PROFESSOR: All right, my trusty $5 Timex watch says it's five after the hour. So I guess we might as well begin. I have corrected all the problems that were turned in-- the puzzles, not problems, puzzles. And I just yesterday got hold of the photographs of you few people. And I'd like to hand out the papers individually so I can start to put names together with faces.

So I'll turn them back individually next time. I have one additional problem set that for you. And again, this is in the form of a puzzle. Be assured that this is the last entertaining problem set. From here on, we move into drudgery-- tedious, drawn out things that hopefully will teach you something. But this is the last one that's done with a little bit of tongue in cheek humor.

This one is a little more subtle than the other two. And for this one, I will hand out a solution because I would like to use these problems to make a couple of basic points about the nature of symmetry theory in crystals. So see how you do on this one.

OK, also before we begin, he gets a little bit uneasy before a crowd. But I brought a little friend in with me today. And could someone hold the box, please, so I could take him out?

Sorry. He gets a little bit nervous before a crowd. Come on, take it easy. Take it easy. This is Rodney. He's an old friend of mine.

Rodney is a crystallographic abomination. Rodney has five-fold symmetry-- impossible in crystallography, an abomination. Why did Rodney decide to do this? Anybody have any idea? Why not six fold symmetry?

He's got five-fold symmetry. Yuck! But why? There's always a reason for things if you look carefully enough and know what questions to ask. Yeah?

AUDIENCE: He had narrow symmetry.

PROFESSOR: Yeah, he had narrow symmetry. But he could do that-- what if you were square? Why not a square starfish? Why five? He did this for a reason. He's been doing this for probably a couple of million years, he and his ancestors. There has to be a reason.

OK, let me turn him over. And maybe that will give you a clue. The backside is not nearly as nice as the front side. And in fact, if you look at Rodney carefully, Rodney has sutures between these five arms.

OK, this is what he looks like from the top. But if you look at him from the bottom, there are sutures between these arms that do this. And these sutures are weak points in his structure. And by having five-fold symmetry, he puts the suture between this pair of arms opposite the midpoint of the opposite rib. And that gives him a great deal more structural integrity.

So when a large fish comes along and gloms onto one of his arms and shakes, he doesn't split in half and end his career prematurely. What happens instead, at very worst, the tip of the arm breaks off. And then, Rodney gets to scamper away and make more starfish or do whatever his purpose in our environment is. If he had an even-fold symmetry, that wouldn't happen. He would really be done in by a fish grabbing hold and shaking.

AUDIENCE: What if [INAUDIBLE]?

PROFESSOR: That's a very good question. If he had only three arms and a fish glomed on and one arm broke off, his mobility would be greatly impaired. In a way, he would go hip, hop, flop. Hip, hip, flop. Hip, hip, flop. Hip, hip, flop. It'd be a pathetic thing. So five is much more advantageous than three.

AUDIENCE: Why not seven?

PROFESSOR: Why not seven? That's a good question, too. Would you believe that on the Australian Barrier Reef, there are starfish with nine-fold symmetry? Quite odd. That's really a creepy thing with all those arms flopping around. But as many things in our environment in our natural world, there's a reason for things if you can only think what it might be.

So we gave you another example of a non-crystallographic symmetry, the large cacti of the Southwest, the saguaros. And there are lots of other ribbed casts. They have symmetries that range between 19 and 23 fold. Why do you think they have this rib structure? Why can't they have a nice, smooth trunk like a birch tree?

Any idea there? I'll give you a hint. They live out in the desert. There's not much water.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Hm?

AUDIENCE: More surface area for the [INAUDIBLE].

PROFESSOR: Yeah, but that's bad. That means they would lose their water. So things that don't want to lose water through their surface-- you're right-- try to minimize their surface. But you're onto something. And it is exactly the reverse of that.

Water is scarce. So when it rains, this cactus wants to soak up the moisture and store it. If he has this pleated structure, he's able to expand. If he had a smooth surface with minimal area, he would just pop. And that would be the end of him or her, as the case may be. "It" is probably the appropriate term. But anyway, there's the reason for these many, many ribs. It gives them the ability to expand, store water, and then contract when the water evaporates.

So again, it's always interesting to look at the world around us and see examples of things that are strange because it's the strange things that very often can lead to penetrating conclusions. All right, last time we had talked about chirality in finite atomic structures, molecules. And I had pointed out-- but did not make the obvious observation-- I cannot think of a more dramatic connection between structure and properties, which is what material science is all about, than the very, very different properties of molecules that are of opposite handedness.

