PROFESSOR: --questions about what we've done. I think it's been fast, but hopefully if you understand the principles, we didn't go too fast. But any questions on what we've done? I guess you haven't had a chance to think of questions yet.

Let me take care of our oddball symmetry at the end of the chart that I handed out-- and this 4 bar-- and ask what we can do there. Having discovered the four bar operation in 2, 2, 2 with diagonal mirror planes, we can consider that as a new type of symmetry element. And this is the symbol for it.

And this would take a pair of objects and repeat them by a 180 degree rotation. And then another pair of opposite handedness, opposite chirality would be rotated 90 degrees and inverted, rotated 90 degrees and inverted. And I mentioned last time that a solid that has this shape is something called a sphenoid. And the 4 bar axis takes a pair of faces that are up and a pair of faces that are down and skewed by 90 degrees.

Now what can we do? If we add a mirror plane that's perpendicular to the 4 bar axis, the right-handed one goes down, the left-handed one comes up. And it becomes simply 4 over m, which we've already got.

If we add a vertical mirror plane through the 4 bar, if you look a bit earlier at 4 bar 2m, now we've got the 4 bar, now we've got the mirror plane. Not surprisingly, we get the two mirror planes. And so S4v is going to be the same as 4 bar 2m.

And that's something we already have D2d. So that exhausts the possibilities. 4 bar just stands by itself.

At a vertical mirror plane, it's D2d 4 bar 2m. At a horizontal mirror plane, it becomes 4 over m. The diagonal mirror plane is not possible. There's nothing to place the mirror plane diagonal to.

And if you add inversion, it changes into 4 over m. So this one is an odd group. It sits by itself. Except that we could start there, and add a vertical mirror plane, and get 4 bar 2m once more.

That leaves the cubic ones, which are really easy to deal with because a cube has such high symmetry. We all know what cubes look like. But when you show arrangements of motifs, it gets a little bit confusing. So let's take a look at the tetrahedral groups.

T is a combination of twofold axes coming out in directions corresponding to face normals to a cube. So these are the twofold axes. And then there are threefold axes coming out of directions that correspond to the face diagonal. So this is the jack-o'-lantern stereographic projection, which is something I love to draw at this time of year.

What will the pattern look like? Well, it's going to look like 2, 2, 2. So that's a subgroup. So let's draw in a pair of objects on either side of the twofold axis, and that's the pattern of 2, 2, 2.

Now this guy here is lurking off of a threefold axis. So that threefold axis is going to repeat it three locations that are 120 degrees apart. And so there's going to be a triangle of objects up above.

This threefold axis is going to reproduce this into a triangle of objects that are down below. The twofold axis will take this triangle of objects and move it over to here. Also, up. And the twofold axis that's vertical will take this triangle and move it down to three that are below.

So it looks very complicated in projection. But it's simply a planar triangle of atoms here rotated 180 degrees. So you've got one like this, one like this. And then down below, two other planar triangles of objects.

Remembering what the symmetry of a tetrahedron looks like, you can draw this arrangement of symmetry elements relative to a tetrahedron. And the threefold axes now are coming out normal to the faces. The twofold axes coming out normal to the edges.

And imagine that we put a triangle on this face, a similar triangle on the face behind, and a triangle pointing in the other direction down this way. These are the three below. So that's the pattern of 2, 3.

Take a tetrahedron, smack a triangle on the two upper faces, and an equilateral triangle on the two lower faces. And the threefold axis that comes out here comes out of the center of this triangle and out of the center of this triangle.

The next one that you can get from this is Th, the tetrahedral arrangement of axes with a horizontal mirror plane. If you put in a horizontal mirror plane, it's going to take this triangle and reflect it down. It's going to take this triangle and reflect it up. And they will overlap in projection. And so that's the pattern for Th.

There's a mirror plane perpendicular to a twofold axis. That creates an inversion center. So we can call this a 3 bar axis. So that's Th 2 over m 3 bar.

So just have triangles on the upper and lower faces as well. The one remaining one is Td. Here are the twofold axes. Here are the threefold axes.

And now if we put a diagonal mirror plane in, diagonal with respect to what? Well, diagonal with respect to these twofold axes. So this mirror plane goes down like this, passes through a twofold axis. There must be a mirror plane 90 degrees away.

