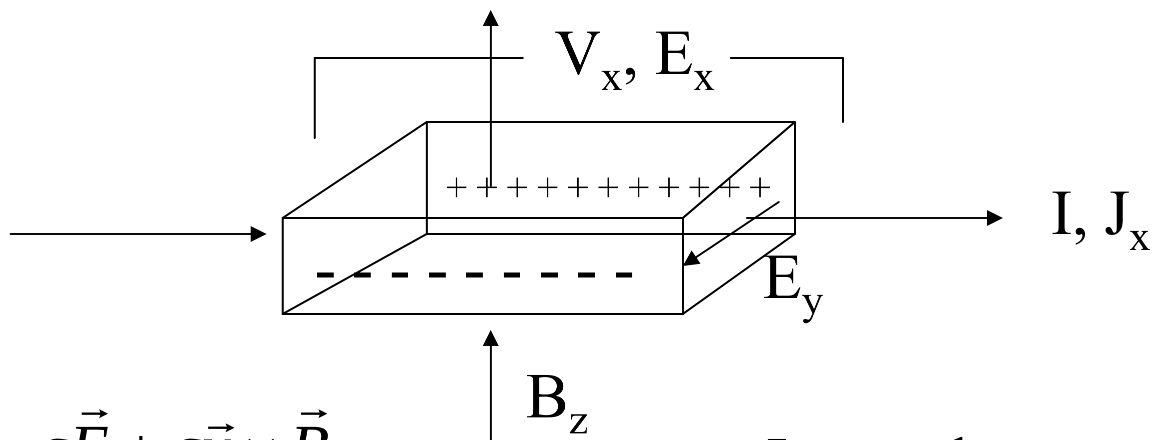
Determining n and μ: The Hall Effect



$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$F_y = -ev_D B_z$$

$$F_y = -eE_y$$

In steady state,

$E_Y = v_D B_Z = E_H$, the Hall Field

Since $v_D = -J_x/en$,

$$E_H = -\frac{1}{ne} J_x B_Z = R_H J_X B_Z$$

$$\boxed{R_H = -\frac{1}{ne}} \qquad \boxed{\sigma = ne\mu}$$

Experimental Hall Results on Metals

- Valence=1 metals look like free-electron Drude metals
- Valence=2 and 3, magnitude and sign suggest problems

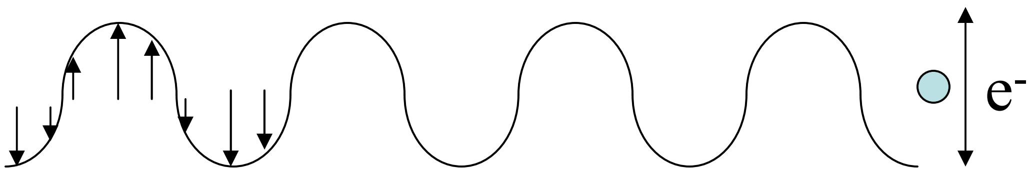
Metal	Valence	$-1/R_H nec$
Li	1	0.8
Na	1	1.2
K	1	1.1
Rb	1	1.0
Cs	1	0.9
Cu	1	1.5
Ag	1	1.3
Au	1	1.5
Be	2	0.2
Mg	2	-0.4
In	3	-0.3
Al	3	0.3

*Hall coefficients of selected elements in moderate to high fields**

** These are roughly the limiting values assumed by R_n as the field becomes very large (of order 10^4 G), and the temperature very low, in carefully prepared specimens. The data are quoted in the form n_0:n. Where n_0 is the density for which the Drude form (1.21) agrees with the measured R_u:n_0 = $-1/R_H ec$. Evidently the alkali metals obey the Drude result reasonably well, the noble metals (Cu. Ag, Au) less well, and the remaining entries, not at all.*

