Example: $D^n x^n$

Let's calculate the n^{th} derivative of x^n

$$D^n x^n = ? \quad (n = 1, 2, 3, ...)$$

Let's start small and look for a pattern:

$$\begin{aligned} Dx^n &= nx^{n-1} \\ D^2x^n &= n(n-1)x^{n-2} \\ D^3x^n &= n(n-1)(n-2)x^{n-3} \\ &\vdots \\ D^{n-1}x^n &= (n(n-1)(n-2)\cdots 2)x^1 \end{aligned}$$

We can guess this $(n-1)^{st}$ derivative from the pattern established by the first three derivatives. The power of x decreases by 1 at every step, so the power of x on the $(n-1)^{st}$ step will be 1. At each step we multiply the derivative by the power of x from the previous step, so at the $(n-1)^{st}$ step we'll be multiplying by the previous power 2 of x.

Differentiating one more time we get:

$$D^n x^n = (n(n-1)(n-2)\cdots 2 \cdot 1)1$$

The number $(n(n-1)(n-2)\cdots 2\cdot 1)$ is written $n!$ and is called "n factorial". What we've just seen forms the basis of a proof by mathematical induction that $D^n x^n = n!$. So $D^n x^n$ is a constant!

The final question for the lecture is: what is $D^{n+1}x^n$?

Answer: It's the derivative of a constant, so it's 0.