

**PROFESSOR:** I think it's time that we got started. I haven't given a problem set out for a couple of days. So I have one that will cover some material that we'll go over with and develop today and some new material that we're not going to introduce for a while.

**AUDIENCE:** [INAUDIBLE]?

**PROFESSOR:** Yes, this is Thursday. And this is for the near future. I've given you two-- two a week before. So it's the adjective you're objecting to not the problem set, of course. Today we're going undertake another major step in our development of three dimensional crystallographic symmetries. We have just completed derivation of the crystallographic point groups, so these are the macroscopic symmetries that crystals can have. And then, by analogy, to what we did in two dimensions, the next step would be to take each of these point groups and hang it at a lattice point of some three dimensional lattice that can accommodate them. We have not as yet developed the three dimensional lattices, and that will be our agenda for today.

Seems like we've already done that, haven't we? We showed that in two dimensions, a cell can be oblique, it can be rectangular, it can be centered rectangular, it can be square, or it can be hexagonal. So these are the different kinds of bases that one can have for the cell, right? So there should be the same number of space lattices. Actually it's more complicated than that. And let me remind you by another handout what the two dimensional space groups, the plane groups, look like assembled in all of their glory on one sheet. And there you see the different ways symmetry can be placed in these five different kinds of lattices-- oblique, rectangular, centered rectangular, square, and hexagonal.

Now a lattice in two dimensions, a potential base for a unit cell, can be rectangular if and only if there is symmetry within that net. That demands that it be exactly rectangular. In other words, if a lattice is to be rectangular in the base of the cell, it has to have in it one of the symmetries that correspond to those in the plane groups that give and require, indeed demand, a rectangular lattice. Similarly a lattice can be truly square, identically square, only if there's a fourfold axis in it. That demands that

it be square. The same could be said for either threefold or sixfold symmetry. One of the groups,  $P3$ , [ $P3$ , ?]  $mP3$ ,  $1m$ ,  $P6$ , and  $P6mm$ , has to be in the base of the cell if that cell is to have the specialization.

So let's give an example with a twofold axis. Let's suppose we have a base of the cell that has an unspecialized shape-- two translations that are unequal in length and an angle between them which is a general obtuse angle. That's about as general as you can get. But suppose that this net has as well a two dimensional-- a twofold axis in it. And suppose we pick our third translation such that it picks up this net which is going to be the base of our three dimensional lattice and translates it over to this point in the base. Well that translation picks up everything. It picks up not only the two translations that we've been calling  $T1$  and  $T2$  to this point, but it also picks up all the twofold axes that are hanging on these lattice points.

So if this is the terminus of  $T3$  and projection, it will have picked up a twofold axis and plopped down a twofold axis here where no twofold axis exists. And we can take one of the theorems that we've seen, namely that the operation  $A\pi$ , followed by translation, is equal to a new rotation operation through  $\pi$  located halfway along the translation. And turn this around and ask what happens when we combine two rotations through 180 degrees about different points. And let's just draw it out to see what happens. Let's say there's a twofold axis here. It takes motif number 1, moves it to number 2. And then here's a second twofold axis and this takes number 2, rotates it over here to number 3. Is number 1 related to 3? But in terms of this first theorem, not surprisingly, we have introduced a translation into the space which is equal to twice the spacing between the twofold axes, which I'll label as  $\delta$ .

So in other words, if we put a twofold axis down in this place in addition to the ones that we already have, we have created, in the base of the cell, a new twofold axis in this location. That just mucks everything up because it's going to rotate the translations that we have, it's going to rotate the twofold axes that we have. And we will not any longer have a group. We will not have a set of operations that closes upon itself. So the conclusion then is that if we're going to safely pick a third translation,  $T3$ , and combine it with a net that has twofold axes in it, in order to get a

group, we are going to have to ensure that the twofold axes line up in every net in the stack.

