ where c is any constant.
2. $\displaystyle\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$ for $n \neq -1$.
3. $\displaystyle\int \frac{dx}{x} = \ln|x| + c$ (This takes care of the exceptional case $n = -1$ in 2.)
4. $\displaystyle\int \sec^2 x\, dx = \tan x + c$.
5. $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ (where $\sin^{-1} x$ denotes "inverse sine" or arcsin, and not $\dfrac{1}{\sin x}$.)
6. $\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1}(x) + c$.