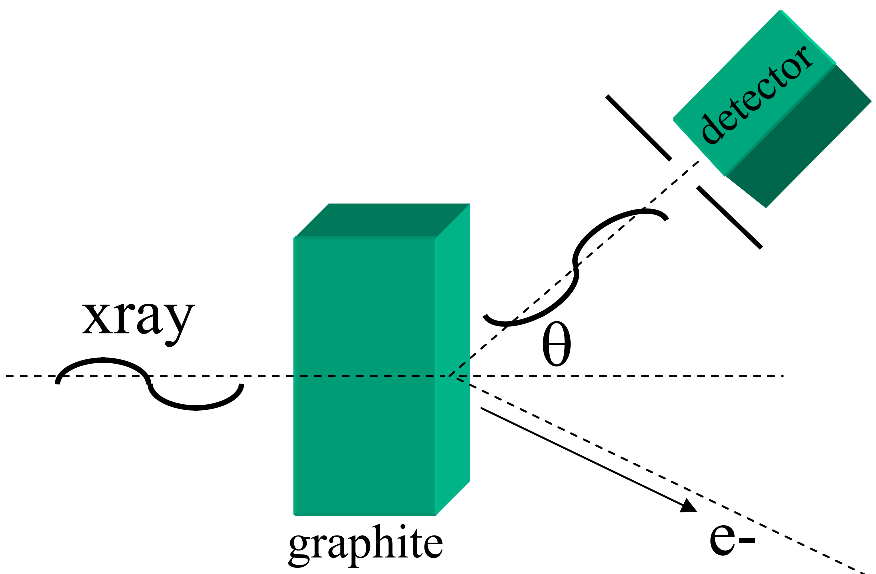
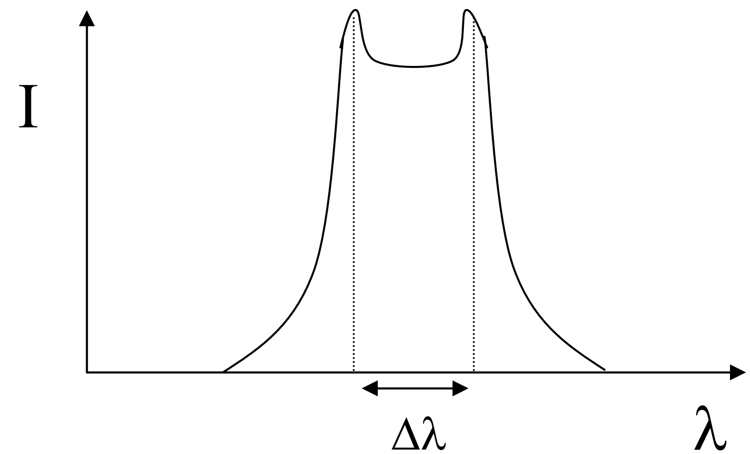
Wave-particle Duality: Electrons are not *just* particles

- Compton, Planck, Einstein
  - light (xrays) can be ‘particle-like’
- DeBroglie
  - matter can act like it has a ‘wave-nature’
- Schrodinger, Born
  - Unification of wave-particle duality, Schrodinger Equation

Light has momentum: Compton

- No way for xray to change λ after interacting classically
- Experimentally: Compton Shift in λ
- Photons are ‘particle-like’: transfer momentum

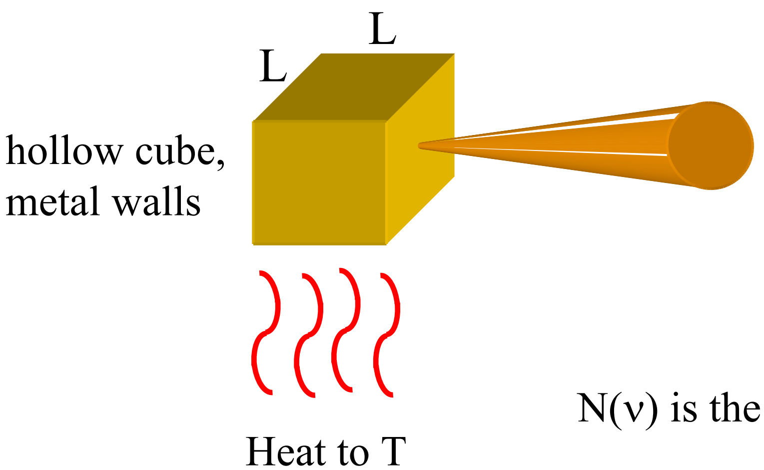




$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta) = \lambda_c(1-\cos\theta)$$

Light is Quantized: Planck

- Blackbody radiation: energy density at a given ν (or λ) should be predictable
- Missing higher frequencies! (ultra-violet catastrophe)



ρ(ν)dν=energy per volume being emitted in ν+dν

$$\rho(\nu)d\nu = \frac{N(\nu)d\nu \cdot E_{wave}}{volume}$$

N(ν) is the number density, i.e. number of waves in ν+dν (#/frequency)