And the threefold axis is going to repeat these mirror planes so that we get mirror planes that are at an angle with respect to the vertical twofold axis. And this is Td. And this, in the international notation, is called-- if we can figure it out-- the diagonal mirror planes with respect to these twofold axes have changed them into 4 bar axes. So this is called 4 bar 3m, which doesn't look cubic at all. So that's a little bit deceptive.

I am foolhardy for even trying to do this. And so I don't think I'm going to do it. 4, 3, 2, we know what that looks like. That is fourfold axes coming out in directions that correspond to the face normals to a cube.

A threefold axis coming out of body diagonals. Twofold axes between all of the fourfold axes that are normal to the edges of the cube. So all sorts of rotational symmetry.

That, if you want a pattern, has a triangle that's on about each of the threefold axes. But the one that is up, points in the opposite orientation of the one that's on the threefold axis coming out of the other end of the cube. So you have one triangle that's like this, up. And another triangle like this that's on the other end of the threefold axis that points down into the blackboard.

What can you add as other extenders? You can put in a mirror plane that is perpendicular to the fourfold axis. And that leaves everything unchanged.

Now we've got a mirror plane passing through a fourfold axis, so we have to have mirror planes at 45 degree intervals. And that will create mirror planes there. This horizontal mirror plane goes through this fourfold axis, so there must be mirror planes at 45 degree intervals.

So there's another one like this and 90 degrees away as well. The fourfold axis is perpendicular to a mirror plane, as are these other twofold axes, so there's an inversion center. Here's a fourfold axis with a vertical mirror plane going through it. And it has to have a vertical mirror plane going like this at 45 degrees away.

So that's the symmetry. This is the regular symmetry of a cube or an octahedron. And I will not, even if pressured, try to draw a pattern that conforms to that.

We've got the fourfold axis perpendicular to a mirror plane. And this is called O sub h, O with a horizontal mirror plane. The fourfold axis in 4, 3, 2 has got a mirror plane perpendicular to it. The twofold axes all pick up mirror planes perpendicular to them.

There's an inversion center at the point of intersection. So we label this axis a 3 bar axis. So that's O sub h, 4 over m, 3 bar, 2 over m. And this, as I say, is the symmetry of a regular cube or an octahedron.

Nothing else we can do here. There's so many symmetry axes all over the place that there's no way we can snake in any diagonal mirror plane. Because there is no second axis of the same kind adjacent to either the twofold, the threefold, or the fourfold axis. So we can't put a mirror plane in here between the threefold and the twofold.

We can't put a mirror plane in here between the fourfold and the twofold. So we're done. And this is the final and 30 second combination. So there are 32 crystallographic point groups.

AUDIENCE: Professor?

PROFESSOR: Yes, sir.

AUDIENCE: So if m3m is a regular operation, are there symbols [INAUDIBLE]?

PROFESSOR: O is the symbol for 4, 3, 2. And what we added as an extender is a mirror plane perpendicular to the fourfold axis. That is just one way of adding an extender. This is also Oi. If we add an inversion center, we get all this.

And if you look at the ones have been honored by being designated by a symbol, the horizontal mirror plane takes the precedence. So for example, for 2 over m, 2 over m, 2 over m, you've got two different vertical mirror planes, and you've got one horizontal mirror plane. But it's called 2 over m, 2 over m, 2 over m. You can call it 2, 2, 2, m, m, m. That's also a possible symbol, but much more of a mouthful.

There's some arbitrariness to the symbol because the arrangement of symmetry elements is what's real. And we decide how we want to devise a notation to label them. And as we've seen, there are two different people who adopted a different code for giving them names. So if it's rational and informative, it's a good notation.

Interestingly, the international notation, on the one hand, and the Schoenflies notation, on the other, complimentary. The international notation tells you unambiguously what you have. So 2, 2, 2 over m, m, m tells you three orthogonal twofold axes if you remember what came out of [INAUDIBLE] construction. And perpendicular to each of them is a mirror plane. And that's what you've got.

Schoenflies tells you how you derived it. You took D2, and that's the dihedral group

2, 2, 2, and you added an h, a horizontal mirror plane. And all hell broke loose. And you got mirror planes perpendicular to all the twofold axes.

