Problem 1.1

(a) We approximate the passenger cabin as a cylinder of radius r and thickness t. Inside the cabin there is a pressure p. The longitudinal stress and hoop stress are given, respectively, by

$$\sigma_{\text{long}} = \frac{pr}{2t}, \quad \sigma_{\text{hoop}} = \frac{pr}{t}$$

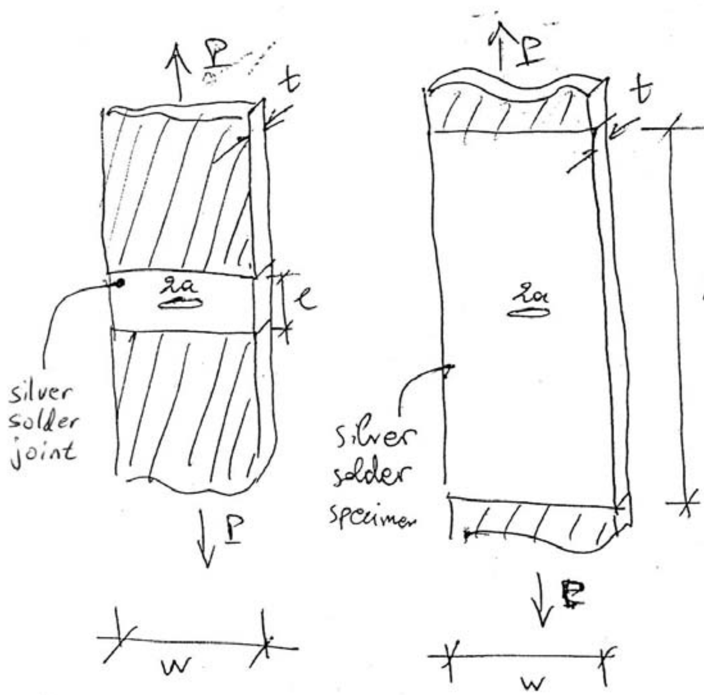
Considering the higher hoop stress as the design stress, we would like to preserve the value of σ_{hoop} in the new designs. Suppose r is increased by 50%, then $\sigma_{\text{hoop}} = 1.5pr/t$ and the thickness t must be increased by 50% in order to keep σ_{hoop} constant. Increasing r by 50% could allow (for example) 9 seats to fit across the diameter of the cabin instead of 6, so the cabin could be made shorter by 33% if the number of passengers is kept constant. The total amount of material surrounding the cabin is approximately given by $2\pi rtL$, where L is the length of the fuselage. With the new dimensions, the total volume of material becomes $2\pi\,(1.5r)\,(1.5t)\,(0.67L)$ or $3\pi rtL$. Thus, using the same material for the fuselage, the weight would be increased by 50% to achieve the same strength. This would in turn increase the fuel consumption, as would the increased drag due to larger cross-sectional area. In addition, the extra weight could require larger wings to achieve more lift. Larger wings create a larger moment on the attachment point of the wings to the fuselage, so the strength at that location would need to be increased, either with reinforcements of a stronger material or through a tapered wing design which is thicker near the point of attachment to the fuselage and decreasing in weight with distance from the fuselage.

(b) If the wall thickness of the fuselage is increased with the radius to maintain constant strength, the volume of skin material will be larger, but plane stress conditions would still prevail in the fuselage skin, since the radius of the fuselage is much greater than the thickness. Since airplanes are generally designed using a defect-tolerant approach, they are assumed to have a distribution of flaws up to the smallest size that can be detected through non-destructive evaluation methods. The larger volume of the aircraft would have more total flaws, but since the design of aircraft allows for flaws to exist, this would not change the approach to damage tolerance.

Depending on the shape of flaws and growing cracks, however, the thickness of the fuselage could have implications for fatigue damage in the aircraft. A thinner skin would be more likely to "leak before break" whereas a thicker skin could allow a larger crack to exist, approaching the critical crack length. However, depending on frequency of maintenance and inspection, the larger flaws could likely be detected in time.