Finding N(ν): Inside box, metal walls are perfect reflectors for the E-M waves

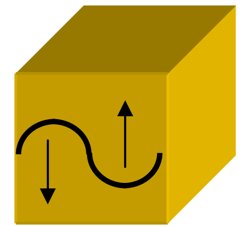
$$E_i = E_{oi}e^{i(\omega t - kz)}; E_r = E_{or}e^{i(\omega t + kz)}$$ Perfect reflection, $E_{oi}=-E_{or}$

$$E_{tot} = E_{oi}e^{i\omega t}\left[e^{-ikz} - e^{ikz}\right] = -2iE_{oi}e^{i\omega t}\sin kz$$

Light is Quantized: Planck

$$\text{Real}\{E_{tot}\} = 2E_{oi}\sin\omega t\sin kz$$

Standing Waves



E-field inside metal wall is zero (due to high conductivity)

Therefore, sinkz must equal zero at z=0 and z=L

$$\sin kL = 0; kL = \pi n; k = \frac{\pi n}{L}$$

Also, since k=2π/λ,

$$n = \frac{2L}{\lambda} \text{ or } \lambda = \frac{2L}{n}$$

In 3-D,

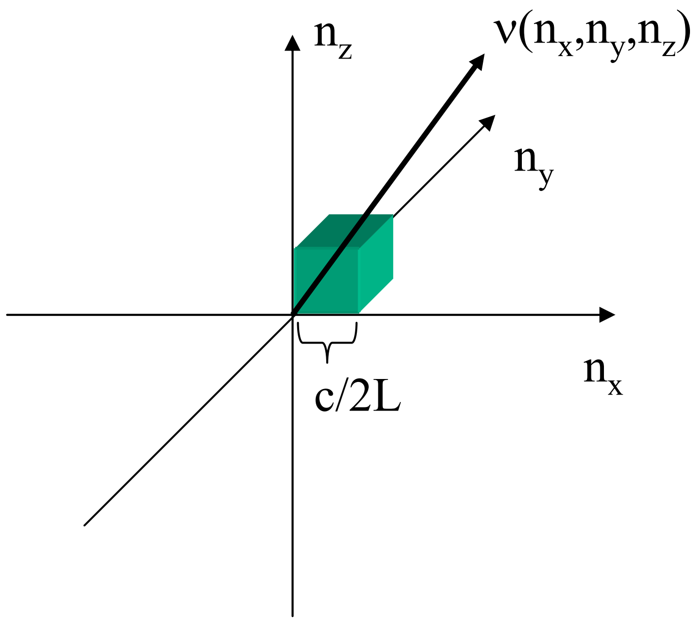
$$\lambda = \frac{2L}{\sqrt{n_x^2 + n_y^2 + n_z^2}}$$

Note that the wavelength for E-M waves is 'quantized' classically just by applying a confining boundary condition

$$\nu = \frac{c\sqrt{n_x^2 + n_y^2 + n_z^2}}{2L} = \frac{c|\vec{n}|}{2L}$$

$$\vec{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$$

Light is Quantized: Planck



1 state (i.e. 1 wavelength or frequency) in $(c/2L)^3$ volume in 'n-space'

2 possible wave polarizations for each state

(Note also that postive octant is only active one since n is positive: shows as 1/8 factor below)

Using the assumption that $\nu \gg c/2L$,

$$N = \frac{\frac{1}{8}\frac{4\pi\nu^3}{3}}{\frac{1}{2}\left(\frac{c}{2L}\right)^3} = \frac{8L^3\nu^3\pi}{3c^3}$$

$$N(\nu) = \frac{dN}{d\nu} = \frac{8L^3\nu^2\pi}{c^3}$$

Light is Quantized: Planck

Now that N(ν), the number of E-M waves expected in ν+dν, has been determined simply by boundary conditions, the energy of a wave must be determined for deriving ρ(ν)

$$\rho(\nu) = \frac{N(\nu)E_{wave}}{volume} = \frac{\frac{8\pi\nu^2 L^3}{c^3} kT}{L^3} = \frac{8\pi\nu^2 kT}{c^3}$$

The classical assumption was used, i.e. $E_{wave}=k_bT$
This results in a ρ(ν) that goes as ν^2

At higher frequencies, blackbody radiation deviates substantially from this dependence

Missing higher frequencies

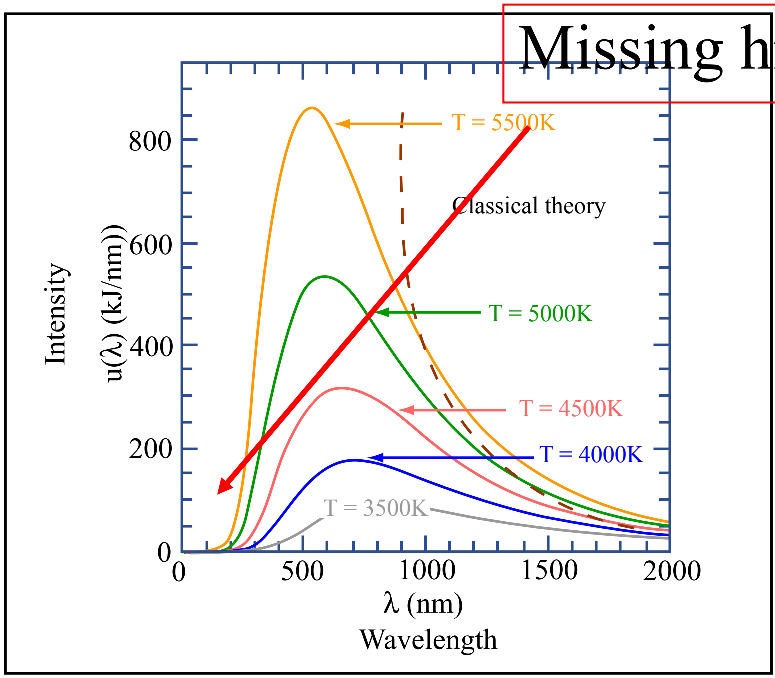


Figure by MIT OpenCourseWare.