Response of free e- to AC Electric Fields

- Microscopic picture



$$E_Z = E_O e^{-i\omega t}$$

B=0 in conductor,

and $\vec{F}(\vec{E}) >> \vec{F}(\vec{B})$

$$\frac{dp(t)}{dt} = -\frac{p(t)}{\tau} - eE_0 e^{-i\omega t}$$

try $p(t) = p_0 e^{-i\omega t}$

$$-i\omega p_0 = -\frac{p_0}{\tau} - eE_0$$

$$p_0 = \frac{-eE_0}{\frac{1}{\tau} - i\omega}$$

ω>>1/τ, p out of phase with E

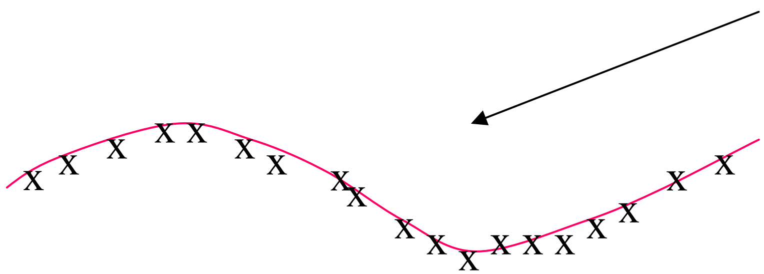
$$p_0 = \frac{eE_0}{i\omega} \qquad \omega \to \infty, p \to 0$$

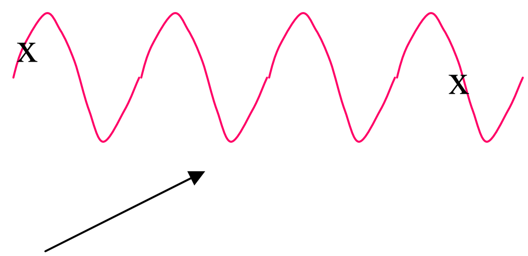
ω<<1/τ, p in phase with E

$$p_0 = -eE_0\tau$$

What if $\omega\tau \gg 1$?

When will $J = \sigma E$ break down? It depends on electrons undergoing many collisions, on the average a collision time τ apart. As long as there are many collisions per cycle of the AC field ($\omega\tau \ll 1$), the AC σ will be $\approx$ the DC σ.





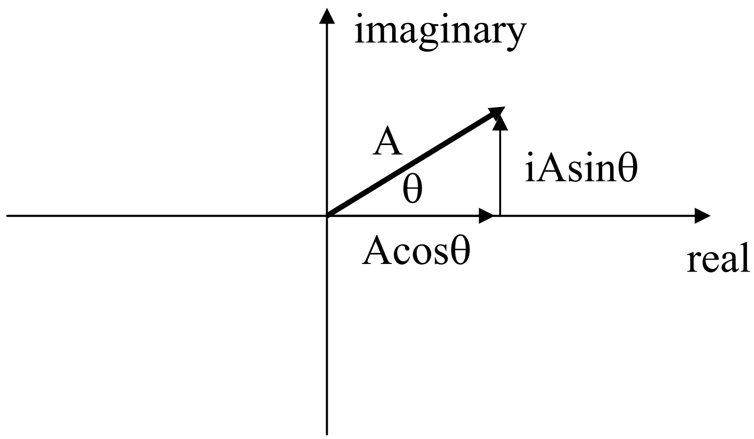
But consider the other limit: $\omega\tau \gg 1$.

Now there will be many cycles of the field between collisions. In this limit, the electrons will behave more like electrons in vacuum, and the relation between J and E will be different x x

Complex Representation of Waves

sin(kx-ωt), cos(kx-ωt), and $e^{-i(kx-\omega t)}$ are all waves

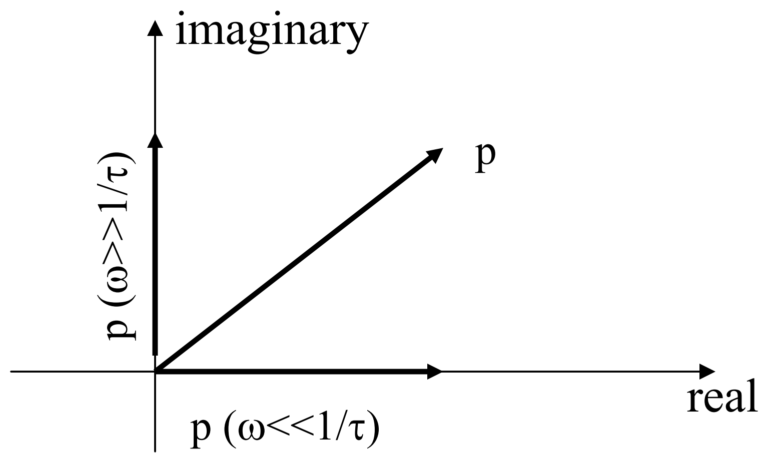
$e^{-i(kx-\omega t)}$ is the complex one and is the most general