And there could be several ways of doing that. We could pick T3 so that it goes straight up. And then the twofold axes are just all slit up parallel to themselves, and they remain in coincidence. Or we could pick T3 so that it terminates over the point that's halfway along T2. And then this twofold axis gets picked up and put down over one that's already there. This one gets picked up and put down over one that's already there and so on. So that's an OK choice. That's a safe choice as well.

Similarly we could pick T3 such that, within the plane of the net, it had a component that was one half of T1, so that it picked up this net and slid it over to this location. So once again all the twofold axes lined up. Or, in the same fashion, let it terminate over the center of the cell. So those are the four ways we can combine an oblique net that contains a twofold axis in such a way that the lattice points in the existing twofold axes remain invariant and are not moved into locations that create new twofold axes.

OK so let's back off a little bit and start with plane group P1-- no symmetry at all. So this is T1 and T2. If there's no symmetry whatsoever in the base of the cell, we are free to pick the third translation in any orientation that we choose. So trying to sketch it in three dimensions. If this is T1, and this is T2, T3 can be in any orientation such that not only is this angle general, but this angle is general, and this angle general as well. And if I would try to carefully complete the parallelepiped so that its geometry looks reasonable. This is going to be the general oblique parallelepiped with three translations that are unequal in magnitude, and three angles between these translations. Let me write them as the angle between T1 and T2-- not equal to the angle between T2 and T3 necessarily, not equal to the angle between T3 and T1.

And all of these angles are completely general and can assume any values that they like. So this then is the lowest common denominator. This is a totally unspecialized space lattice. It has the shape of the general parallelepiped-- no special relations between any of the translations whatsoever.

**AUDIENCE:** [INAUDIBLE]

**PROFESSOR:** I say they're-- Oh I'm sorry, magnitude are not equal. [INAUDIBLE] I meant to put not equal signs there. Yeah, they can have any length they wish. They're not equivalent. Any other questions?

OK let's go back then to the case of an oblique net to which we've decided to add a twofold axis. And that gives twofold axes in the locations that we've become familiar with in the plane group. And if I pick the translation so that it goes directly normal to the base of the cell, I'm going to have a new kind of lattice. The lattice is going to have two angles between axes that are exactly 90 degrees, not approximately so, but exactly so, because only if that translation, T3, is exactly normal to the base of the cell will the twofold axes in all of these nets lineup exactly. So here we have a degree of specialization.

The translations remain unequal in magnitude. They can be any length they wish and not change things, but two of the angles-- axial angles, T1 to T2, is identically 90 degrees. The angle between T2 and T3 is identically 90 degrees. And the third angle, the angle-- I'm sorry, T1 and T2 is my general angle. So this can be anything. So that can be any general angle. T2 and T3 and T1 and T3, want to be exactly 90 degrees. So we're getting space lattices that have a degree of precise specialization. And the reason is that the geometry of the base of the cell is inseparable from symmetry in the base of the cell that demands the degree of specialization that makes the space lattice unique.

So let us look at a couple of the other choices for the third translation that will leave the twofold axes in coincidence. Another choice would be to pick the third translation such that it terminated exactly over the midpoint of T2. So here's T1. Here's T2. Now what I'm going to do is something I have not done to this point. Having this translation T3 terminate exactly over the midpoint of a translation [INAUDIBLE] was clearly a specialization. If I were to go up two translations, T3, I would be directly over the end of T2, and that is a much more convenient way of demonstrating that special feature.

So let me pick a new T3 prime. It goes up exactly normal to the base of the cell, and in so doing, I will have defined a cell which is a double cell. And the catch is an extra lattice point in the middle of one of the pair of faces. So I'll refer to this general sort of situation-- this is a double cell. And I'll call this, in words, a side-centered cell. And it has, with this redefinition of translations, exactly the same degree of specialization as before-- magnitude of T1 not equal to the magnitude of T2, not equal to the magnitude of T3. And it has the same specialization of the angles. The angle between T1 and T3 is exactly 90 degrees. The angle between T2 and T3 is exactly 90 degrees. And the angle between T1 and T2 is general. General, but by convention we will assume the obtuse angle rather than the acute angle.