- Low ν OK: $E=k_bT$
- At high ν, E goes to zero (i.e. no waves up there!)

Light is Quantized: Planck

- Classical $E=k_bT$ comes from assumption that Boltzmann distribution determines number of waves at a particular E for a given T
- Since $N(\nu)$ can not the problem with $\rho(\nu)$, it must be E
- E must be a function of ν in order to have the experimental data work out

Origin of $E=k_bT$

Boltzmann distribution is $P'(E) = Ae^{-\frac{E}{k_bT}}$

Normalized distribution is $P(E) = \int_0^\infty P'(E)dE = 1; A = \frac{1}{k_bT}$

Average energy of particles/waves with this distribution:

$$\overline{E} = \frac{\int_0^\infty EP(E)dE}{\int_0^\infty P(E)dE} = \text{if P(E) is normalized} = \int_0^\infty EP(E)dE = k_bT$$

Light is Quantized: Planck

- If P(E) were to decrease at higher E, than $\rho(\nu)$ would not have ν^2 dependence at higher ν
- P(E) will decrease at higher E if E is a function of ν
- Experimental fit to data suggests that E is a linear function in ν, therefore $E=nh\nu$ where h is some constant

$$\bar{E} = \frac{\sum_0^\infty \frac{nh\nu}{k_b T} e^{-\frac{nh\nu}{k_b T}}}{\sum_0^\infty \frac{1}{k_b T} e^{-\frac{nh\nu}{k_b T}}} = \frac{h\nu}{e^{\frac{h\nu}{k_b T}} - 1}$$

Note: the integrals need to be removed in the average and replaced with sums since the spacing of energies becomes greater as E increases

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{k_b T}} - 1}$$

h determined by an experimental fit and equals

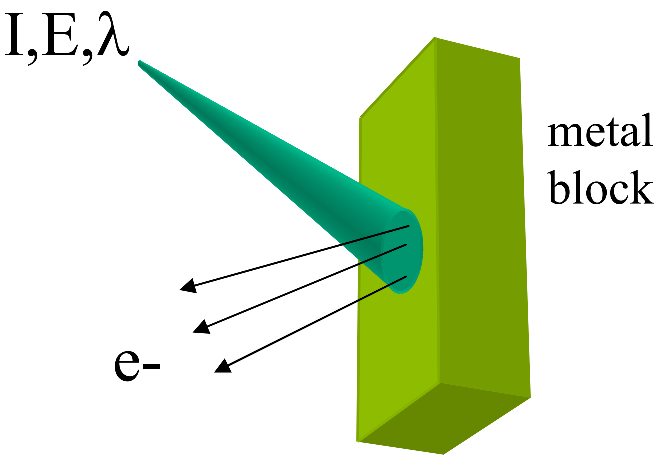
At small $h\nu/k_bT$, $e^{h\nu/kT} \sim 1 + h\nu/k_bT$ and $\rho(\nu) \sim k_bT$
At large $h\nu/k_bT$, $\sim h\nu e^{-h\nu/kT}$ which goes to 0 at high E

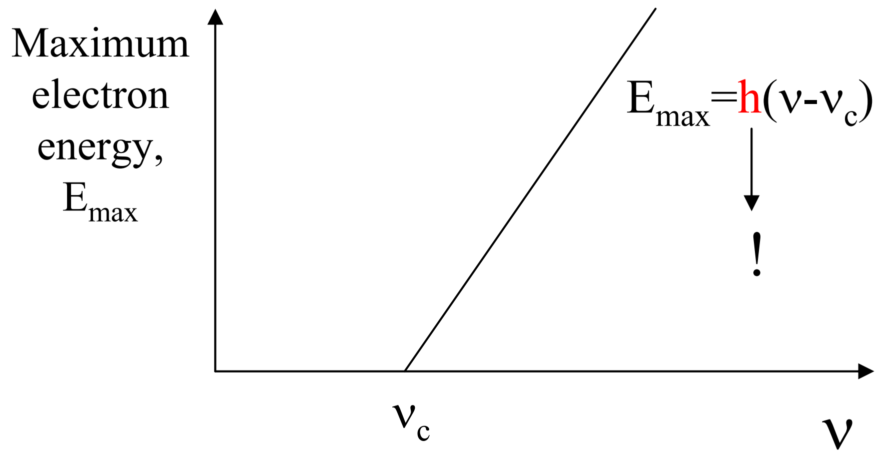
Light is Quantized: Planck

- Lessons from Planck Blackbody
  - waves which are confined with boundary conditions have only certain λ available: quantized
  - $E=nh\nu$, and therefore E-M waves must come in chunks of energy: photon $E=h\nu$. Energy is therefore quantized as well
  - Quantized energy can affect properties in non-classical situations; classical effects still hold in other situations

Light is always quantized: Photoelectric effect (Einstein)