Problem 3.3

The solder joint has a smaller volume than a tensile specimen. It is less likely to contain flaws and defects that could lead to fracture.



(small effect of thermal strains)

Characteristic strain in the joint and the specimen : $\frac{P}{Etw}$

(E : elastic modulus of the solder)

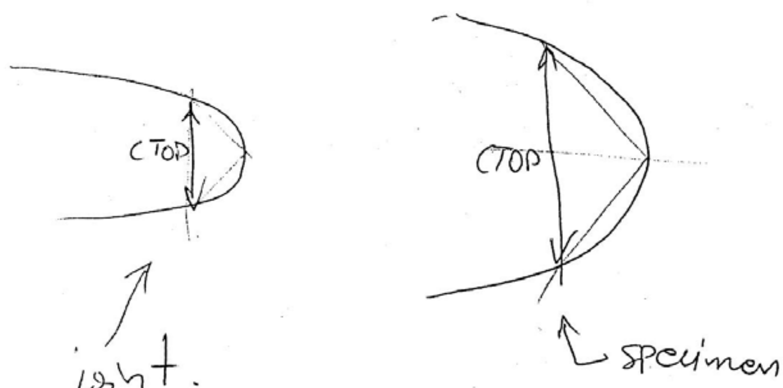
Displacement : (a) in the joint $\frac{P}{Etw} \cdot l$

(b) in the specimen $\frac{P}{Etw} L$

It is expected that the joint will constrain the crack-tip opening displacement and hence strengthen the joint! (The yield strength of the solder is smaller that that of the connecting materials)

Remember that $CTOD \sim J$

$$\Rightarrow CTOD_c \sim J_c$$



- Regarding deformation, the joint will be more constraint elastically and will resist yielding more than the specimen. Possible reasons:
  1) Thermal effects → cotrol the grain size → control the yield strength (rapid cooling → fine grains)
  2) Slip systems → activate fewer slip systems and inhibit dislocation motion

Problem 10.7

Piston: $d_i = 9 \times 10^{-2}\ m$ $d_o = 11 \times 10^{-2}\ m$

Material: $\sigma_y = 550\ MPa$ $K_{IC} = 30\ MPa\sqrt{m}$

Defect: $2c = 4.5\ mm \Rightarrow c = 2.25\ mm$

$a = 1.45\ mm$

Stresses: $r \sim 5\ cm$

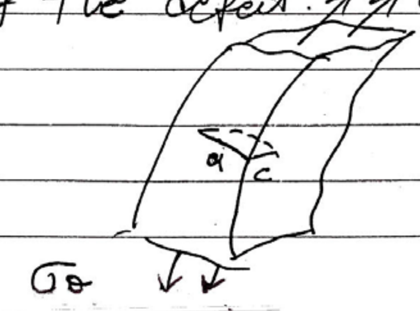
$t = 1\ cm$ (thickness)

Radial stress: $\sigma_r = \frac{r_i^2 P}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right]$

Hoop stress: $\sigma_\theta = \frac{r_i^2 P}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right]$

If closed ended piston: axial stress: $\sigma_a = \frac{P r_i^2}{r_o^2 - r_i^2}$

Loading of the defect: σ_θ



The hoop stress σ_θ is normal to the crack.

From Appendix: $Q = \psi^2 - 0.212 \dfrac{\sigma_\theta^2}{\sigma_y^2}$

$\psi = \dfrac{3\pi}{8} + \dfrac{\pi}{8}\left(\dfrac{a^2}{c^2}\right) = 1.3412$

$K_I = \dfrac{1.12\, \sigma_\theta \sqrt{\pi a}}{\sqrt{Q}}$ stress intensity factor

$Q = 1.7988 - 7.0083 \times 10^{-7}\, \sigma_\theta^2$ (σ: MPa)

Failure occurs when $K_I \geq K_{IC}$

$\Rightarrow$ σ_θ critical $= 505.1$ MPa

$\Rightarrow$ **$p = 104$ MPa**

if $r = r_i + a$

(or 100 MPa if $r = r_i$)

This should have been the pressure, if the piston have failed without prior fatigue crack growth! But the defect may have grown via fatigue!