OK so by picking this double cell, we've defined a cell that has a twofold redundancy. But in doing so, we have gained the advantage of showing that this specialized geometry is exactly the same as this one-- in the same sense that in two dimensions, the primitive rectangular cell and the centered rectangular cell were both cousins. They were both had orthogonal translations. But the other thing that we gain is that even though we can't have an orthogonal coordinate system, if we use the three translations as the basis of a coordinate system, at least we have two 90 degree angles on our coordinate system. And operationally, that is going to be an enormous advantage. Yes, sir.

**AUDIENCE:** So is T3 [INAUDIBLE]

**PROFESSOR:** T3 was selected so that we picked it as  $2 T_2$ . I'm sorry,  $2 T_3$  minus  $T_2$ . That was my new T3 prime. I went up two translations in T2 and then back T2, and that put me directly over the lattice point from which my translations emanate.

**AUDIENCE:** So your T3 is actually [INAUDIBLE]

**PROFESSOR:** It goes to the next level up in the stack of nets. If I would draw this in projection, which is not nearly as clear in some respects, this would be the base of the cell. Let me use x's for the next level up. So here's the net offset by half of T2. And then if I go up two of these T3's and then back by T2, my next layer up is directly over the

first one-- the third layer up is directly over the first.

**AUDIENCE:** Shouldn't it be T3 prime then?

**PROFESSOR:** Yes if I were to be consistent, I should call this T3 prime, which I did here. But now I should call that T3 as well. I got a little bit careless because we're going to forget that we ever had this T3-- to find this selected as a stacking vector. OK well in P2 there are four different kinds of twofold axes. So we could have the third translation terminate over either of the remaining pair of twofold axes as well. I think that it's apparent that if I pick T3, the original T3, as one half of T1, so that the next layer up terminates in lattice points in this orientation. And then I pick a T3 prime, which is equal to  $2 T1$  minus--  $2 T3$  minus  $T1$ , I will have gotten again a side-centered lattice.

The only difference is that it's going to be a different pair of faces. It's going to be the O1Oa faces that have the extra lattice point. So that is not fundamentally a different sort of lattice. It is also side-centered. And differs from our second result only in whether it is the side face with the shortest translation or the longest translations of the base that bears the extra lattice point.

One final choice though does result in a new lattice with special features. And this would be one where I pick the third translation such that it terminates over the center of the net below. So let's let this be T1, this be T2, and pick T3 so that it moves the origin lattice point directly over the center of the parallelogram face below. And then if I go up two translations, T3, and then subtract off T1, and subtract off T2, I will again have a T3 prime, which is exactly normal to the base of the cell. OK this is a type of lattice that you've come to know and love. In the cubic system, this is referred to as a body-centered lattice.

**AUDIENCE:** So then, T3 is just [INAUDIBLE]

**PROFESSOR:** T3, the original choice, would pick this original net up and move it so that that net was moved-- such that it's corner was over the center of the original net. So if we go up two translations, we have lattice point over lattice point, and we redefine T3 by

going back  $T_1$  and minus  $T_2$ . And that makes it exactly normal to the base.

**AUDIENCE:** Not on the sides?

**PROFESSOR:** Nothing on the sides. Now in fact, if we try to do that, we would have something that's not a lattice. If we tried to have some additional lattice points in the center and the edges of this upper level, we would have destroyed the original  $T_1$  and  $T_2$  in the base of the cell. Let me do one or two more and then I think the way in which one proceeds should be quite clear. If you take this list of all of the plane groups, if we would like to have a three dimensional cell that has a rectangular base, the base can be rectangular only if there is a mirror plane in that net or a twofold axis with the mirror plane. The two situations are different.