We pointed out, for example, that many pharmaceuticals are of chiral molecules that exist in right handed and left handed forms. And very often, one form of one handedness has a very different set of properties, and in particular pharmacological action, then it it's enan enantiomorph. So in the case of thalidomide, one handedness of the molecule acts as a tranquilizer. The other handedness acts as a source of birth defects.

One of the other ones less familiar to me, but this is [? ethambutol. ?] It's used as a drug to treat tuberculosis. The molecule of opposite handedness creates blindness.

The drug Ritalin that is used to treat hyperactivity, one handedness of the molecule is absolutely useless. So you only use half of what you pay for. So it's of great interest to pharmaceutical companies to be able to synthesize a molecule of one particular handedness with little if none of the other one.

And I looked up a number since we met last time. It turns out that in 1997, enantiomorph pure drugs were a $400 billion dollar industry. So again, this is structure property relations in action.

So let me ask you another question. It turns out that products that are derived from nature-- a good example being sugar, which is produced by sugar beets or sugar cane-- produces a molecule only of one handedness. They are [? enantiomer ?] pure. But every synthetic compound that can exist in right handed and left handed forms occurs with equal probability.

So how do you make a pharmaceutical of one particular handedness? And this is an interesting question because I think it was two years ago the Nobel Prize in chemistry was given to three people for their work in being able to synthesize molecules of a given handedness. Anybody an idea how you do it?

AUDIENCE: I don't have an idea how you do it, but what do you mean by one handedness versus--?

PROFESSOR: OK, left handed or right handed. So my left hand is exactly the same as my right hand, but I cannot match them into congruence with one another. And we don't know how to describe this other than we used the term derived from our physiology. We say left handed and right handed.

So when I was referring to a molecule of one handedness-- and I'll bring in a Xerox copy of a couple of examples. There are many examples of fairly common molecules which can exist in one form in which the hydroxyl group points out this way, then another form where the hydroxyl groups points out this way. Sorry, you should have asked that earlier.

AUDIENCE: So if it's not the same molecule, it would be that molecule but--?

PROFESSOR: Chemically, it is exactly the same concept. But the way in which those component models are configured has the same number of side groups, the same number of appendages. But they, don't have a better word. One is left handed. One is right handed. I'll give you some examples next time. Somebody else, you had a question?

AUDIENCE: I was just going to see if I could venture a guess how they do it. I was going to guess they use an electric field that has its bearings to set the right handed course [INAUDIBLE] that would make [? enantiomorphs ?] or more advantageous than [INAUDIBLE].

PROFESSOR: That's not a bad idea. Anything that could bias the energy of one configuration over the other would do it. But actually, no reason why you should know how to do it. If you knew how to do it, maybe you could have gotten the Nobel Prize instead of these other guys.

But actually, you do it through catalysis. And you use a catalyst which itself has chirality. And that chirality favors the formation of one molecule over the other almost exclusively. And that's why living plants make-- or animals-- make chiral molecules in just one handedness as well. So it's using a catalyst that also has some chirality.

AUDIENCE: How do you get the one hundred catalysts [INAUDIBLE]?

PROFESSOR: How do you have it--? OK, there are lots of materials. Any crystalline material that does not have a mirror plane in it can exist in chiral forms. And one of the classic examples of this is quartz. Quartz is SiO2. And quartz has actually only a threefold symmetry. But it forms crystals that look prismatic, as though they were hexagonal prisms.

And if I draw two of these prisms side by side, you could not tell which prism was left handed and which was right handed. But quartz ends up growing with a couple of little facets on it that look like this. They spiral up this way in a crystal of one handedness and they spiral up in the opposite direction for a crystal of opposite handedness. So the external faces on the crystal show you that this crystal shape cannot be mapped one into another. They are of opposite handedness.

And they have very different properties. And so here's an example. I'm not answer your question. I'm talking around it very adroitly in the hope that if I talk enough you will forget what you asked. But how do you make a chiral catalyst? Well, there are some materials if you prepare them in a single crystal form, once you nucleate one enantiomorph, that will continue to grow. So I would presume one could do exactly the same with a catalyst.