$$e^{i\theta}=\cos\theta+i\sin\theta$$

Response of e- to AC Electric Fields

- Momentum represented in the complex plane



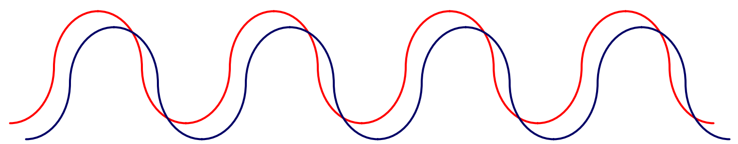
Instead of a complex momentum, we can go back to macroscopic and create a complex J and σ

$$J(t) = J_0 e^{-i\omega\tau} \qquad J_0 = -nev = \frac{-nep_0}{m} = \frac{ne^2}{m(\frac{1}{\tau} - i\omega)} E_0$$

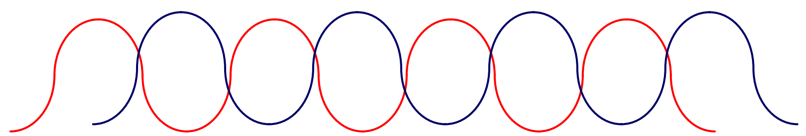
$$\sigma = \frac{\sigma_0}{1 - i\omega\tau}, \sigma_0 = \frac{ne^2\tau}{m}$$

Response of e- to AC Electric Fields

- Low frequency ($\omega \ll 1/\tau$)
  - electron has many collisions before direction change
  - Ohm's Law: J follows E, σ real



- High frequency ($\omega \gg 1/\tau$)
  - electron has nearly 1 collision or less when direction is changed
  - J imaginary and 90 degrees out of phase with E, σ is imaginary



Qualitatively:

$\omega\tau \ll 1$, electrons in phase, re-irradiate, $E_i = E_r + E_t$, *reflection*

$\omega\tau \gg 1$, electrons out of phase, electrons too slow, less interaction, *transmission* $\varepsilon = \varepsilon_r \varepsilon_0$ $\varepsilon_r = 1$

$$\tau \approx 10^{-14}\ \sec, \nu\lambda = c, \nu = \frac{3x10^{10}\ cm/\sec}{5000x10^{-8}\ cm} \approx 10^{14}\ Hz$$

E-fields with frequencies greater than visible light frequency expected to be beyond influence of free electrons

Response of light to interaction with material

- Need Maxwell's equations
  - from experiments: Gauss, Faraday, Ampere's laws
  - second term in Ampere's is from the unification
  - electromagnetic waves!

SI Units (MKS)

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu \vec{H}$$

$$\mu = \mu_r \mu_0 ; \varepsilon = \varepsilon_r \varepsilon_0$$

Gaussian Units (CGS)

$$\nabla \bullet \vec{D} = 4\pi\rho$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla x \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla x \vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\vec{B} = \vec{H} + 4\pi\vec{M}$$

Waves in Materials

- Non-magnetic material, $\mu=\mu_0$
- Polarization non-existent or swamped by free electrons, P=0

$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla x\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla x(\nabla x\vec{E}) = -\frac{\partial \nabla x\vec{B}}{\partial t}$$

$$-\nabla^2 E = -\frac{\partial}{\partial t}[\mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}]$$

$$\nabla^2 E = \mu_0 \sigma \frac{\partial E}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

For a typical wave,

$$E = E_0 e^{i(k\bullet r - \varpi t)} = E_0 e^{ik\bullet r} e^{-i\varpi t} = E(r) e^{-i\varpi t}$$

$$\nabla^2 E(r) = -i\varpi\mu_0 \sigma E(r) - \mu_0 \varepsilon_0 \omega^2 E(r)$$

$$\boxed{\nabla^2 E(r) = -\frac{\omega^2}{c^2}\varepsilon(\omega) E(r)}$$ Wave Equation