So let's examine first the case of a rectangular net that has either a mirror plane or a glide plane in it. And they're both going to have the same sort of constraint. If I have a mirror plane, that-- let me use a slightly wiggly line so the mirror line is not confused with the edges of the cell. So here are two translations,  $T_1$  and  $T_2$ . And if I pick a  $T_3$  that moves this rectangular net up and over itself, we have to do so in such a way that the mirror planes coincide. Because if I have a mirror plane in space that passes in proximity to a lattice point, I can turn my theorem around and say that a translation,  $T_1$ , combined with a mirror plane that is a reflection operation that's removed from the lattice point, I would have to introduce a new translation to  $\delta$  which is going to be incommensurate with  $T_1$ .

So I have to pick my third translation in one of two ways. Let me indicate that by drawing a circle.  $T_3$  can terminate anywhere over this mirror plane. I could pick this net up, slide it parallel to  $T_2$ , and as long as I put it down so that this is the lattice point in the next layer up, that will be a legitimate placement of the net. Because all the mirror planes are simply slid into coincidence with one another. So if I sketch that thing in three dimensions-- this is the base of the cell and it has a rectangular shape. And the third translation goes up at an angle. That's going to be exactly something that I've made before. And the only difference is that the oblique angle is now on the vertical plane but that's a right angle. And this stays a right angle.

So that's exactly a lattice of the shape that we had last time with twofold axes, except this cell is sitting on one of its rectangular sides rather than sitting on its oblique base. So that is something that's not new. And if I would pick the translations so that  $T_3$  had a component that was one half of  $T_1$ , plus some amount  $z$  that is perpendicular to the base, I can redefine that in terms of a cell that has one pair of faces inclined to one another. And the difference is going to be that the lattice point would be caught in the center of the cell. This would be my original translation  $T_3$  and I'll redefine that as a  $T_3$  prime. And that gives me two right angles and one general angle. So those are exactly the same lattices that I obtained from stacking a net with a twofold axis in it.

Let me introduce now another piece of jargon as we go along. The relative angles between the translations are going to determine the shape of the coordinate system that we pick along those translations to specify the geometry of features within the lattice. So we've had a first case, and this was just a general oblique net. So we had a  $T_1$  not equal to a  $T_2$ , not equal to  $T_3$ , and all three angles were general. All three pair of axes are inclined. And this coordinate system, if we want to refer to it in terms of words, is called a triclinic. This is the triclinic system. And the triclinic system is short for coordinate system.

So this is as general as things get. And if this is the shape of the lattice there is no symmetry. Which is the same as saying that the point group, the only point group that can fit into such a lattice, is point group 1. And then we hit another sort of geometry. And that is where the three translations had a general angle-- a third translation that was exactly normal to the plane of the first two. And one pair of axes is inclined. And you can see that this notation is a little strange, perhaps, but consistent. And it says something about the nature of the arrangement of cell edges. So one pair of axes inclined is called the monoclinic system.

And this can contain lattices that are either primitive, side-centered-- with either the longest or shortest translation in the parallelogram that's coming out of the face that's centered. Or we saw there was one final possibility. And that was body-centered. So we'll refer to these lattices subsequently as monoclinic primitive,

monoclinic side-centered, or monoclinic body-centered. These lattices were compatible with point group 2 or point group m. We obtained a lattice of the same shape with point group m or with point group 2. If it works for 2 and it works for m, why not both at the same time. It would also work for 2 over m-- twofold axis perpendicular to a mirror plane. Any question at this point? Yeah?

**AUDIENCE:** Also 2m, right? [INAUDIBLE] Yeah

**PROFESSOR:** No, because as soon as you got a mirror plane, then you have to have a rectangular net. So it's going to be-- it's going to fall into this family. You're right, a mirror plane by itself and we've got that here. That's where the mirror plane falls in the side of the thing. You're right. But if it has a rectangular shape, then we have to have either orthogonal mirror planes and a twofold axis. The base, if the base-- yeah.