Actually, the properties of quartz are strikingly different for these two different forms. If you would subject the crystal to compression, this crystal would develop a positive charge on the surface. This crystal would develop a negative charge on the surface. That's a property that we'll discuss in detail later on. That's something called piezoelectricity, literally pressure electricity. So the nature of the charge induced on the crystal is different for the crystals of the two chiralities. OK, any other comment or question?

OK, last time we had taken the first small steps in what will turn out to be a long and protracted process of synthesis. We've covered by now some of the general notions of operations that can exist in a pattern, two dimensional pattern of wallpaper or fabric or a three dimensional pattern that constitutes a crystal. And we said that in three dimensions, there are four basic different operations that can exist that tell you how one part of the system is related to another.

There is an operation of translation. And this has all the characteristics of a vector, magnitude and direction. We summarize that translational periodicity by a set of fiducial markers that are called lattice points. And analytically, the transformation takes the coordinate x, y, z. And if you do the operation once, maps this to x plus a, y plus b, z plus c where the translation has components a, b, and c in x, y, and z directions.

Another operation is the operation of reflection. And what this operation does is to take the coordinates of an initial part of the space atom or location and maps it into some location where one direction is reversed in sign. Which direction that is or whether it's a pair of directions will depend on how the reflection locus is located relative to the coordinate system.

Next, there was an operation of rotation. And this involves repeating things at angular intervals. Take the space or the location of the atom and rotate it through some angle, alpha, about some rotation, a. And in a sense, this is a periodicity which is reminiscent of translation except translation strings things out in a line. Rotation wraps up the directions about some central axis. But it's also periodic.

Then finally, in three dimensions, there is another operation called inversion. And what this does is-- to could describe it as such in literal terms-- take something and turns it inside out to a point. So if the inversion point were at the origin, it would take x, y, z, and map it into minus x, minus y, minus z.

And then, we concluded last time with a definition of a vocabulary for indicating analytically a single operation, the set of operations that constitutes the symmetry element, and then a geometric symbol for the locus of that operation. And for translation, we could represent a single operation by t. The set of operations could be-- and I'll use this customary set of braces to indicate the set-- this would be a one dimensional lattice, a set of operations, p, where p is an integer times t.

The geometric symbol, it's convenient to use an arrow extending from one lattice point to another. But I emphasize that there's no unique origin to the translation. We cannot be entitled to say it sits here as opposed to down here or down here. There are an infinite number of parallel directions that could specify a given interval between lattice points.

For rotation operations, the notation for a single operation in as much as we must specify two things, the point about which we rotate and then the angular interval about which we perform the rotation. And so we need to specify two things, the location of the axis, a point a, and the angle, alpha, through which we rotate.

The set of operations, the rotation axis, is indicated by an integer, n, where the rotation part of the operation is 2 pi divided by the integer, n. So we would use, for example, 2 to represent a rotation axis where things were moved through 180 degrees, 3 for a symmetry where things were mapped through intervals of 120 degrees, and 22 for my good friend, the Saguaro cactus, which is left invariant perhaps by a rotation through 1/22 of 2 pi.

For the geometric symbol for a rotation axis, we will use a little polyhedron. Triangle for a threefold axis. Square for a fourfold axis. For a twofold axis, the polyhedron that had that symmetry would be a line segment. So we'll take a little artistic license and let the middle of the line segment bulge out a little bit.

Finally for inversion, for reasons that will not become clear for another meeting or two, the single operation is denoted one bar. The set of operations of which there are only two, inverting and coming back again, also indicated by the symbol one bar. And the geometric symbol for the locus is a little, tiny circle that's open large enough to be noticed, but not so large that it looks like an atom in an atomic arrangement.

So there's our basic bag of tricks. And last time, we had already used some simple geometry to come to a rather astonishing conclusion. Yes, sir?

AUDIENCE: Reflection?

PROFESSOR: Reflection, that's this middle one. Whoops, not going to help. Thank you very much.

I get excited and carried away.

OK, reflection, the second operation. Single operation, although crystallographers don't often use it in condensed matter physics, a single operation is represented by the symbol sigma. Symbol for a mirror plane is m, easily appreciated. And the geometric locus across which things are reflected left to right and vice versa is done with a bold, solid line when you're looking down parallel to the surface of the reflection locus.