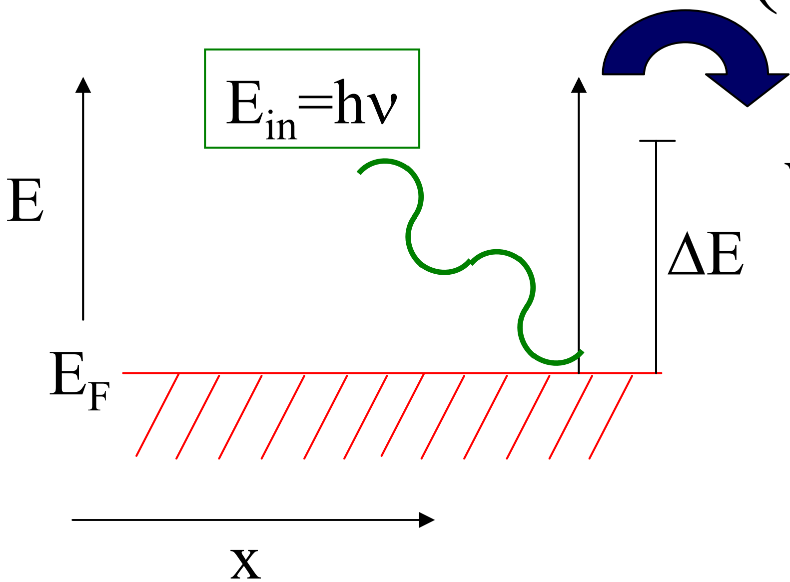
- Planck (and others) really doubted fit, and didn't initially believe h was a universal constant
- Photoelectric effect shows that $E=h\nu$ even outside the box





For light with $\nu<\nu_c$, no matter what the intensity, no e-

Light is always quantized: Photoelectric effect (Einstein)



$$E_{vac}=E_{in}-\Delta E$$

$$E_{max}=E_{in}-\Delta E=h\nu-h\nu_c$$

$$E_{in}=h\nu!$$

Strange consequence of Compton plus E=hν: light has momentum but no mass

$$\lambda = \frac{hc}{E} = \frac{h}{p} \text{ since } \mathrm{E} = cp \text{ for a photon}$$

DeBroglie: Matter is Wave

- His PhD thesis!
- $\lambda = h/p$ also for matter
- To verify, need very light matter (p small) so λ is large enough
- Need small periodic structure on scale of λ to see if wave is there (diffraction)
- Solution:electron diffraction from a crystal

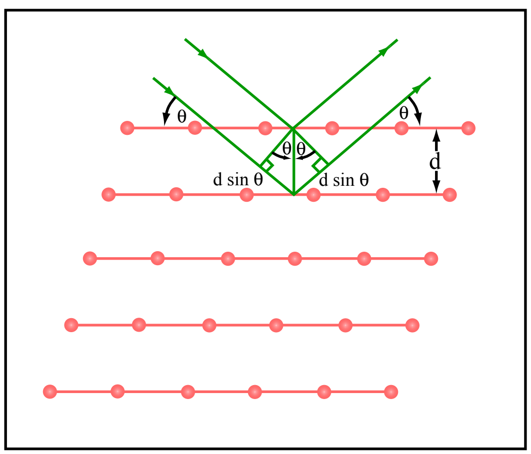


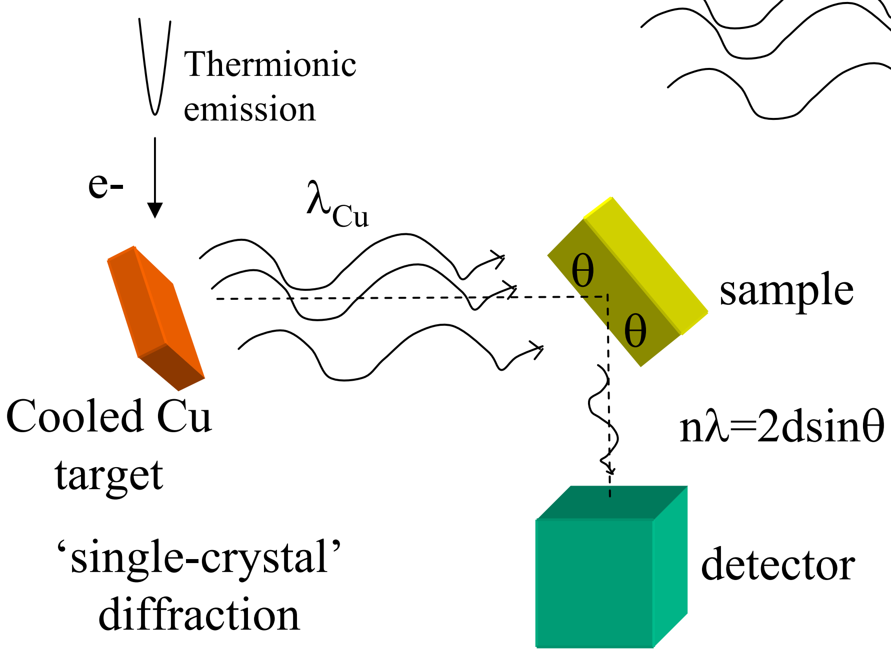
Figure by MIT OpenCourseWare.

