Quotient Rule Practice

Find the derivatives of the following rational functions.

a) $\dfrac{x^2}{x+1}$

b) $\dfrac{x^4+1}{x^2}$

c) $\dfrac{\sin(x)}{x}$

Solution

a) $\dfrac{x^2}{x+1}$

The quotient rule tells us that if $u(x)$ and $v(x)$ are differentiable functions, and $v(x)$ is non-zero, then:

$$\left(\frac{u(x)}{v(x)}\right)' = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{(v(x))^2}$$

In this problem $u = x^2$ and $v = x + 1$, so $u' = 2x$ and $v' = 1$. Applying the quotient rule, we see that:

$$\begin{aligned}\left(\frac{x^2}{x+1}\right)' &= \frac{2x \cdot (x+1) - x^2 \cdot 1}{(x+1)^2} \\ &= \frac{2x^2 + 2x - x^2}{(x+1)^2} \\ &= \frac{x^2 + 2x}{(x+1)^2}.\end{aligned}$$

b) $\dfrac{x^4+1}{x^2}$

Here $u(x) = x^4 + 1$, $u'(x) = 4x^3$, $v(x) = x^2$ and $v'(x) = 2x$. The quotient rule tells us that:

$$\begin{aligned}\left(\frac{x^4+1}{x^2}\right)' &= \frac{4x^3 \cdot x^2 - (x^4+1) \cdot 2x}{(x^2)^2} \\ &= \frac{4x^5 - 2x^5 - 2x}{x^4} \\ &= \frac{2x^4 - 2}{x^3}.\end{aligned}$$

c) $\frac{\sin(x)}{x}$

The derivative of $\sin(x)$ is $\cos(x)$ and the derivative of x is 1, so the quotient rule tells us that:

$$\begin{aligned}\left(\frac{\sin(x)}{x}\right)' &= \frac{(\cos(x))\cdot x - (\sin(x))\cdot 1}{x^2} \\ &= \frac{x\cos(x) - \sin(x)}{x^2}.\end{aligned}$$

When we learn to evaluate this expression at $x = 0$, it will tell us that the slope of the graph of $\operatorname{sinc}(x) = \frac{\sin x}{x}$ is 0 when $x = 0$.