Then, I will now be caught up. And I can set off on something new. We had said that in a pattern, very often more than one of these operations are combined in a single space. And one of the things that must be present-- if the symmetry of the entity we are discussing is a crystal-- one of the things that must be present is a translation.

And so I asked last time the question, what if we want to say that our space has a translational periodicity described by t. And then, I add to one lattice point a rotation operation, A alpha, that has to get translated to another location where that operation, A alpha, exists. And if A alpha exists and I repeat that translation by a rotation alpha, I'm going to have a sheath of translations all separated by a distance alpha.

If I single out the one that is removed from the first by an operation of rotation A alpha in a counterclockwise direction and single out from that sheath another one that is alpha away in the clockwise direction. Then, I have lattice points at the end of all of these translations. But these two guys up here are also lattice points. And therefore, I have mucked up to the entire construction unless I can claim that this distance between these two lattice points is either T or some multiple P times T, where P is an integer.

And then lickety split, before you could catch your breath in wonder, I dropped a perpendicular down to the original translation. This distance in here was PT. This distance on either side was T times the cosine of alpha. And that led me to a constraint that alpha, that rotation operation A alpha, if it were to be combined with translation, would be restricted to values such that the cosine of alpha was equal to

1 minus an integer P divided by 2.

Wow, simple, but so incredibly profound. What came out of this was that alpha could either be 0 or 360 degrees. That was a one-fold axis. Alpha could be 180 degrees. That was a two-fold axis. 120, which would be the angular rotation of a three-fold axis. 90, that would be the operation of a four-fold axis. Or 60, that would be the operation of a six-fold axis.

It's incredible. Anything, any pattern, any crystal that is based on a lattice can contain only these five rotational symmetries including no symmetry at all. So this says a lot about what the morphology of crystalline materials are permitted to be.

Then, we looked at the nature of the lattices that were described by these angular rotations. And we found that there were only three types of lattices that were required by rotation. These were two dimensional lattice nets. One was a general oblique net which had two different translations, some arbitrary angle between them, two translations, T1 and T2, which were not equal to one another. We would call this the oblique net.

Then, there was another net that was square in the sense that two translations, T1 and T2, were identical to one another-- not close, but identical. And the angle between them was identically 90 degrees. This oblique net could accept either a one-fold axis or a two-fold axis. This was the specialized shape that was required by a four-fold axis.

And then finally, there was another net that looked oblique except it was a specialized oblique net. It had two translations, T1 and T2, which were identical just as in the square net and an angle between them, which was exactly 120 degrees. And this same sort of net could be compatible with either a three-fold axis or a six-fold axis.

Then, we took the only other operation that could be present in a two dimensional pattern, and this was the operation of reflection, a mirror point, m. And we saw that this by itself could require two different sorts of specializations in a lattice, a lattice in which two directions were exactly orthogonal but the lengths of the translations were unequal. And then, a diamond shaped net-- and this was a new wrinkle. This was a net that had two translations that were Identical in length and an arbitrary angle between them.

Or alternatively, if we chose to pick a redundant lattice, one that had two translations, T1 and T2 prime which were different but an angle between them that was exactly 90 degrees, so this one we would refer to as a rectangular lattice, a rectangular net, and this one as a centered rectangular net-- same shape, but an extra lattice point in the middle.

Why pick a double cell, which is redundant, takes up more, and doesn't convey any more information? And the advantage comes when you want to analytically describe the relation between different directions or the positions of atoms within the cell. In that case, an orthogonal coordinate system, even if it's not Cartesian, is of immense advantage as opposed to an oblique system.

OK, slightly out of breath, but that is everything we've covered to this point. And you can find this material discussed in the first chapter, in the introduction, and then half of the second chapter of Berger's book if you want to read over it. OK, any questions or comments?

All right, I wanted to say a little bit more about lattices and something with which you are no doubt familiar, but perhaps have some questions about why one does it in such an obscure, difficult way. And let me introduce what's done by an analogy which we have around us in everyday life.

Let me draw a system of streets and avenues for a city that might be one like Boston where the streets were laid out by cows wandering around in search of forage. And suppose somebody met you on the street and said, may I stop you here at point A and ask you a question? How do I get from point A to point B? That's not a vector. That's a point. How do I do that?