$$N\lambda = 2d\sin\theta$$

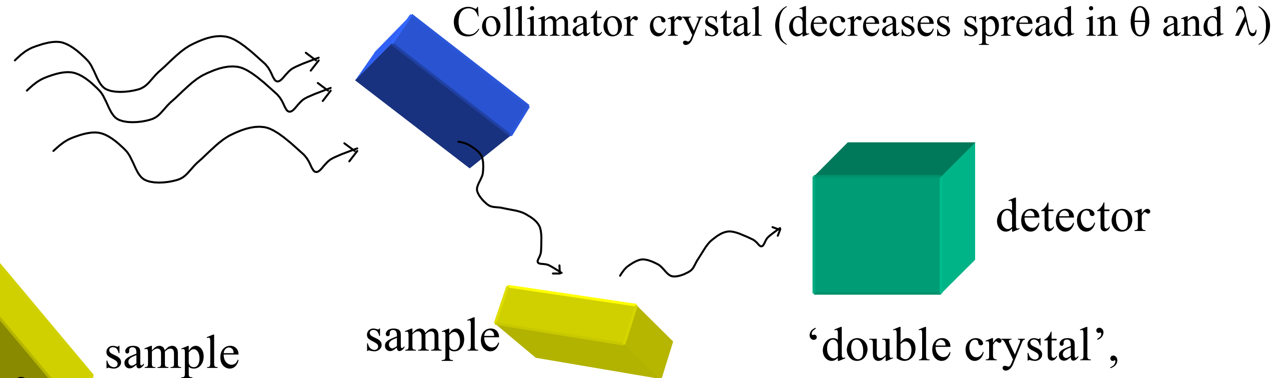
For small θ, $\theta \sim \lambda/d$, so λ must be on order of d in order to measure easily

Diffraction

- Incoming λ must be on the order of the lattice constant a or so (λ<~ few tenths of a nanometers)
- x-rays will work (later, show electrons are waves also and they can be used for diffraction also)
- x-rays generated by core e- transitions in atoms
  - distinct energies: $E=hc/\lambda$; E~ 10keV or so (core e- binding energies)