So there's a certain complementarity to the two different notations. And the people who do diffraction and crystallography for the most part follow the international notation, the Hermann-Mauguin notation. The people who do condensed matter physics use the Schoenflies notation.

Because it's more inscrutable. And condensed matter physicists like to be inscrutable because it's how you gain respect. So both notation survive, but are more prevalent in some disciplines than in others and vice versa.

Let's take a brief look ahead. What remains to be done? We now have the 32 crystallographic point groups. These are the way things can be arranged by symmetry about a fixed point in space.

If we were to proceed in the same way that we did for the two-dimensional space groups, what we should do next is decide what sort of three-dimensional space lattices these symmetries will require. And then proceed to drop each of the point groups into each of the lattices that can accommodate them.

And then use the tricks that we've used in two dimensions, take mirror planes and replace them by glide planes. And then having done that, we would take the mirror planes and the rotation axes and interweave them. And lest that job seem too daunting, let me point out that we already have 17 of the space groups.

All we have to do is take the plane groups, take a translation that's perpendicular to the plane of the plane group, and let all the rotation axes and mirror planes extend up indefinitely in three dimensions parallel to that translation. So we already have 17 of the three-dimensional space groups. They look just like the plane groups except they extend in a direction that's perpendicular to the plane of our original two-dimensional group.

So we're already a long way towards deriving a three-dimensional space group without really knowing it. But what I would like to consider next is the lattices that are required for three dimensions. And I've already given you how one can approach this problem.

Take the lattices that have been required in the two-dimensional plane groups. And if the presence of a threefold axis and the base of the cell require, say, net that is hexagonal with a1 equal to a2 identically in magnitude and exactly at 120 degrees with respect to one another. Let's let the third translation be perpendicular to the base.

If it's hexagonal, we have to have at least a threefold axis here. And we've found that adding a threefold axis to that net gave rise to two other threefold axes in the center of the triangle. So one lattice would be one in which the third translation, which we'll label c, is straight up. In other words, perpendicular to the net that we derived in two dimensions.

What we will have as a constraint is that if we pick a certain translation that is not normal to the plane group, and let's say it terminates here in projection. So the third translation goes up and over. If there's a threefold axis at this end of the translation, there must be a threefold axis at the end of that translation.

And that mucks everything up, and we no longer have a group. We can't have another threefold axis poking down through the plane of this two-dimensional plane group. But it is perfectly OK if the third translation moves this threefold axis and puts down a lattice point and another threefold axis directly over this one.

Or alternatively, another choice for T3 would be this one. That would put the lattice point in a threefold axis directly over this one. So again, we have threefold axes extending normal to the plane of my drawing. And I've created no threefold axes.

So with a threefold axis, there are three potentially different space lattices, one in which the third translation is normal to the plane of the plane group, another one where the translation terminates over the other two twofold axes. And so this is the way in which one would proceed to derive the space lattices.

Buerger does not do it this way. He doesn't look at the plane groups. He looks just at where the axes sit in the nets. No mirror planes whatsoever.

And I'll return to the point. And the place where I think he's wrong is that if you add a twofold axis to a centered net, you get twofold axes in all of these locations. These are the twofold axes in C2mm.

And if that's all you look at, there's no reason why this should be the plane group. And it looks as though you can make T3 terminate over this twofold axis. And that would give you a peculiar triple cell with three lattice points along the long diagonal of a rectangular net.

And you say, wow, 15 space lattices. We're going to be famous, if not rich. You can't do that because this plane group can exist only if there are mirror planes in here like this. And then through these twofold axes, these have to be glides. So you can't take 2mm and put it on top of 2gg. It's impossible.

So there are 14 space lattices. The 15th doesn't exist. But it looks as though it's possible in Buerger's treatment when he looks just at the placement of the rotation axes alone and not the location of any mirror planes that are in the plane groups.

So that is where we're going to go next. And that is a process that actually is surprisingly simple. And we will get the space lattices very quickly.

We'll get a number of space groups along the way, as we've just seen. And then we will cut to the bottom line and just look at how this information is tabulated in tables for you in a fashion that's analogous to the representation of the plane groups. So we're pretty close. We'll be wrapping things up in another two or three meetings.

What I would like to do in the time that remains though, the point groups are very difficult to visualize unless you look at real crystals. So what I'm going to do is pass around a collection of models of actual crystals. I'm not sure I can tell you what every one of these models represents.