OK. Part of the amusing part of this game is that you can, upon appropriate redefinition of cells, come up with a given coordinate system that defines more than one lattice. And I'll give you-- because you get a little space doing this-- I'll give you a two dimensional example. Here is one cell. This is T1, this is T2. This is a primitive cell. Nobody should bother to write this down. Here's a T1, here's a T2. So anybody know what that is? This is a primitive cell. This is a jail cell. Couple sniggers, thank you.

Still another one-- this is T1, this is T2. Anybody guess what that is? That's a soft sell. I got a million of them. Know what that is? It's an underground cell. Here's one that's kind of dated-- I don't think anybody will get this one. Does anybody know what that is? That was a misadventure of the Ford Motor Company that was named the Ed cell. Well, I'll give you equal time. Anybody who, after intermission, wants to respond in kind, I'll let you have an opportunity to do so. But obviously these are not standard crystallographic cells, but they make for a smile in an otherwise dreary business.

I don't know how to proceed from here. I have a set of notes for you which carries all this out in incredible thoroughness. So let me pass this around. Let you take a copy of this and we won't go over every single thing, but I think just to outline quickly

the sort of choices that we have. We haven't yet finished with the rectangular cell, but if it's got something like 2mm in it. with twofold axes here and mirror planes running this way-- P2mm in the base, P2gm, P2gg, can be stacked only in such a fashion that T3 brings twofold axes and mirror planes into coincidence with one another.

So T3 could be  $0T1$  plus  $0T2$  plus some amount  $z$  which is straight up. Or it could be equal to one half of  $T1$ ,  $0T2$ , plus  $z$ . That would be making the origin twofold axis fall over this location. Or T3 could be equal to  $0$  plus one half of  $T2$  plus  $z$ . And they're both going to, upon redefinition, give the same result. Or  $T2$  could be equal to one half of  $T1$  plus one half of  $T2$  plus  $z$  straight up. And upon redefinition, the first one that would be a brick-shaped unit cell. That's going to be a primitive cell.

Picking T3 with a component of either one half of  $T1$  or one half of  $T2$  is going to be something that gives us a cell, which can again be redefined. This was the original T3 in terms of a T3 prime, which is equal to  $2T1$  or  $2T2$  minus  $[? 2T3 ?]$  minus  $T1$  or  $T2$ . This is going to be side-centered. And the final choice where it is a T3 that's one half of  $T1$  plus one half of  $T2$  can once again be defined in terms of-- redefined in terms of a body-centered lattice.

OK this is a coordinate system where all interaxial angles are 90 degrees. The translations, however, have arbitrary magnitude with respect to one another. All of these cells could be described as rhombuses. In this case all angles in the rhombus are 90 degrees, so this is called the orthorhombic system-- orthorhombic coordinate system. All the angles between the axes are 90 degrees. So we can get that out of any one of the rectangular groups that has twofold axes and symmetry planes in it. And conversely, take one of these lattices and let the symmetry elements that we found in the plane group extend through them and you've got a three dimensional space group. So without emphasizing the fact is we go along, we're picking up a healthy number of three dimensional space groups as we go along.

**AUDIENCE:** [INAUDIBLE]?

**PROFESSOR:** This is primitive. It's also orthorhombic -- same dimensionality, three translations,

and they're all mutually orthogonal. That's true of these other cells except they're double cells in the case of the side-centered and double as well for the case of the body-centered.

It's time we introduced some other conventions for notation in describing lattices. We've been using T1, T2, and T3 to designate the three translations that are used to define the cell. One has to have some sort of rules for picking a standard unit cell so that two people can do an x-ray diffraction experiment, let's say, on a particular material and end up defining the lattice in exactly the same way. So let me now list, so that we can use these labels as we go along further, conventions for selecting cell edges.

