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18.01 Single Variable Calculus  
Fall 2006

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## 18.01 Problem Set 8B

Due Friday 12/08/06, 1:55 pm

### Part I (20 points)

**Lecture 34.** Fri. Dec 1 Indeterminate forms; L'Hospital's rule, growth rate of functions.

Read: 12.2, 12.3 (Examples 1-3, remark 1) Work: 6A-1befgj, 5, 6c

**Lecture 35.** Tue. Dec. 5 Improper integrals.

Read: 12.4, Notes INT Work: 6B-1,2,7afkm, 8c

**Lecture 36.** Thurs. Dec. 7 Infinite series; simple convergence tests

Geometric series; harmonic series. Read: pp. 439-442(top)

Comparison tests. pp. 451-3 (skip proof in Example 3)

Integral test. pp. 455-457(top)

Work: 7A-1abc; 7B-1abf 7B-2acde

**Lecture 37.** Fri. Dec. 8 Taylor series.

Read: 14.4 through p. 498 (bottom); skip everything involving the remainder term  $R_n(x)$ .

Differentiation and integration of series. Read: 14.3-p.490(top); Examples 1-5.

Work: see handout with remarks about the final exam

**Lecture 38.** Tues. Dec. 12 Final Review.

### Part II (17 points + 5 extra)

**Directions:** Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

**0.** (not until due date; 3 pts) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". (See full explanation on PS1).

**1.** (Lecs 34-36, 6 + 5 pts: 1 + 2 + 2 + 1 + (5 extra))

a) Use L'Hospital's rule to evaluate

$$\lim_{x \rightarrow \infty} x^m e^{-x}$$

b) Use part (a) and limit comparison to show that the improper integral  $\int_0^{\infty} x^n e^{-x} dx$  converges for  $n \geq 0$ . (Do not integrate by parts.)

c) Denote  $A(n) = \int_0^{\infty} x^n e^{-x} dx$ . Use integration by parts to find the constant  $c_n$  for which

$$A(n+1) = c_n A(n)$$

(Explain what happens at the infinite limit using part (a).)

d) Find  $A(0)$  and deduce from part (c) the formula for  $A(n)$ ,  $n = 0, 1, 2, \dots$

e) (optional; 5pts extra credit) Show that the improper integral representing  $A(-1/2)$  converges and evaluate it using a change of variables.

**2.** (Review problem using solids and surfaces of revolution; 9pts: 4 + 2 + 3)

a) Show that the volume  $V$  of the solid of revolution enclosed on the top by a spherical cap of radius  $r$  of height  $h$  and underneath by a horizontal plane is

$$V = \pi \left( rh^2 - \frac{h^3}{3} \right)$$

Draw a picture. Doublecheck this answer in the three cases: the empty set ( $h = 0$ ); the half ball ( $h = r$ ); the whole ball ( $h = 2r$ ). We will always assume that  $0 \leq h \leq 2r$ . Why?

b) Find the surface area of the spherical cap of radius  $r$  and height  $h$ . Doublecheck your answer by evaluating it in the three cases  $h = 0$ ,  $h = r$  and  $h = 2r$ .

c) We will now find the shape of a soap bubble sitting on a table. Suppose that the bubble is a portion of a sphere as above, that it encloses a fixed volume  $V$ , and that its surface area is as small as possible (area of the curved surface, not counting the flat bottom = table). Find the minimizing shape, and describe it geometrically. (Hint: find  $h/r$ .)