These are curious things. I would beg you not to drop them on one of the sharp corners. These are made out of wood. They're made out of pear wood in the Black Forest by elves. No, that last part is not true.

But they are incredibly expensive. Because the angles in these things have to be exactly right. So they are made on the same sort of machine that you use for faceting diamonds.

It's something that has two degrees of freedom, so you can get a surface that you want on the material exactly parallel to a grinding surface. And then you have a provision for advancing it normal to that direction by a controlled amount. If it were not precise, you would not see sharp edges and sharp corners.

So these are incredibly expensive. I, therefore, ask it that you hold them tightly with both hands. Don't drop them on the floor.

And I'll take around a couple of handfuls of these. I don't know if I have enough for everyone. With the tables of the point groups in front of you, why don't you try to identify the point group that's possible in each of these models.

This is a big one. So I'm sure you won't drop that one. I don't think I have enough for everyone, so I'll start passing them out to every other person. Maybe you can look on with the person adjacent to you.

Take one of these. Pass those over. Here you go.

I gave you that just to be mean. That is an example of something that's called a twinned crystal. You can't have reentrant faces. Yeah, that's actually two crystals that are intergrown.

AUDIENCE: So we can't identify the [INAUDIBLE]?

PROFESSOR: You can look. Block one of them out in your mind, and try to imagine what it would look like.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah. Here's an example of another one. That's actually gypsum. And that's a very common twin in gypsum.

And if you could take this and rotate it-- this one doesn't rotate-- rotate it 90 degrees, then you would have a crystal that had a parallelogram shape. And actually that's been rotated by 90 degrees about an axis. That's within the base.

AUDIENCE: No, no, I think that's just 4m or 4 over m.

AUDIENCE: Yeah, there's no mirror there. That's a 4 bar, I think.

AUDIENCE: Oh, is that the inversion?

PROFESSOR: There's no-- neither. I am a touchy-feely guy. If I see something up here, I look down here and try to feel something that's parallel to it. But you're right, you're right.

If I rotate 90 degrees-- remember rotoinversion is the same as rotoreflection. So I could rotate to here and then reflect down, and I get this one.

AUDIENCE: So it's 4 bar.

PROFESSOR: It's 4 bar, yeah. And you see in the cross section, it is square. There's a little square on top here, so there is a fourfold aspect to it when you look at planes in a special orientation.

AUDIENCE: I have a question about 4 bar. 4 bar in planes are twofold rotation.

PROFESSOR: A twofold pure rotation. Probably the best example of 4 bar is a tetrahedron. Two faces on top, two faces twisted 90 degrees, and inverted down.

AUDIENCE: So that means an 8 bar would be a fourfold proper rotation. So why cannot I put an 8 bar in the point group? Because it's only a fourfold rotation, and that's allowed.

PROFESSOR: The reason is that there's no origin to translations. So if you had translations in an 8 bar crystal, you would have four translations that were repeated by rotation and then another translation 45 degrees and inverted down. But you can assemble those at this same point. So you would have eight translations 45 degrees apart.

And that's impossible in a lattice. Need another one?

AUDIENCE: I'm sorry.

PROFESSOR: Do you need another one to look at?

AUDIENCE: No, I'm good.

PROFESSOR: OK. I can't have you just sitting there doing nothing.

AUDIENCE: So is this half of this [INAUDIBLE] material?

PROFESSOR: Yeah, that's twinned by a 180 degree rotation on the 1, 1, 1 plane.

AUDIENCE: Oh, yeah, the 1, 1, 1 [INAUDIBLE].

PROFESSOR: Yeah. That is actually half of an octahedron. This is a-- whoops, no, it's not. Yes, it is.

No, I think that's just the threefold-- 3, 3, 3. 3 bar, 2 over m. And it's been rotated by 180 degrees, which is not a symmetry transformation. So the 3 bar is rotated 180 degrees.

AUDIENCE: That's a 3 bar, 2 over m?

PROFESSOR: I think so. I think it's a 3 bar here, a face here, and a face down below, inverted, 60 degrees away. And they're mirror planes. And there two full axes perpendicular to those mirror planes, I do believe.

AUDIENCE: I don't see the twofold axes.