Using thin shell analysis $\sigma_{\theta\theta} = \frac{p\bar{r}_i}{t}$

For the regular use $p = 55$ MPa
$t = 2$ cm, $\bar{r}_i = \frac{11+9}{2} = 10$ cm

$\Rightarrow \sigma_{\theta\theta} = 275$ MPa

$\psi = 1.34$ remains invariant if a/c = constant

$$Q = \psi^2 - 0.212 \frac{\sigma_{\theta\theta}^2}{\sigma_y^2} = 1.799 - 0.106 = 1.693$$

Load ratio: $R = 0$

$$K_{max} = \frac{1.12 \cdot \sigma_\theta \sqrt{\pi a}}{\sqrt{Q}}$$

For $a = 1.45$ mm $\Rightarrow K_{max} = \frac{1.12 \times 275 \times \sqrt{\pi \times 10^{-3}\, 1.45}}{1.301} = 16$ MPa$\sqrt{m}$

Then $\Delta K = 16$ MPa$\sqrt{m}$

~~7075~~-T6511
From Fig. 10.12 of the Book $\Rightarrow$ $\Delta K_{th} = 5$ MPa$\sqrt{m}$

Therefore: $5 < \Delta K < 30$ MPa$\sqrt{m}$ $\Rightarrow$
inside the Paris regime!

For 7075-T6511

Paris law: $\frac{da}{dN} \approx 2\times10^{-10} (\Delta K)^3$ $\begin{cases} a \text{ in m} \\ \Delta K \text{ in MPa}\sqrt{m} \end{cases}$

$\Delta K = \frac{1.12 \times 275 \sqrt{\pi \times a}}{1.301} = 419.6\sqrt{a}$ (MPa $\sqrt{m}$)

$\frac{da}{dN} = 0.0148\, a^{3/2}$

Integrate: $N = 67.7\,(-2)\left[a^{-1/2} - a_0^{-1/2}\right]$

(initial crack: $a_0 = 1.45\times10^{-3}$ m)

$\Rightarrow a^{-1/2} = a_0^{-1/2} + \frac{N}{135.4}$

$\Rightarrow a = \left(29.6 + \frac{N}{135.4}\right)^{-2}$

N	P (MPa)
1	100
1000	84
3000	48

Therefore, P depends on when the accident happened.

12.3

(a) Start with the definition of compliance, $C = \delta/P$, which gives $C(a) = a^3/24EI$ in this case. Then we have that

$$2\mathcal{G} = \frac{P^2}{2B}\frac{dC}{da} = \frac{P^2}{2B}\frac{a^2}{8EI} = \frac{P^2a^2}{16BEI}$$

