18.01 Single Variable Calculus
Fall 2006

18.01 Problem Set 7

Due Friday 11/17/06, 1:55 pm

Part I   (20 points)

Lecture 25. (We will begin here on Nov 9.)   Trigonometric integrals. Direct substitution.
   Read 10.2, 10.3   Work: 5B-9, 11, 13, 16; 5C-5, 7, 9, 11 (moved here from PS6)

Lecture 27. Thurs. Nov. 9   Inverse substitution. Completing the square.
   Read 10.4   Work: 5D-1, 2, 7, 10

Lecture 28. Tues. Nov. 14   Integrating rational functions; partial fractions.
   Read 10.6, Notes F   Work: 5E-2, 3, 5, 6, 10h (complete the square)

Lecture 29. Thurs. Nov. 16   Integration by parts. Reduction formulas.
   Read 10.7   Work: 5F-1a, 2d then 2b, 3

Lecture 30. Tues. Nov. 16   Parametric equations; arclength. Surface area
   Read 17.1, 7.5   Work will be assigned on PS8

Part II   (22 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

0. (not until due date; 3 pts) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". (See full explanation on PS1).

1. (Lec 27, 4pts) (from PS6) The voltage V of house current is given by

$$V(t) = C \sin(120\pi t)$$

where t is time, in seconds and C is a constant amplitude. The square root of the average value of V^2 over one period of $V(t)$ (or cycle) is called the *root-mean-square* voltage, abbreviated RMS. This is what the voltage meter on a house records. For house current, find the RMS in terms of the constant C. (The peak voltage delivered to the house is $\pm C$. The units of V^2 are square volts; when we take the square root again after averaging, the units become volts again.)

2. (Lec 27, 4pts) The solid torus is the figure obtained by rotating the disk $(x - b)^2 + y^2 \leq a^2$ around the y-axis. Find its volume by the method of shells. (Hint: Substitute for $x - b$. As noted p. 229/11, the answer happens to be the area of the disk multiplied by the distance travelled by the center as it revolves.)

3. (Lec 27, 4pts: 2 + 2)

a) For any integer $n \geq 0$, use the substitution $\tan^2 x = \sec^2 x - 1$ to show that

$$\int \tan^{n+2} x \, dx = \frac{1}{n+1} \tan^{n+1} x - \int \tan^n x \, dx$$

b) Deduce a formula for $\int \tan^4 x \, dx$.

4. (Lec 28, 4pts: 3 + 1)

a) Derive a formula for $\int \sec x \, dx$ by writing $\sec x = \dfrac{\cos x}{1 - \sin^2 x}$ (verify this), and then making a substitution for $\sin x$ and using partial fractions. (Your final answer must be expressed in terms of x.)

b) Convert the formula into the more familiar one by multiplying the fraction in the answer on both top and bottom by $1 + \sin x$. (Note that $(1/2) \ln u = \ln \sqrt{u}$.)

5. (Lec 29, 3pts) Find the volume under the first hump of the function $y = \cos x$ rotated around the y-axis by the method of shells.