PROFESSOR: You're only seeing half the crystal, and I think that's the reason why. This is the mirror plane here. And this edge is parallel to that mirror plane. And there's a twofold axis that comes out of the middle of that edge.

You can't see it. That edge. And the mirror plane is exactly parallel to that. It's tough to see because you're only seeing half of it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah.

AUDIENCE: This [INAUDIBLE]?

PROFESSOR: Yeah. This is a fourfold axis perpendicular to a mirror plane. There's a threefold axis coming out here and a twofold axis coming out here. And there is a mirror plane perpendicular to the fourfold axis and perpendicular to the twofold axis also under the mirror plane. So this is 4 over m, 3 bar, 2 over m.

AUDIENCE: [INAUDIBLE].

PROFESSOR: That's the rotational symmetry. But you get all sorts of mirror planes coming through here. So really, it's this one.

Mirror plane this way, mirror plane this way, mirror plane this way. So if we set it up relative to this, there's the fourfold coming out here, here's the fourfold coming out here. And there are mirror planes this way and also this way, 45 degrees away.

Nothing to do? Can I steal one that you're not working with? There's a--

AUDIENCE: This one is [INAUDIBLE].

PROFESSOR: That's 2, 3 with no mirror planes. Oh, no, no, that's not true. No, that is 4 bar, 2m, believe it or not.

Look at the three different directions. These two corners are the same. There's a little, tiny line segment there.

AUDIENCE: Yeah, so there is a mirror plane right here?

PROFESSOR: Yeah, but there's no mirror plane going through the other edges. So there's a mirror plane here, mirror plane here. And then there is twofold axes coming out of this little straight line segment here.

AUDIENCE: Really?

PROFESSOR: Yeah. And that's different from this. So these two edges are the same.

AUDIENCE: Where is the principle axis?

PROFESSOR: The principle axis would be this one.

AUDIENCE: This one?

PROFESSOR: Yeah.

AUDIENCE: So basically, I have this mirror plane right here. Then twofold axis in the middle of the circle. So now here, where is it?

PROFESSOR: Well, it's hexagonal. So you want to back up a little bit. Right there.

AUDIENCE: This one?

PROFESSOR: No, sorry. This one. Threefold axis, twofold. That's perpendicular to a mirror plane.

AUDIENCE: So here's my mirror plane.

PROFESSOR: Coming right out of this little edge here.

AUDIENCE: Let's say like this. No mirror plane?

PROFESSOR: No.

AUDIENCE: Which one is this one? It's like this one.

PROFESSOR: Let's put it right here. Here is--

AUDIENCE: Like this?

PROFESSOR: No, you've got to get it up like this. OK. There is a mirror plane.

AUDIENCE: No, this is not. No, the mirror plane is right here.

PROFESSOR: Yeah, OK. It goes up like-- oh. Let me get it set up here. OK. There is the mirror plane. This face is different from the others.

AUDIENCE: And the twofold axes are on the edges.

PROFESSOR: Yeah.

AUDIENCE: So basically, there are three for one mirror plane.

PROFESSOR: Here are the mirror planes coming through this way, this way, this way. They're 60 degrees apart.

AUDIENCE: You say there are three mirror planes?

PROFESSOR: Yeah, this one, this one, and this one.

AUDIENCE: I don't agree. This one is no mirror plane.

PROFESSOR: You're right. You're right.

AUDIENCE: There is only one when you look from above. But there are two for just [INAUDIBLE] 45 degrees-- or, not 45 degrees. That's a triangle.

PROFESSOR: It's a terrible one.

AUDIENCE: Because if we decide that the top is one, the principle--

PROFESSOR: There's a twofold axis. That's a mirror plane, and that's a mirror plane. And it looks like it is 2nn. I think it's 2nn.

AUDIENCE: OK. There's nothing further?

PROFESSOR: Nothing further. Got that one?

AUDIENCE: 4 bar 2m? I had to look to find a twofold axis. That was the tricky part.

PROFESSOR: No, no threefold axis.

AUDIENCE: Well, twofold.

PROFESSOR: Yeah.

AUDIENCE: Finding those was [INAUDIBLE].

PROFESSOR: That's it.

AUDIENCE: I can see the mirror planes.

PROFESSOR: That's the twofold axis.

AUDIENCE: Yeah.