OK one convention is that one should select the shortest translations in the lattice. And these shortest translations, if there's no symmetry, will be [? dignified ?] by calling them a, b and, c-- standing for the x, y, and z directions. One picks always, if there's nothing special about the translations in the triclinic system, the magnitude of b greater than a. And in a rational world-- every so often you come to a point like this and it feels almost the silliest talking about underground cells and jail cells-- in a logical, rational thinking world, you would do this.

This is not what one does-- for reasons that have to do with the perverse nature of crystals. Take b greater than a, one does that always, but you pick c as the shortest translation. Why? It has something to do with the peculiar behavior of crystals. If you were working with a crystal that had the shape of a needle, in other words, greatly elongated in one direction so that the crystal looks something like this. How would you draw a picture of that? Would you put it straight up and down like this or would you put it on its side. Takes up a lot of room this way.

You'd almost certainly instinctively put it this way. Now if you didn't know anything about the translations down inside the innards of that crystal-- and this was the position that the people who did crystallography solely on the basis of crystal shape and morphology-- and had no idea if there was even a lattice inside of the crystal that caused this regular shape. When you draw a coordinate system x, y, z, z

always goes up. Who draws a Cartesian coordinate system with z going this way, x going this way, and y going this way? It's obscene. We always put z going straight up and down.

You do the same if you don't know what the lattice translations are with the labels on the axes of the crystal. So this is a, this is b, and this is c. And there were tons of pages published with drawings of crystal morphology on them. And then along came x-rays. And when they begin to determine lattice constants of a the material that looked like this, almost invariably they found the translations inside of a needle-like crystal corresponded to a unit cell in the shape of the pancake. Crystals, when they have a very, very short lattice vector, almost always grow most rapidly in that direction. So this is a terrible pickle. Here are reams of books that published descriptions of crystals that had a needle-like-- an acicular shape, if you want to use a fancy term, which had c as a direction which corresponded to the shortest translation in the crystal.

So what do you do? You don't want to have to redraw all these figures and redefine all these directions in the crystal. So what you do is cop out. And you make c the shortest direction in the crystal. You think this field is rigorous but people wimp out and they do something that's contrary to the logical thing to do. Interesting question is why crystals should grow most rapidly in the direction of the shortest lattice vector. And I think I know the answer to that-- I've never seen anybody express it in the writing. The clue comes from some early work that looked at the growth of the crystal from solution. And this was done first with-- I think it was some sort of iodide. I can look it up-- something like cadmium iodide. And this material in solution grew very, very slowly in the form of hexagonal plates. And continued to grow most rapidly in this direction.

This was the direction of the shortest lattice translation. Then all a sudden something happened. And suddenly the rate of growth and the nature of the morphology changed. The crystal took off like a bat in these directions and finally ended up in a needle-like fashion like this. This was an observation that led to the dislocation theory of crystal growth. The interpretation was that in the early stages

of growth, the crystal is growing by accretion of atoms on to these planar surfaces. And the probability of an atom sticking is less high when the atom is on the flat surface than when an atom is on the surface that might have a little crevice in it so it can bind to two planes at once.

So the original postulate-- and now we all take this for granted-- is that if there was a flaw such that the crystal developed a dislocation on this surface, and an exposed ledge, atoms would accrete and stick more tenaciously to this exposed ledge of the dislocation. And this is the so-called dislocation-- screw dislocation mechanism-- for crystal growth. And after the crystal has grown for a while, you can see spiral steps on the surface of the crystal that correspond to this dislocation winding around.

OK so why should crystals grow most rapidly along the direction of the shortest translation? It's going to be a lot easier to create a screw dislocation with a Burgers vector for a translation that is short, rather than the translation that's 10 or 20 angstroms that the Burgers vector would have to involve-- many, many layers of atoms. If the translation is very small, just two or three atoms, it doesn't cost you much work to make a screw dislocation. So I think that is the reason why crystals with very short lattice translations grow most rapidly in that direction, because it's easier to make the screw dislocation with a Burgers vector that's parallel to that translation.

