18.01 Single Variable Calculus
Fall 2006

Exam 3 Review

Integration

1. Evaluate definite integrals. Substitution, first fundamental theorem of calculus (FTC 1), (and hints?)
2. FTC 2:
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(t)$$
If $F(x) = \displaystyle\int_a^x f(t)\,dt$, find the graph of F, estimate F, and change variables.
3. Riemann sums; trapezoidal and Simpson's rules.
4. Areas, volumes.
5. Other cumulative sums: average value, probability, work, etc.

There are two types of volume problems:

1. solids of revolution
2. other (do by slices)

In these problems, there will be something you can draw in 2D, to be able to see what's going on in that one plane.

In solid of revolution problems, the solid is formed by revolution around the x-axis or the y-axis. You will have to decide how to chop up the solid: into shells or disks. Put another way, you must decide whether to integrate with dx or dy. After making that choice, the rest of the procedure is systematically determined. For example, consider a shape rotated *around the y-axis.*

- *Shells*: height $y_2 - y_1$, circumference $2\pi x$, thickness dx
- *Disks (washers)*: area πx^2 (or $\pi x_2^2 - \pi x_1^2$), thickness dy; integrate dy.

Work

$$\text{Work} = \text{Force} \cdot \text{Distance}$$

We need to use an integral if the force is variable.

Example 1: Pendulum. See Figure 1
Consider a pendulum of length L, with mass m at angle θ. The vertical force of gravity is mg (g = gravitational coefficient on Earth's surface)

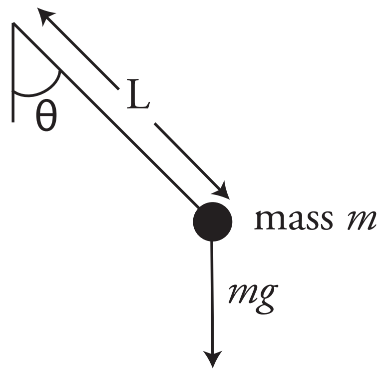

Figure 1: Pendulum.

In Figure 2, we find the component of gravitational force acting along the pendulum's path $F = mg \sin\theta$.

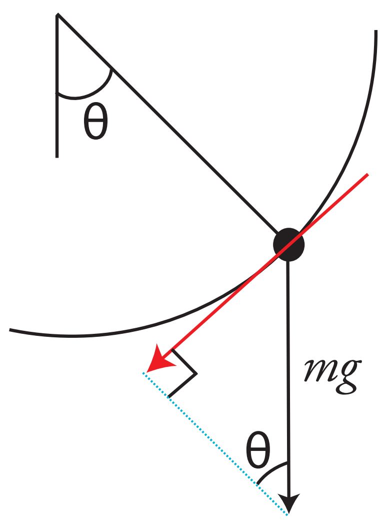

Figure 2: $F = mg \sin\theta$ (force tangent to path of motion).

Is it possible to build a perpetual motion machine? Let's think about a simple pendulum, and how much work gravity performs in pulling the pendulum from θ_0 to the bottom of the pendulum's arc.

Notice that F varies. That's why we have to use an integral for this problem.

$$W = \int_0^{\theta_0} (\text{Force}) \cdot (\text{Distance}) = \int_0^{\theta_0} (mg \sin\theta)(L\,d\theta)$$

$$W = -Lmg\cos\theta\Big|_0^{\theta_0} = -Lmg(\cos\theta_0 - 1) = mg\left[L(1-\cos\theta_0)\right]$$

In Figure 3, we see that the work performed by gravity moving the pendulum down a distance $L(1-\cos\theta)$ is the same as if it went straight down.

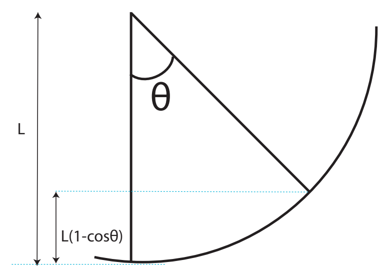

Figure 3: Effect of gravity on a pendulum.

In other words, the amount of work required depends only on how far down the pendulum goes. It doesn't matter what path it takes to get there. So, there's no free (energy) lunch, no perpetual motion machine.