$$2\mathcal{G} = \frac{12P^2a^2}{16B^2H^3E}$$

Now solve for the load P_c and find that

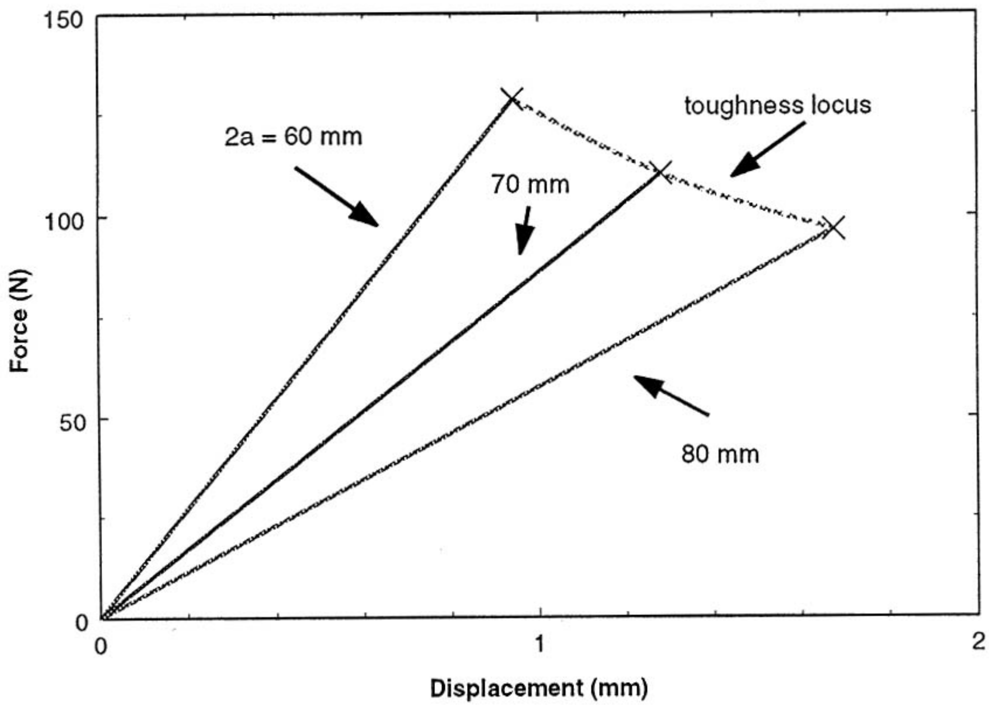
$$P_c = \left(\frac{8\mathcal{G}B^2H^3E}{3a^2}\right)^{1/2}$$

Substututite in the known values for the toughness and the geometric parameters in the problem and find that $P = 128.6$ N for $2a = 60$ mm

(b) From the equation derived above, $P = 110.2$ N for $2a = 70$ mm and $P = 96.5$ N for $2a = 80$ mm. The load P and the displacement δ are related by the expression:

$$P = \frac{24EI}{a^3}\delta$$

So the values of δ at failure can be calculated from the known values of P_{crit} to be 0.942 mm, 1.281 mm and 1.675 mm for $2a =$ 60, 70 and 80 mm, respectively. The plot showing the toughness locus is shown below.



(c) Before we were asked to calculate the values of P (and the associated values of δ), given that the toughness $\mathcal{G}_c$ was known. In this question, we assume that we have only experimental data in the form of various values of P and δ (the toughness locus), and are asked to calculate the value of the toughness $\mathcal{G}_c$. By estimating the area bounded by the toughness locus and the $P-\delta$ curves for $2a = 70$ mm and 80 mm, which represents mechanical potential energy lost which goes into crack growth, we may write that

$$2\mathcal{G}_c B \Delta a \approx \text{area bounded}$$

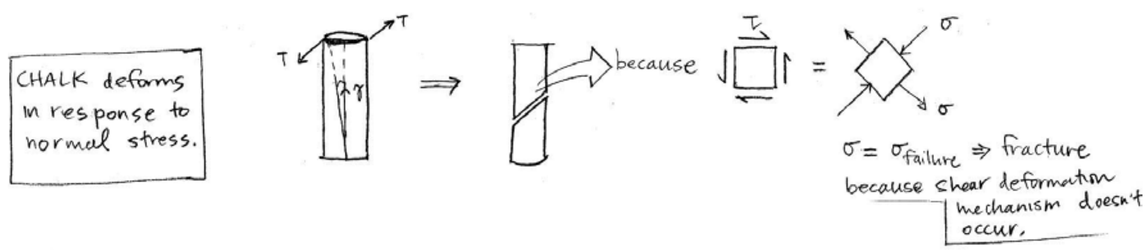
Where $\Delta a = 5$ mm. I found that the area bounded was approximately 29.6×10^{-3} J so that $\mathcal{G}_c \approx 293$ Pa·m, which is close to the value used to derive the toughness locus.

