Repeated Differentiation of Sine and Cosine

a) For which of the functions whose graphs are displayed by the Creating the Derivative mathlet is it true that $f''(x) = -f(x)$?

b) Can you think of any other functions for which $f''(x) = -f(x)$?

Solution

a) For which of the functions whose graphs can be displayed by the Creating the Derivative mathlet is it true that $f''(x) = -f(x)$?

By selecting various functions from the pull-down menu on the lower left, checking the box next to the green $f''(x)$ and moving the slider (or pushing the >> button) we see that $f''(x) = -f(x)$ only for the two functions $\sin(x)$ and $\cos(x)$ out of the five available functions.

b) Can you think of any other functions for which $f''(x) = -f(x)$?

If you thought of $f(x) = 0$ or of something like $f(x) = a\cos(x) + b\sin(x)$ you have done very well.

If neither of these ideas occurred to you, do not be discouraged; functions with this property are rare. It can be extremely difficult to solve "differential equations" like $f''(x) = -f(x)$; MIT offers an entire course on differential equations.

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