'single-crystal' diffraction

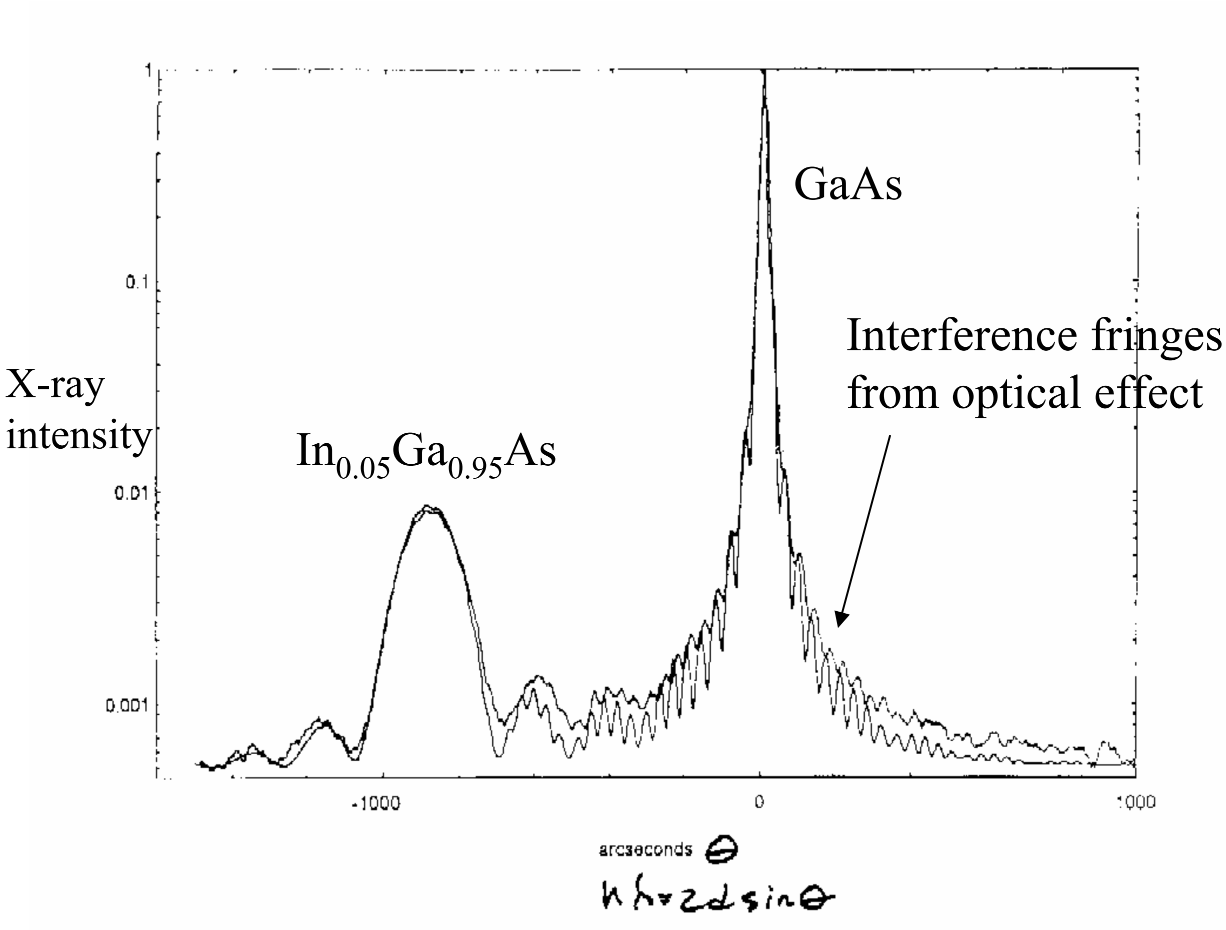


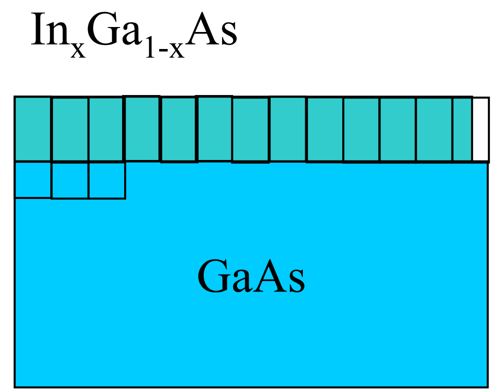
'double crystal', 'double axis' diffraction

Add a channel crystal (also called analyzer crystal) after the sample and it is called triple axis diffraction

Example of Diffraction from Thin Film of Different Lattice Constant

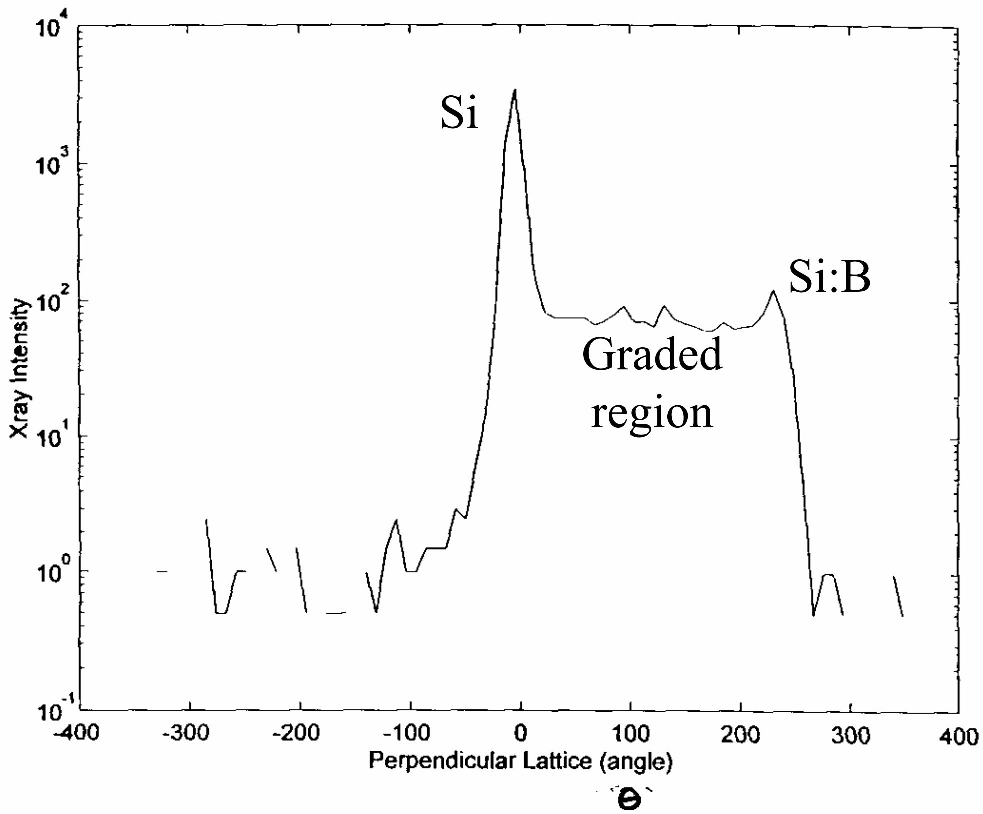
- InGaAs on GaAs deposited by molecular beam epitaxy (MBE)
- Can determine lattice constant (In concentration) and film thickness from interference fringes



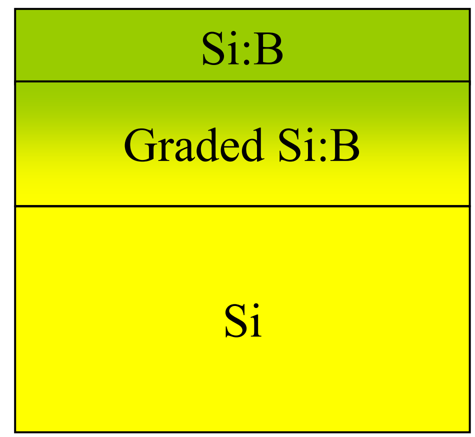


Example: Heavily B-diffused Si

- B diffusion from borosilicate glass
- creates p++ Si used in micromachining
- gradients created in B concentration and misfit dislocations