By the way, students who are very familiar with beam theory may have noticed that there is a typo in the text. The statement of the problem and the equation refer to the deflection of the beam δ; in these expressions δ should be replaced by $\delta/2$. This will change the values of P_c and δ you obtain, and your plot of the toughness locus will be different, but you will obtain the same result for the toughness using the method described in part (b).

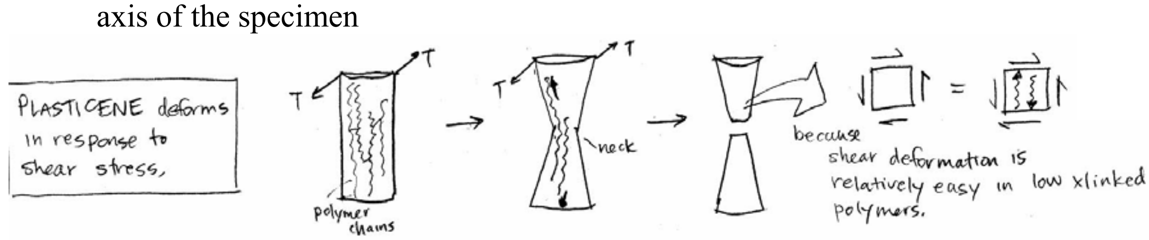
5a. Increase in the ductile-brittle transition temperature (DBTT) of a low strength plain carbon steel:

1) *Chemical environment*: Increasing amounts of embrittling agents, such as hydrogen, increases the DBTT.
2) *Strain Rate*: Increasing strain rate will lead to an increase in DBTT because dislocation motion (required for ductile transition) is not given sufficient time to occur in response to the applied load.
3) *Alloy Composition*: Increasing the concentration of carbon increases the DBTT. Carbon can inhibit the dislocation motion required for ductile deformation.
4) *Geometry*: Plane stress vs. Plane strain. Thicker plates (plane strain) have a higher DBTT. For thin plates (plane stress) the deformation zone size is roughly equal to the thickness. Fracture can occur by ductile tearing, etc.
5) *Work Hardening*

5b. Chalk is an ionic solid with no significant dislocation mobility. Therefore, it is very brittle and the principal crack-driving stresses are those that are normal to the crack face (i.e. those which are oriented 45° to the axis of the specimen).



In contrast, Plasticene is a polymer capable of significant plastic deformation. Failure is ductile and occurs via shear. Shear stresses are maximum normal to the axis of the specimen



5c. Tempering of glass is a thermomechanical processing step that induces a residual compressive stress in the surface of the glass. The residual compressive stress suppresses the growth of cracks on the surface of the glass. When glass is loaded in tension, failure almost always initiates from surface flaws. For the case of tempered glass, before the applied tensile stress can exert any tensile load on the cracks, it must first overcome the residual compressive stress that exists due to the tempering process, and thus tempered glass is more resistant to tensile fracture than ordinary glass, and can sustain larger tensile loads.

The process of tempering occurs by cooling the glass surface (using jets of cool air or water) while it is still hot and able to flow. The cooling causes the outer surface to contract, while the inner core is still hot and flows along with the contracting outer surfaces. When the core finally cools, it contracts and puts the outer surface into residual compression, while the core is put into residual tension. Tempering of glass is somewhat analogous to shot-peening of metals as it represents a processing step which induces a beneficial compressive residual stress in the surface layer. Tempered glass has been used for many applications (e.g. automobile windshields).

Problem 6.

