Prob. 2.7

(a) Relation for derivatives:

$$\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial S}{\partial L}\right)_T \Big/ \left(\frac{\partial S}{\partial T}\right)_L \qquad (1)$$

Second law along with heat content as mass *M* specific heat *c* temperature change *dT:*

$$dS = dQ / T, \quad dQ = McdT = TdS \Rightarrow \left(\frac{\partial S}{\partial L}\right)_L = \frac{Mc}{T} \qquad (2)$$

Substituting (2) into (1);

$$\boxed{\left(\frac{\partial T}{\partial L}\right)_S = \frac{-T}{Mc}\left(\frac{\partial S}{\partial L}\right)_T} \qquad (3)$$

(b) First and second law, with *dU* = 0 for an ideal rubber:

$$dU = 0 = dQ + dW = TdS + FdL \Rightarrow \frac{\partial S}{\partial L} = \frac{-F}{T}$$

Using this in Eq. (3) of the previous problem:

$$\frac{\partial T}{\partial L} = \frac{F}{Mc} \qquad (4)$$

Extension ratio: $\lambda = L / L_0 \Rightarrow dL = L_0 d\lambda$. Using this in Eq. (4):

$$\frac{\partial T}{\partial \lambda} = \frac{FL_0}{Mc} = \frac{FAL}{A_0 Mc} \qquad (5)$$

Now substituting

$$\frac{AL}{M} = \frac{V}{M} = \frac{1}{\rho}$$

where *V* is volume and ρ is density, along with the engineering stress $\sigma = F / A_0$ into (5):

$$\frac{\partial T}{\partial \lambda} = \frac{\sigma}{\rho c} \Rightarrow dT = \frac{\sigma}{\rho c} d\lambda$$