So here's the first flagrant bit of logic that flies in our face. Let me give a few more definitions and then we'll return to this. If we have a crystal in which two translations are equivalent by symmetry-- and we would have such a situation if we picked a translation that was perpendicular to a square net. And would we call the base of the cell something defined by translations  $a$  and  $b$ ? If this is a fourfold axis, what goes on in this direction is identical to what goes on in this direction by symmetry. And if these two directions are identical, why call one of them  $a$  and one of them  $b$ ?

So if two directions are equivalent, and this is true of the square net, what you do is call one  $a_1$  and you call one  $a_2$ -- because they're the same thing. So let's call them both  $a$ . And the third direction then is  $c$ . And in such a situation here,  $c$  is always

defined as the unique direction. What do I mean by a unique direction? This is the direction of high symmetry in crystals that are based on square bases and hexagonal bases. Yes sir?

**AUDIENCE:** You pretty much contradict yourself if  $c$  is the longest [INAUDIBLE].

**PROFESSOR:** Not always, not always.

**AUDIENCE:** Well, I mean in that picture.

**PROFESSOR:** Well I drew it that way. You know why? Because if I draw it like this, the top of my cell doesn't get in the way of the bottom of my cell. And if I want to draw a small  $c$  translation, then I get into geometrical complications that doesn't look nearly as nice.

**AUDIENCE:** Well, then I'm just saying two [INAUDIBLE].

**PROFESSOR:** No, no, no. It's different for different systems. This is what is true for a triclinic system. And in the case of a brick-shaped unit cell-- orthorhombic. So this is done for triclinic. It's also done for orthorhombic. Not for monoclinic. For monoclinic, there is a special direction because there is a direction of a twofold axis or maybe a twofold axis perpendicular to a mirror plane. In that case, if we were rational, we'd call that  $1c$ . OK

I'll close with an opinion poll. I will invariably, and I'm sure most people do, draw a crystal with a square base, with the  $c$  axis the longest. And if we have someone drawing a sketch of a cell in which the base of the cell is a hexagonal net, we'd again call one  $a_1$ , one  $a_2$ . And perpendicular to that would be the same direction. Say look, there I did it again. I made  $c$  longer than  $a_1$  and  $a_2$ .

All right. Let's look at hexagonal crystals. Let me ask for a poll. What percent of all crystals with threefold and sixfold axes do in fact have the length of  $c$  greater than the length of  $a$ ? 50/50? How many would say equally probable to have it the longest or the shortest? No opinion. Let me tell you the astounding fact. Seventy percent of all hexagonal crystals have  $c$  greater than  $a$ . Astounding.

Let's look at the next-- haven't really defined these terms-- tetragonal crystals. These are crystals that have a fourfold axis in them. What percentage have  $c$  greater than  $a$ ? This is almost a first cousin of the cubic crystal or all three translations are equal. How many would say more have  $c$  greater than  $a$ , fewer have  $c$  greater than  $a$ ? One vote. More? That's a good answer, but it's only about 60 some percent. So let's go down to orthorhombic crystals. And orthorhombic crystals are the ones that have a brick-shaped unit cell. How many have  $c$  greater than  $a$ ? It's declining. So if you extrapolate 50/50?

**AUDIENCE:** Zero?

**PROFESSOR:** Who said that? Are you a wise guy? You weren't supposed to give the right answer. It's 0 percent. It's 0 percent because there's nothing that makes one direction more special than any other. So you decide on the labelling of axes on the basis of their relative lengths. And you, by definition, make  $c$  the smallest axis. Congratulations. To the rest of you, gotcha. So that's a good note on which to quit. And a few remaining mysteries of coordinate systems and lattices will be expounded in the second half of our lecture.