Main Formula

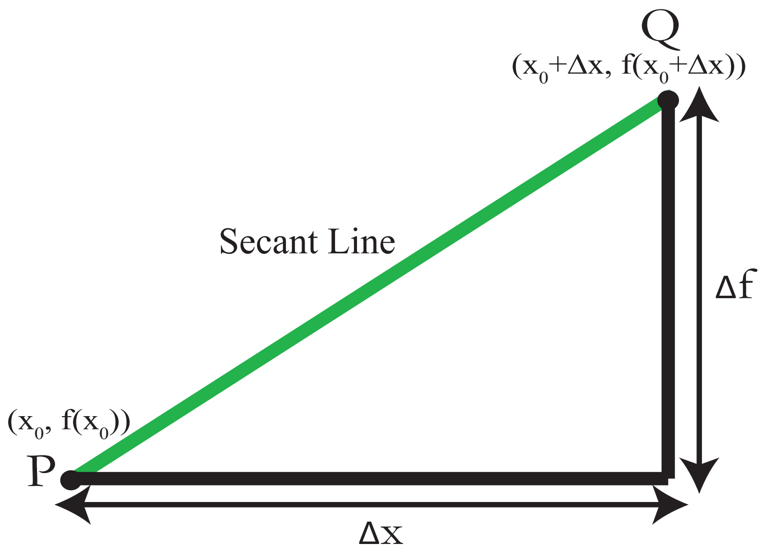

Figure 1: Geometric definition of the derivative

We started with a point P on the graph of $y = f(x)$ which had coordinates $(x_0, f(x_0))$. We then found a point Q on the the graph which was Δx units to the right of P. The coordinates of Q must be $(x_0 + \Delta x, f(x_0 + \Delta x))$. We can now write the following formula for the derivative:

$$m = \underbrace{f'(x_0)}_{\text{derivative of } f \text{ at } x_0} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \underbrace{\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}}_{\text{difference quotient}}$$

This is by far the most important formula in Lecture 1; it is the formula that we use to compute the derivative $f'(x_0)$, which equals the slope of the tangent line to the graph at P. A machine could use this formula together with the coordinates $(x_0, f(x_0))$ of the point P to draw the tangent line to the graph of $y = f(x)$ at the point P.