$$\varepsilon(\omega) = 1 + \frac{i\sigma}{\varepsilon_0 \omega}$$

$$E(r) = E_0 e^{ik\bullet r}$$

$$k^2 = \frac{\omega^2}{c^2}\varepsilon(\omega)$$

$$v = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon(\omega)}}$$

Waves in Materials

- Waves slow down in materials (depends on $\varepsilon(\omega)$)
- Wavelength decreases (depends on $\varepsilon(\omega)$)
- Frequency dependence in $\varepsilon(\omega)$

$$\varepsilon(\omega) = 1 + \frac{i\sigma}{\varepsilon_0 \omega} = 1 + \frac{i\sigma_0}{\varepsilon_0 \omega (1 - i\omega\tau)}$$

$$\varepsilon(\omega) = 1 + \frac{i\omega_p^2 \tau}{\omega - i\omega^2 \tau}$$

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m} \quad \text{Plasma Frequency}$$

For $\omega\tau >>> 1$, $\varepsilon(\omega)$ goes to 1

<u>For an excellent conductor (σ_0 large), ignore 1, look at case for $\omega\tau << 1$</u>

$$\varepsilon(\omega) \approx \frac{i\omega_p^2 \tau}{\omega - i\omega^2 \tau} \approx \frac{i\omega_p^2 \tau}{\omega}$$

Waves in Materials

$$k = \frac{\omega}{c}\sqrt{\varepsilon(\omega)} = \frac{\omega}{c}\sqrt{i}\sqrt{\frac{\sigma_0}{\omega\varepsilon_0}}$$

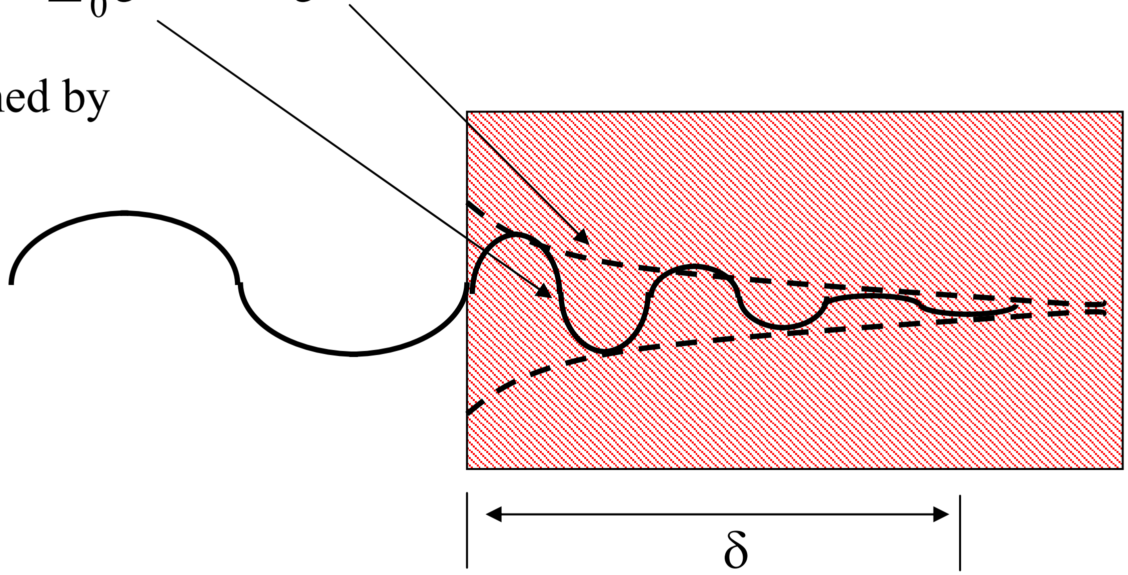
$$k = \frac{\omega}{c}\left(\frac{1+i}{\sqrt{2}}\right)\sqrt{\frac{\sigma_0}{\omega\varepsilon_0}} = \left(\sqrt{\frac{\sigma_0\omega}{2\varepsilon_0 c^2}} + i\sqrt{\frac{\sigma_0\omega}{2\varepsilon_0 c^2}}\right)$$

For a wave $E = E_0 e^{i(kz-\omega t)}$ Let $k = k_{real} + k_{imaginary} = k_r + ik_i$

$$E = E_0 e^{i[k_r z-\omega t]} e^{-|k_i| z}$$

The skin depth can be defined by

$$\delta = \frac{1}{|k_i|} = \sqrt{\frac{2\varepsilon_o c^2}{\sigma\omega_o}} = \sqrt{\frac{2}{\sigma_o\mu_o\omega}}$$