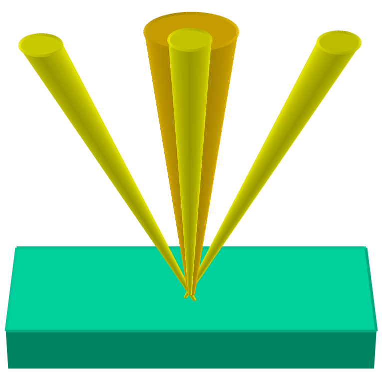
$$n\lambda = 2d \sin\theta$$

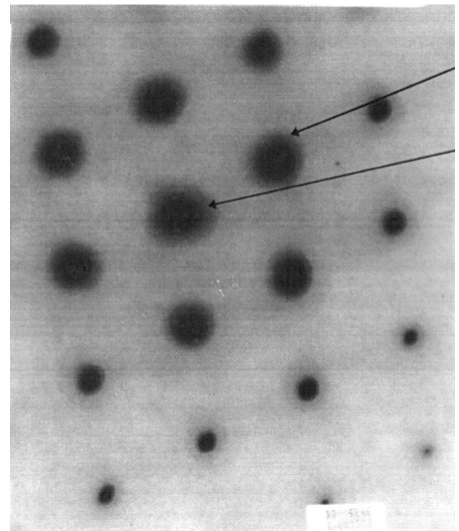


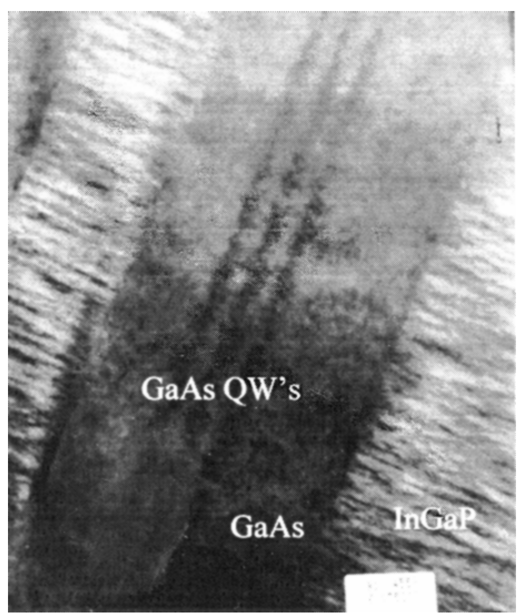
DeBroglie: Matter is Wave

Proof electron was wave by transmission and beackscattered experiments, almost simultaneously

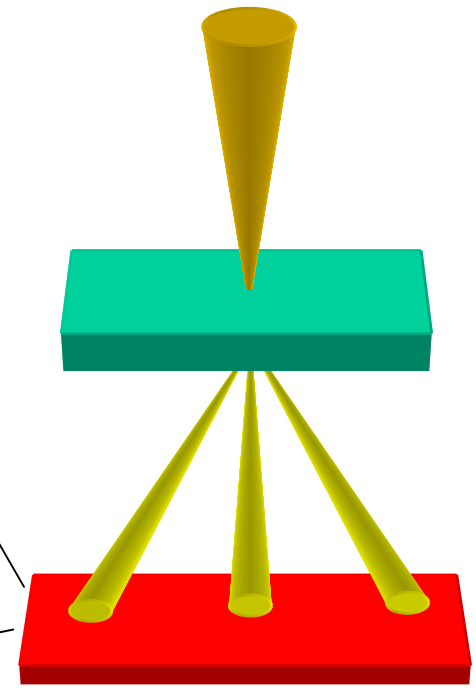
Backscattered





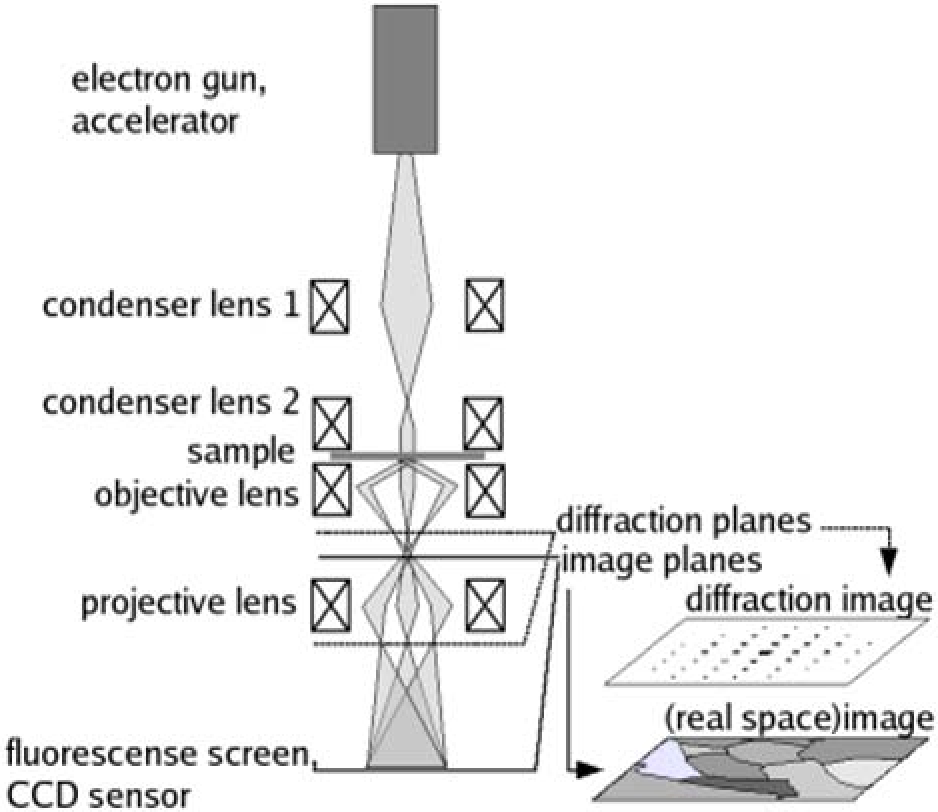


Transmission



DeBroglie: Matter is Wave

Modern TEM



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Modern SEM

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Please see any schematic of a scanning electron microscope, such as http://commons.wikimedia.org/wiki/Image:MicroscopesOverview.jpg