You would not say, go 342 meters to the east and 26.4 meters to the south. That's perfectly logical that is admirably Cartesian but you wouldn't do it that way what we just said you would say go one block straight ahead, and then turn one block to your left, and then go down the street to your right and turn to your right again, and you'll be right there. You can't miss it. Of course, the person in Boston would miss it and would have to stop a second time and ask again and by a method of successive approximations eventually get to where he or she would like to be.

I submit if a system gives you a grid, it makes sense to use that grid as the basis vectors of coordinate system even if the intervals are different, in two different directions, and even though the intervals might be not vectors that are orthogonal to one another. If the system is periodic and has this grid work to it, it makes sense to use that as the basis of a coordinate system.

So if we have a lattice that is oblique and there are lattice points defined by these two translations, T1 and T2, it makes great sense if you want to specify the vector that runs from this lattice point to this lattice point to do that by saying that this is a lattice point that is at the end of the vector 1 T1 plus 2 T2. And that, just as a system of streets and avenues, is a lot more efficient than saying go 12.2 angstroms in the x direction and go 15.3 angstroms in the y direction.

What we have defined here is something that is called a rational direction. And a rational direction is something that is going to be a direction that extends between lattice points. And what's rational about it is that the coefficients in front of T1 and T2 that would appear would be integers. And an integers is sometimes referred to as a rational number.

That would be quite different from a vector that extended from a lattice point to, let's say, some atom that is within the boundaries of this cell. And this might be a radial vector from the origin to one lattice. And that could be used to specify the atomic coordinates. This is going to complete the dichotomy, a feature that is referred to as irrational. Why? Because the components of the steps along T1, x, and along T2, y, are going to be fractional numbers.

So you'll hit irrational notation for directions that go to different locations with a unit cell. You'll use rational vectors in directions that extend between integral numbers of lattice points.

These directions occur all the time when one discusses the physical properties of crystals. I'm sure in any class that you've had that's discussed mechanical properties, one of the favorite topics is to discuss the ways in which close packed metals can deform. And this can be discussed easily in terms of one layer of close packed spheres sliding over another.

And one of the ways that the layer can slide to go from one set of close packed hollows to another is in a direction like this. And that turns out to be a rational direction. And these are directions in which a close packed metal will easily deform. So directions of easy plastic deformations almost always involve rational directions.

So there is a unique application of these sorts of features in the discussion of mechanical properties. Another thing that can happen is that a particular structure, if it is weakly bonded or relatively weakly bonded in one direction, if you come down with a chisel on that plane and give it a little tunk with a hammer, the crystal will fall exactly in half and give you two very smooth surfaces on either side of some plane.

And the reason is if you look down into the guts of the crystal and look at the atoms and how they're bonded together, there may be some direction of weak bonding between atoms that is very easily separated by some sort of mechanical force. This is true even for crystals that are very strongly bonded.

The way in which people begin to facet diamonds, for example, is to sit for a month and study the diamond and then decide how you're going to put a straight edge to it and give it a little tunk so that it falls neatly into two pieces at the sides that you want to facet. Doesn't always work, so you sit for a long time to figure out just what you're going to do. I think diamond cutters burn out early and have a useful career that last perhaps 3 and 1/2 years. And then, they're burned out. I don't know that for sure. But it must be a nerve wracking way of making an existence.

Not a far-fetched story. At one time, I was doing a diffraction study on a material which was a low temperature face, very interesting, but formed stable only at such low temperatures that you could not make it by cooking the components. But it had been found as a mineral in the Swiss Alps, all sorts of exotica attached to this particular material.

There was a particular place up in the Alps which had a very unusual chemistry involving lead and arsenic and thallium and sulfur-- not what you find in your usual backyard rock pile. And there was something like 37 unique minerals that were found in this little spot that were known nowhere else on the face of the Earth. And I was very interested in the atomic arrangements in one of them.

So I wrote to the British Museum that had found the sole supplier of this material that-- back in about 1901 or so-- I wrote and I said, do you have any crystals of this stuff? Yes. Would you like to study it? And I said yes, I would. Well, we'd be happy to send it. But please be careful not to damage the morphology because it is a very rare material.

So I said fine. I'm as careful as the next meticulous guy. They sent two crystals. The crystals measured-- one of them was 2/10 of a millimeter in diameter. One of them was about 0.15 millimeters in diameter. And I was supposed to take off a piece for study without damaging the morphology.

Man, I felt just like one of these Dutch diamond cutters. I sat and I looked at those two little suckers under the microscope for, I think, a week. And then finally, the moment of truth, and I took a needle and popped off a corner.