PROFESSOR: And this two on top and two underneath skewed by 90 degrees, that's a 4 bar. So that's 4 bar 2m. You have nothing to do? Oh, you're talking with them, OK.

AUDIENCE: What is this one actually? There is 4, 4, 2, 4, 3, 4, 3.

PROFESSOR: Right, so it's cubic. And mirror planes are going down through the twofold axis. So that's enough to tell you that it is this.

So let's set it up here. Fourfold axes are coming out of the points. So you've got mirror planes this way, mirror point 45 degrees.

Here is this twofold axis. Here is this twofold axis. And here are the twofold axes that are in the same plane. And here's the other fourfold axis.

AUDIENCE: I see. That one's not fair. This is actually two crystals that are grown together by an operation that is not a symmetry element of the crystal. And actually these two crystals have symmetry 2 over m.

And they are rotated relative to one another by a 180 degree rotation. It's not a symmetry element. So this is something that's called a twinned crystal.

AUDIENCE: Twinned?

PROFESSOR: Twinned crystal. So it's really two of them. And if I could twist this one around so that the crystal continued on in that direction, it would be something that had a lozenge-like shape. So I should put that one away. That's only confusing people.

AUDIENCE: Professor Wuensch?

PROFESSOR: Yes, sir.

AUDIENCE: Is that a 6 over mmm?

PROFESSOR: Yes, that's 6 over m, 2 over m, 2 over m. Six folds. Two kinds of twofold, one out of the edge, one out of the corner. And a mirror plane perpendicular to each of those. 6 over m, 2 over m, 2 over m.

AUDIENCE: So that's the same thing [INAUDIBLE]. So the 6 over m comes from this way. And then the 2 over m is there.

AUDIENCE: Yeah, so it's [INAUDIBLE].

AUDIENCE: Yeah, exactly.

PROFESSOR: Let me put that one away. It's just confusing people.

AUDIENCE: Oh, that one was easy.

PROFESSOR: That's a twinned crystal. It was easy? You found it easy?

AUDIENCE: Yeah, so 3 over m, right?

PROFESSOR: Yeah, I guess. This looks as though it might be the corner of an octahedron because you don't see much of it. But these two faces and these two faces are not related by a fourfold rotation.

AUDIENCE: Well, either way, there wouldn't be anywhere to put a fourfold rotation.

PROFESSOR: Yeah, right. So if you look at the whole thing, this is actually two intergrown crystals when you see this sort of reentrant angle. This is two crystals grown together by an operation which is not a symmetry operation. So this is something that's called a twin, a twinned crystal.

AUDIENCE: But the symmetry is nevertheless 3 over m, correct?

PROFESSOR: Of the whole thing? Yeah, yeah, of the whole thing.

AUDIENCE: And this is just 3.

PROFESSOR: No, this is 3mm. There's a mirror plane down there and a mirror plane down here.

AUDIENCE: You're right.

PROFESSOR: No twofold axis because this end is different from this end.

AUDIENCE: So this is a regular solid, right? What is it called?

PROFESSOR: This is a rhombic dodecahedron.

AUDIENCE: Oh, the one you were talking about the other day.

PROFESSOR: Yeah. This is 4, 3, 2. And then mirror planes too all over the place.

AUDIENCE: So all of these solids do not have to be members of this list, right? Because these are finite objects. They can be [? twelvefold ?] and--

PROFESSOR: They could be. These actually, in fact, are models of real crystalline materials. And they're made with the faces and the relative sizes that these minerals actually have.

AUDIENCE: OK.

PROFESSOR: So somewhere floating around is something that's very characteristically quartz. And there's one that's a flat one with a reentrant angle in it, that's gypsum.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, that's fourfold. Twofold here. This goes into this. No mirror planes because these things are inclined. So it's 4, 2, 2. Good.

It's easy when you know what to look for. When you say, they are only a few possibilities. And if I see a fourfold axis in it, that narrows it down to two or three.

AUDIENCE: Like that. That's an inversion center running through here. And there's no rotation there [INAUDIBLE].

AUDIENCE: [INAUDIBLE] here? Here through this way?

AUDIENCE: No, because this doesn't map [INAUDIBLE]. Does that map that by a mirror? The mirror [INAUDIBLE]?

AUDIENCE: It cuts across this way.

AUDIENCE: Yeah, I could buy that.

