Notations

Calculus, rather like English or any other language, was developed by several people. As a result, just as there are many ways to express the same thing, there are many notations for the derivative.

Since $y = f(x)$, it's natural to write

$$\Delta y = \Delta f = f(x) - f(x_0) = f(x_0 + \Delta x) - f(x_0)$$

We say "Delta y" or "Delta f" or the "change in y".

If we divide both sides by $\Delta x = x - x_0$, we get two expressions for the difference quotient:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta x}$$

Taking the limit as $\Delta x \to 0$, we get

$$\begin{aligned} \frac{\Delta y}{\Delta x} &\to \frac{dy}{dx} \text{ (Leibniz' notation)} \\ \frac{\Delta f}{\Delta x} &\to f'(x_0) \text{ (Newton's notation)} \end{aligned}$$

In Leibniz' notation we might also write $\frac{df}{dx}$, $\frac{d}{dx}f$ or $\frac{d}{dx}y$. Notice that Leibniz' notation doesn't specify where you're evaluating the derivative. In the example of $f(x) = \frac{1}{x}$ we were evaluating the derivative at $x = x_0$.

Other, equally valid notations for the derivative of a function f include f' and Df.