Imaging Defects in TEM utilizing Diffraction

- The change in θ of the planes around a defect changes the Bragg condition
- Aperture after sample can be used to filter out beams deflected by defect planes: defect contrast

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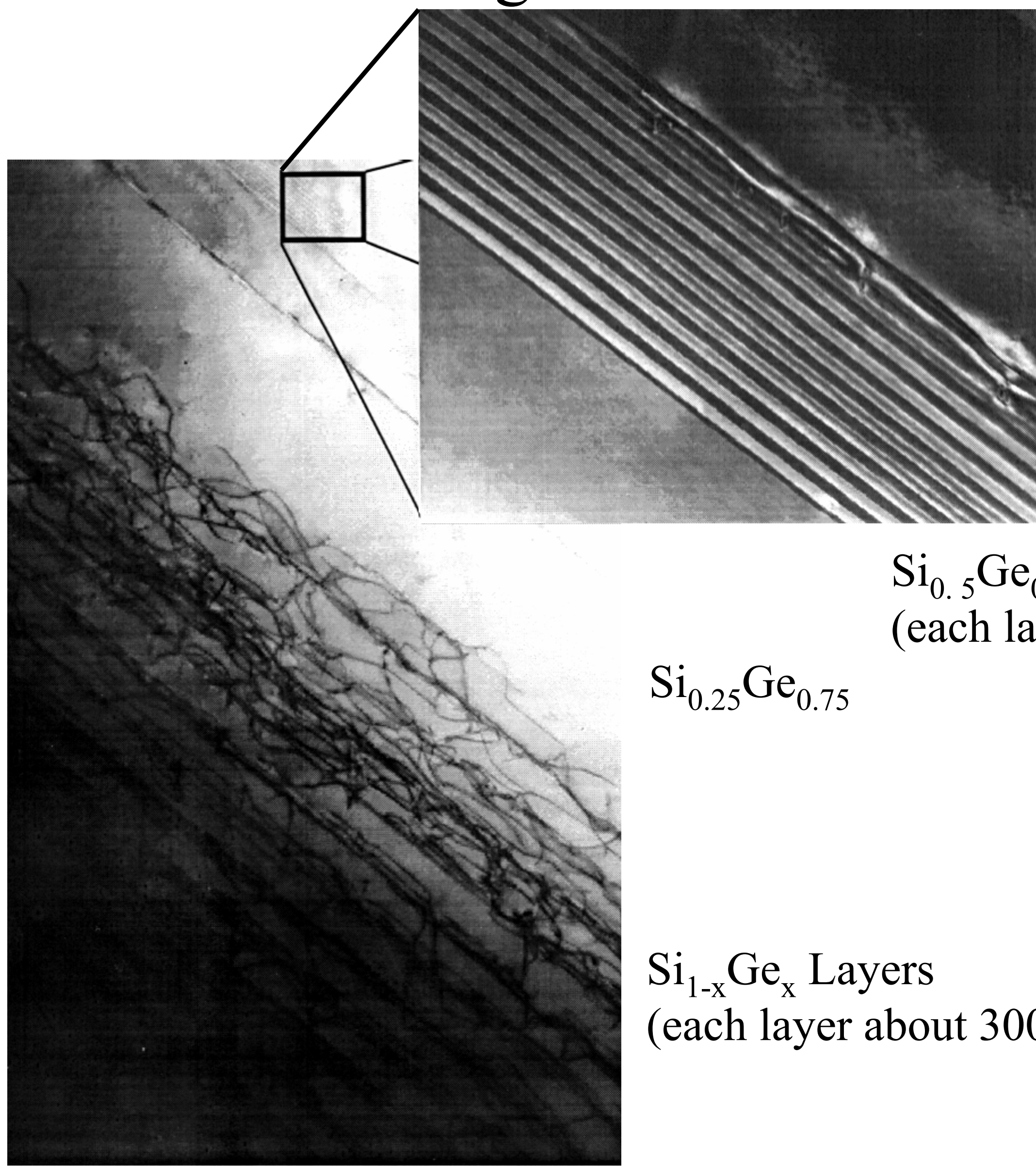
Please see any explanation of detecting dislocations via TEM, such as
http://www.uni-saarland.de/fak8/wwm/research/dip_welsch/ecci-defect-scheme_e.png

Imaging Defects and Man-made Epitaxial Structures in TEM utilizing Diffraction



$Si_{0.5}Ge_{0.5}$/Ge superlattice
(each layer ~100A)

$Si_{0.25}Ge_{0.75}$

$Si_{1-x}Ge_x$ Layers
(each layer about 3000A)