Prob. 15.1

Write out the two-dimensional compliance matrix **S** and stiffness matrix **D** for a single ply of graphite/epoxy composite with its fibers aligned along the *x* axes.

Define compliance matrix Eq. 15.11):

Digits:=4;with(linalg);

S:=matrix(3,3,[[1/E[1], -nu[21]/E[2], 0],[-nu[12]/E[1],1/E[2],0],[0,0,1/G[12]]]);

$$S := \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Numerical parameters for graphite-epoxy:

unprotect(E); E[1]:=230e9; E[2]:=6.6e9; G[12]:=4.8e9; nu[12]:=.25;

nu[21]:=nu[12]*E[2]/E[1];

Evaluate compliance matrix:

S_bar:=map(eval,S);

$$S\_bar := \begin{bmatrix} .4348\ 10^{-11} & -.1087\ 10^{-11} & 0 \\ -.1087\ 10^{-11} & .1515\ 10^{-9} & 0 \\ 0 & 0 & .2083\ 10^{-9} \end{bmatrix}$$

Obtain stiffness matrix by inversion:

D_bar:=inverse(S_bar);

$$D\_bar := \begin{bmatrix} .2304\ 10^{12} & .1653\ 10^{10} & 0 \\ .1653\ 10^{10} & .6611\ 10^{10} & 0 \\ 0 & 0 & .4799\ 10^{10} \end{bmatrix}$$

Prob. 15.2

Write out the *x-y* two-dimensional compliance matrix $\bar{S}$ and stiffness matrix $\bar{D}$ (Eqn. 15.12) for a single ply of graphite/epoxy composite with its fibers aligned 30° from the *x* axis.

Compliance matrix (Eq. 15.11):

Digits:=4;with(linalg); S:=matrix(3,3,[[1/E[1], -nu[21]/E[2], 0], [-nu[12]/E[1] ,1/E[2],0],[0,0,1/G[12]]]);

$$S := \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Numerical parameters for graphite/epoxy:

```
unprotect(E); E[1]:=230e9; E[2]:=6.6e9; G[12]:=4.8e9; nu[12]:=.25;
nu[21]:=nu[12]*E[2]/E[1];
```

Compliance matrix evaluated:

```
S2:=map(eval,S);
```

$$S2 := \begin{bmatrix} .4348\ 10^{-11} & -.1087\ 10^{-11} & 0 \\ -.1087\ 10^{-11} & .1515\ 10^{-9} & 0 \\ 0 & 0 & .2083\ 10^{-9} \end{bmatrix}$$

Transformation matrix (Eq. 10.4):

```
A:=matrix(3,3,[[c^2,s^2,2*s*c],[s^2,c^2,-2*s*c],[-s*c,s*c,c^2-s^2]]);
```

$$A := \begin{bmatrix} c^2 & s^2 & 2\,s\,c \\ s^2 & c^2 & -2\,s\,c \\ -s\,c & s\,c & c^2 - s^2 \end{bmatrix}$$

Trigonometric relations and angle:

```
s:=sin(theta);c:=cos(theta);theta:=30*Pi/180;
```

Transformation matrix evaluated:

```
A2:=evalf(map(eval,A));
```

$$A2 := \begin{bmatrix} .7500 & .2500 & .8660 \\ .2500 & .7500 & -.8660 \\ -.4330 & .4330 & .5000 \end{bmatrix}$$

Reuter's matrix (Eq. 10.7):

```
R:=matrix(3,3,[[1,0,0],[0,1,0],[0,0,2]]);
```

$$R := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Transformed compliance matrix (Eq. 11.12);

```
Sbar:=evalf(evalm( R &* inverse(A2) &* inverse(R) &* S2 &* A2 ));
```

$$Sbar := \begin{vmatrix} .5056\ 10^{-10} & -.1050\ 10^{-10} & -.7460\ 10^{-10} \\ -.1052\ 10^{-10} & .1241\ 10^{-9} & -.5278\ 10^{-10} \\ -.7462\ 10^{-10} & -.5278\ 10^{-10} & .1706\ 10^{-9} \end{vmatrix}$$

Transformed stiffness matrix (inverse of compliance matrix):

Dbar:=inverse(Sbar);

$$Dbar := \begin{vmatrix} .1344\ 10^{12} & .4187\ 10^{11} & .7173\ 10^{11} \\ .4191\ 10^{11} & .2236\ 10^{11} & .2525\ 10^{11} \\ .7175\ 10^{11} & .2524\ 10^{11} & .4506\ 10^{11} \end{vmatrix}$$