The Tresca criterion states that yield occurs when the maximum shear stress reaches a critical value, i.e.

$$\tau_{\max} = \frac{1}{2}\left|\sigma_{\max} - \sigma_{\min}\right| = \frac{\sigma_{\rm YS}}{2}$$

So the only trick in the problem is to find out what are the maximum and minimum stresses. Looking at the form of the principal stresses (see page 7 in the notes on plastic zone size), it can be seen that for θ between -180 and 180 degrees, σ_1 is always $\sigma_{\max}$. What about $\sigma_{\min}$? For the *plane stress* case, $\sigma_3 = 0$, (and σ_2 is never negative) and thus $\sigma_3 = \sigma_{\min}$ so that

$$\frac{1}{2}\left|\sigma_{\max} - \sigma_{\min}\right| = \frac{1}{2}\left|\sigma_1 - 0\right| = \frac{K_I}{\sqrt{2\pi r_p}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right) = \frac{\sigma_{\rm YS}}{2}$$

solve for r_p

$$\frac{r_p}{r_p^*} = \cos^2\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)^2$$

where $r_p^* = K_I^2/(2\pi\sigma_{\rm YS}^2)$ is the approximate plastic zone size (see page 2 on the notes on the plastic zone size). For the *plane strain* case, we need to decide which stress (σ_2 or σ_3) is the minimum stress. First evaluate the stresses σ_2 and σ_3 at $\theta = 0°$. We find that

$$\sigma_2 = \frac{K_I}{2\pi r}, \quad \sigma_3 = 2\nu\frac{K_I}{2\pi r}$$

For a reasonable value of ν (say $\nu = 1/3$) we see that $\sigma_3 < \sigma_2$ so $\sigma_3 = \sigma_{\min}$. But note that at some critical angle θ_c, σ_3 is equal to σ_2, and beyond that angle σ_2 is now σ_{min}. So the minimum stress σ_{min} is *either* σ_2 or σ_3, depending on the angle. We can find the critical angle by setting $\sigma_3 = \sigma_2$:

$$\frac{K_I}{2\pi r}\cos\frac{\theta_c}{2}\left(1-\sin\frac{\theta_c}{2}\right) = \frac{K_I}{2\pi r}2\nu\cos\frac{\theta_c}{2}$$

or

$$\left(1-\sin\frac{\theta_c}{2}\right) = 2\nu$$

Solve for θ_c:

$$\theta_c = 2\sin^{-1}(1-2\nu)$$

For $\theta < \theta_c$, $\sigma_3 = \sigma_{\text{min}}$ and the plastic zone radius is given by:

$$\frac{r_p}{r_p^*} = \cos^2\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}-2\nu\right)^2$$

and for $\theta > \theta_c$, $\sigma_2 = \sigma_{\text{min}}$ and the plastic zone radius is given by:

$$\frac{r_p}{r_p^*} = 4\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$$

In the problem we are also asked to consider the ratio of the plastic zone radius for plane strain and plane stress for $\theta = 0°$ and $\theta = 45°$. For the calculation, I assumed that ν was equal to 1/3 and thus $\theta_c =$. The plastic zone radii are evaluated below:

$$\frac{r_p}{r_p^*} = 1\,(\text{Plane Stress}, \theta = 0°)$$

$$\frac{r_p}{r_p^*} = 1.63\,(\text{Plane Stress}, \theta = 45°)$$

$$\frac{r_p}{r_p^*} = \frac{1}{9} = 0.111\,(\text{Plane Strain}, \theta = 0°)$$

$$\frac{r_p}{r_p^*} = 0.5 \,(\text{Plane Strain}, \theta = 45^\circ)$$

and thus the ratios are given as:

$$\frac{r_p\,(\text{Plane Strain})}{r_p\,(\text{Plane Stress})} = \frac{1}{9} = 0.111\,(\theta = 0^\circ)$$

$$\frac{r_p\,(\text{Plane Strain})}{r_p\,(\text{Plane Stress})} = \frac{0.5}{1.63} = 0.307\,(\theta = 45^\circ)$$