Introduction to Antiderivatives

This is a new notation and also a new concept. $G(x) = \int g(x)dx$ is the *antiderivative* of g. Other ways of saying this are:

$$G'(x) = g(x) \quad \text{or,} \quad dG = g(x)dx$$

There are a few things to notice about this definition. It includes a differential dx. It also includes the symbol $\int$, called an *integral sign*; the expression $\int g(x)dx$ is an *integral*. Another name for the antiderivative of g is the *indefinite integral* of g. (We'll learn what "indefinite" means in this context very shortly.)

If $G(x)$ is the antiderivative of $g(x)$ then $G'(x) = g(x)$. To find the antiderivative of a function g (to integrate g), we need to find a function whose derivative is g. In practice, finding antiderivatives is not as easy as finding derivatives, but we want to be able to integrate as many things as possible. We'll start with some examples.

Example: $\sin x$

We start with the integral of $g(x) = \sin x$. This is a function whose derivative is $\sin x$. What function has $\sin x$ as its derivative?

Student: $-\cos x$

Because the derivative of $-\cos x$ is $\sin x$, this is an antiderivative of $\sin x$. If:

$$\begin{aligned} G(x) &= -\cos x, \quad \text{then} \\ G'(x) &= \sin x \end{aligned}$$

On the other hand, if we had instead chosen $G(x) = -\cos x + 7$ we would still have had $G'(x) = \sin x$. Because the derivative of a constant is 0, we can add any constant to $G(x)$ and still have an antiderivative of $\sin x$. We write:

$$\int \sin x \ dx = -\cos x + c$$

and call this the *indefinite integral* of $\sin x$ because c can be any constant — it's an indefinite value. Whenever we take the antiderivative of something our answer is ambiguous up to a constant.