PROFESSOR: You've reached a consensus?

AUDIENCE: We've got a mirror, and we've got an inversion. But we're not sure what else we've got.

AUDIENCE: While I like the inversion, I'd also like the mirror. I don't see the mirror. All I see is a really an inversion axis there. I think I'm missing that mirror that you're saying.

PROFESSOR: There's a mirror that goes down this way. And there's a twofold axis here.

AUDIENCE: OK, that was the one we missed.

PROFESSOR: So that's 2 over m.

AUDIENCE: Over m. And the inversion just falls out.

PROFESSOR: Yeah, that's in there too. Inversions are nice. You can feel inversions. You put your finger on one face and your finger on the other face.

[INTERPOSING VOICES]

PROFESSOR: Get it?

AUDIENCE: So this is this one, right?

PROFESSOR: Yes, very good. And that is a silicate mineral called garnet.

AUDIENCE: What's that?

PROFESSOR: Garnet. It's actually a silicate mineral, which is used as a gemstone sometimes.

AUDIENCE: So this should be a 4, 3, 2 face.

PROFESSOR: Yep.

AUDIENCE: But I don't know if it's the end of the [INAUDIBLE]. Could be m3m.

AUDIENCE: Where are the-- it doesn't an inversion center.

PROFESSOR: Hmm?

AUDIENCE: It doesn't have an inversion center here.

AUDIENCE: Oh yeah, it does.

PROFESSOR: Oh yes, it does. Yes, it does. Actually, the thing to look for are the high symmetry things. Your eye spots them. That's got a fourfold axis in it. OK

So are there other fourfold axes? Yes, yes. So that has to be based on 4, 3, 2. Because you have three orthogonal fourfold axes. Threefold is here. Twofold axis is here.

AUDIENCE: OK.

PROFESSOR: OK, thanks.

AUDIENCE: There are-- oh, I already saw those ones.

PROFESSOR: You want to look at some more?

AUDIENCE: All right.

PROFESSOR: Enough for one day?

AUDIENCE: Professor Wuensch, I've been staring at this one for such a long time. And I couldn't match it with anything on here.

PROFESSOR: This looks like a tetrahedron except the face is puckered up into this little thing. So the way I would start with saying, OK, this is tetrahedral in which case if I make these things flat, that's a perfect tetrahedron. Four sides.

But what's happened is that this face is not quite 1, 1, 1. The 4 bar axes come out these three directions. And they have got twofold symmetry, mirror planes running through the twofold axes. This is the 4 bar axis with mirror planes running this way, this way.

So this is actually 4 bar. This is still further. It's a cubic crystal. This is based on the tetrahedral symmetry.

Here is the 4 bar axis. Mirror plane this way, mirror plane this way. Another 4 bar coming out of these two edges. And then these are the diagonal mirror planes. So this is it.

AUDIENCE: I think I was looking for a threefold axis in the center.

PROFESSOR: No, the 4 bar comes out of the edge of the tetrahedron. And this is the other 4 bar coming out here. Threefold like this and mirror planes going through the threefold axis like that.

AUDIENCE: I see it.

PROFESSOR: It's hard when you look at this thing. I never saw this thing before. What's here?

But if you know the results have to be 1 of these 32 possibilities, and this is obviously cubic, and it's based on the tetrahedron, that means there are only three possibilities. No mirror planes. If you find one mirror plane, you ask yourself, does the mirror plane go through the threefold axis, or does it miss the threefold axis? It goes through the threefold axis, so it's got to be this. And then you know just what to look for.

AUDIENCE: Is it just [INAUDIBLE] m?

PROFESSOR: This is a dirty one. This is a dirty one. If you look at this very carefully, this face is the same as this face, but it's not the same as that one. It's a little bigger.

So if you say, what's in here, these two things do not come together at a common vertex like these two. There's another little line segment between these faces. So there are no threefold axes coming out of this.

Looks like it might be tetrahedral. Clearly, a mirror plane going this way. And then there is also a twofold axis coming out here. So there's a 2 over m and another 2 over m.

I did this once before with somebody, and this is the hardest one in the whole set. So a mirror plane this way. There's got to be another mirror plane because there's a twofold axis.

4 bar. Up, down, up, down. So this is 4 bar, 2 over m. 4 bar, 2 over m.