And it went into two pieces. One went for a microprobe analysis to determine the composition. One of them went to a single crystal x-ray diffraction study to determine the atomic arrangement. Both analyses can be done on a very small piece of material.

So my story about the diamond cutter agonizing before looking for a cleavage or a fracture surface is not just made up fiction. It is something that, when you work with crystals very often, is encountered.

OK, cleavage planes are also rational directions. The cleavage plane of rock salt is legendary. And it's nice, really, to spend an afternoon splitting it up just to see how neat it falls apart.

OK, how are we going to denote rational planes? Talked about rational and irrational directions, and that's easy. How are we going to describe rational planes in crystals? And this is rather bizarre. So let me tell you what you do, which is something you've probably heard before, and then explain why one does it this way.

Let's suppose we have three vectors that are the vectors we will use to define the lattice. And if there is a plane hanging on one of these lattice points, everything is translationally periodic, so there must be a similar plane in the same orientation that passes through all the lattice points.

So let me look at a very special plane, namely one that cuts a lattice point at an integral number of translations, T3, and an integral number of translations, T2, and an integral number of translations, T1. And I don't want any common factor here, so let's look at this point here. And now, let's connect these lattice points together.

And that will define a plane uniquely. And what is special about this point is that it hits a lattice point on each of these three axes. And this is a plane that I'll call the intercept plane.

There will be a similar plane hanging on all of these other lattice points. But those planes will not hit a lattice point on the directions of T1, T2, and T3. This is the first one out from the origin that does that. Now, we will use this oblique set of vectors, T1, T2, and T3, as the coordinate system for specifying direction and angles.

So let me ask a question now that's going to blow you away. What is the equation for the locus of this plane If I use this oblique coordinate system based on T1, T2, and T3 as my coordinate system? Holy mackerel. What a way to wrap up a Tuesday.

Actually, it's very, very easy. First of all, it has to be a linear equation. So it's going to be linear in x, y, and z. So something times x, something times y, something times z equals a constant. That's going to define, if I use the proper coefficients, the equation for this plane.

And let me determine quite easily what those coefficients are. What is the coordinate of this point, which sits on the plane and therefore must satisfy this equation? Well, y is 0. z is zero. So this is the point-- if I am a translations out from the origin, this is the point x equals A.

So if I'm three translations out from the origin, that integer is the coefficient in front of x because when y and z are 0, if I make the constants on the right hand side equal to 1, this is the point x equals a. And it's nice to have one on the right hand side. That's about as nice and neat and tidy a constant as you could have.

We do the same thing for this point at the end of two translations, T2. let's say in general that for this intercept plane, we are two translations out along T2. So for the point 0-- this point, 0-- if I put the number of translations, b, in front of y, This is the point y equals b when x and z are 0.

And in the same way, if we are C translations out along T3, C times z-- that integer-- is equal to 1.

AUDIENCE: Can you explain it again?

[INAUDIBLE]

PROFESSOR: I'm sorry. What?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, I'm saying I'm going to [INAUDIBLE] my definition at a plane which cuts the direction of T1, which is what I called the variable x, at a translations out. In this case, this is 3. This is 2. And this is 1. So for this particular example, I would have 3x plus 2y plus 1z equals 1 because when y is 0 and z is 0, the point on the surface is-- 1/3. Sorry. Good question.

OK, that's a lot better. I would've gotten in much deeper trouble if you had let me persist in that. So thank you for the correction. But now, I would like to have integers out in front here. So let me multiply both sides through by A times B times C. And now, I do get something with integers in front of x and integers in front of y. And this would be A times C, and integers in front of z. And that's A times B equals-- and now on the right side, I would have a times B times C.

So everything, every coefficient and the term on the right hand side are products of integers. So they are all integers themselves.

OK, let me next ask the question these planes passing through all of the other lattice points that are contained within this little tetrahedron of volume are going to be equally spaced. How will I claim that these planes are all equally spaced? Well, that's easy just in general terms.

Here is a lattice point. There's a plane passing through it. There's some lattice point neighboring it that's out at distance, D. Here is another lattice point. It has a plane passing through it. There must be a plane out here on D. And I'm not going to be able to have the same environment for every lattice point unless all of these planes have the same spacing from one another. So to say that they pass in lattice points in the environment of everybody lattice point is by definition identical is possible only if the planes are equally spaced.