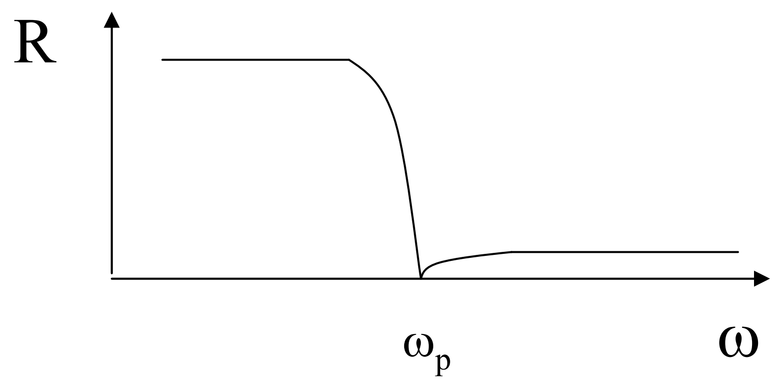
Waves in Materials

For a material with any σ_0, look at case for $\omega\tau >> 1$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$\omega < \omega_p$, ε is negative, $k=k_i$, wave reflected

$\omega > \omega_p$, ε is positive, $k=k_r$, wave propagates



Success and Failure of Free e- Picture

- Success
  - Metal conductivity
  - Hall effect valence=1
  - Skin Depth
  - Wiedmann-Franz law
- Examples of Failure
  - Insulators, Semiconductors
  - Hall effect valence>1
  - Thermoelectric effect
  - Colors of metals

K/σ=thermal conduct./electrical conduct.~CT

$$\mathrm{K} = \frac{1}{3} c_v v_{therm}^2 \tau$$

$$c_v = \left(\frac{\partial E}{\partial T}\right)_v = \frac{3}{2} n k_b ; v_{therm}^2 = \frac{3 k_b T}{m}$$

$$\mathrm{K} = \frac{1}{3}\left(\frac{3}{2} n k_b\right)\left(\frac{3 k_b T}{m}\right)\tau = \frac{3}{2}\frac{n k_b^2 T \tau}{m}$$

$$\sigma = \frac{n e^2 \tau}{m}$$

Therefore : $$\frac{\mathrm{K}}{\sigma} = \frac{3}{2}\left(\frac{k_b}{e}\right)^2 T$$

$\frac{3}{2}\left(\frac{k_b}{e}\right)^2$ ~C!

Luck: $c_{vreal} = c_{vclass}/100$; $v_{real}^2 = v_{class}^2 * 100$

Wiedmann-Franz 'Success'

	273K		373K	
Element	k (watt cm-K)	$k\,\sigma\,T$ (watt-ohm K^2)	k (watt cm-K)	$k\,\sigma\,T$ (watt-ohm K^2)
Li	0.71	2.22 x 10^8	0.73	2.43 x 10^8
Na	1.38	2.12		
K	1.0	2.23		
Rb	0.6	2.42		
Cu	3.85	2.20	3.82	2.29
Ag	4.18	2.31	4.17	2.38
Au	3.1	2.32	3.1	2.36
Be	2.3	2.36	1.7	2.42
Mg	1.5	2.14	1.5	2.25
Nb	0.52	2.90	0.54	2.78
Fe	0.80	2.61	0.73	2.88
Zn	1.13	2.28	1.1	2.30
Cd	1.0	2.49	1.0	
Al	2.38	2.14	2.30	2.19
In	0.88	2.58	0.80	2.60
Tl	0.5	2.75	0.45	2.75
Sn	0.64	2.48	0.60	2.54
Pb	0.38	2.64	0.35	2.53
Bi	0.09	3.53	0.08	3.35
Sb	0.18	2.57	0.17	2.69

Experimental thermal conductivities and Lorenz numbers of selected metals

Table by MIT OpenCourseWare.

Thermoelectric Effect

Exposed Failure when c_v and v^2 are not both in property

$$E = Q\nabla T$$

Thermopower Q is $Q = -\dfrac{c_v}{3ne} = \dfrac{-\frac{3}{2}nk_b}{3ne} = -\dfrac{nk_b}{2e}$

Thermopower is about 100 times too large!