Next question-- and I'm not going to be gutsy enough to try to demonstrate this in three dimensions, so let me do it in two. Let us ask the question, how many planes are there hanging on all the lattice points between the origin and the intercept plane? And let me demonstrate that for a two dimensional case. And I will refer you to a drawing in Berger's book because this is not convincing unless you do it absolutely exactly. And to do that in front of a live audience with chalk is not always the easiest thing in the world.

So here is T1. Here is T2. And let me look at this plane. This is the intercept plane. A is equal to 3 and B is equal to 2. Now, what I'm going to do is to use plus and minus the translation T1 to repeat this intercept plane.

OK, I go minus T1. And that's going to give me a plane in here. I'm going to go minus T1 again. And that's going to give me a plane in here. So I have split the interval between the origin and the intercept plane into A parts. OK so far?

Let me now use the translation T2. And I will use the plus and minus the translation T2 to take this stack of A planes and repeat it B times. And that's going to take the plane at the origin and move it up to here. It's going to move the planes into stacks like this.

I will map the stack of A planes B times. And that is going to give me A times B intervals except if a special condition applies. And then, I will go back to it in a moment. This is the equation of the intercept plane.

I will argue that the three dimensional analogy of what I've done is that there will be, if I let the translations T1, T2, and T3 go to work, there will be A times B times C intervals between the origin and the intercept plane.

So if this is the intercept plane and this is A times B times C times the spacing between the planes out from the origin, then the first plane from the origin is going to happen to intercept which is 1ABC to the first. And ABC divided by ABC turns out to be 1 for any value of A, B, and C.

So the first plane from the origin is going to have the simple equation be BC times x plus AC times y plus AB times z equals 1. And the second plane out from the origin will have the same coefficients except the term on the left will be 2. Third plane will have 3 on the right and finally the integer ABC when we get to the intercept plane.

OK, now I'm ready to make a momentous definition just as we get five of the hour. B times C is an integer. Let me define that integer by a single integer. And just for the heck of it, I'll call it H. A times C is an integer. Let me call that integer K. And AB is an integer. Let me call that integer L.

So using T1 and T2 and T3 as the basis vectors of my coordinate system, the first plane from the origin has the equation Hx plus Ky plus Lz equals 1. And these three integers, H, K, and L, are said to be the Miller indices of this plane.

PROFESSOR: Yes, sir?

AUDIENCE: Just on that figure, [INAUDIBLE] connect to the lattice points. I'm trying to make sense of why did you put the planes in between them?

PROFESSOR: OK, what I did was I started with just these three hanging on the lattice point separated by T1. And then, I move this by T2. OK, so T2 has a plane in the middle of it. And it's going to move that up to here and give me another plane. If you just draw this out-- and you're going to get a problem that asks you to do this-- since these integers are mutually prime, the second translation is going to interleave planes between the first set.

And it gets even worse in three dimensions because the third translation, which is the axis C translations out, is going to interleave that set of AB planes C times. And they have to be at equal distances. Otherwise, the definition of the lattice point having identical environment is violated.

OK, these are the infamous Miller indices that are the downfall of all freshman who take 309.1. It is not necessarily a very straightforward definition. But the advantage of it is that it lets you analytically describe the locus of the lattice planes in a very, very convenient way.

Actually, let me tell you something maybe you have not heard before. These indices were not invented by a fellow named Miller who was an Englishman who wrote a book, an early book on crystallography that incorporated this notation. The guy who invented them and first proposed them was Auguste Bravais, a very famous French crystallographer-- mathematician, actually-- who gave his name to the three dimensional space lattices. These are universally known as the Bravais lattices, 14 of them. It's a great name. It's music. it rolls of the tongue-- Bravis, Bravais.

Actually, Bravias wasn't the first guy to try to derive them. The first guy who tried to drive the lattices was somebody named Frankenheim. He blew it. He didn't get 14. He only found 13. So there's a moral here.

If immortality is your game, get it right the first time because everybody has heard of Bravais, Bravais. Nobody has heard of Frankenheim. And for that, I say thank god. What a name! It makes you think of somebody who's got plugs in his neck and walks around like this and has a green face. Frankenheim-- bah! Bravais, Bravais, it really sings to you.

OK, I'm getting silly so it's time to quit and take a break. Come up and say hello to Rodney.