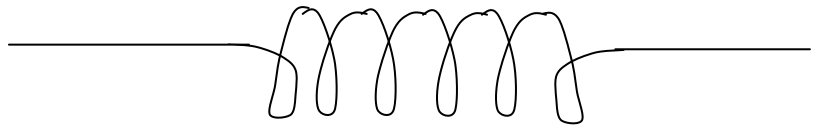
Magnetic Materials

- The inductor



$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \text{ (CGS)}$$

$$\int\int \nabla \times E dS = -\frac{1}{c}\frac{\partial}{\partial t}\left(\int\int B dS\right) = -\frac{1}{c}\frac{\partial \Phi_B}{\partial t}$$

$$\Phi_B \equiv \text{magnetic flux density}$$

$$\int\int \nabla \times E dS = \oint E \cdot d\ell \text{ (Green's Theorem)}$$

$$V = \int E \cdot d\ell = -\frac{1}{c}\frac{\partial \Phi_B}{\partial t} \text{ (explicit Faraday's Law)}$$

$$\Phi_B = LI \text{ (Q = CV)}$$

$$\frac{\partial \Phi_B}{\partial t} = L\frac{\partial I}{\partial t}$$

$$V_{EMF} = -\frac{\partial N\Phi_B}{\partial t} = -L\frac{\partial I}{\partial t}$$

$$V = L\frac{\partial I}{\partial t} \text{ (recall } I = C\frac{\partial V}{\partial t} \text{ for the capacitor)}$$

$$\text{Power} = VI = LI\frac{\partial I}{\partial t}$$

$$\text{Energy} = \int Power \cdot dt = \int LIdI = \frac{1}{2}LI^2 = \frac{1}{2}N\Phi_B I$$

$$\left(capacitor \ \frac{1}{2}CV^2\right)$$

The Inductor

$$\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c}\frac{\partial E}{\partial t}$$

$$\int\int \nabla \times B dS = \oint B \cdot d\ell = \frac{4\pi}{c}\int\int J \cdot dS = \frac{4\pi}{c} I$$

$$B = \frac{4\pi}{c} In$$

$$N = n \cdot length = nl$$

$$L = \frac{N\phi_B}{I} = \frac{N(BA)}{I} = \frac{4\pi}{c} n^2 lA$$

Insert magnetic material

Magnetic dipoles in material can line-up in magnetic field

$$B = H + 4\pi\chi H = H + 4\pi M$$

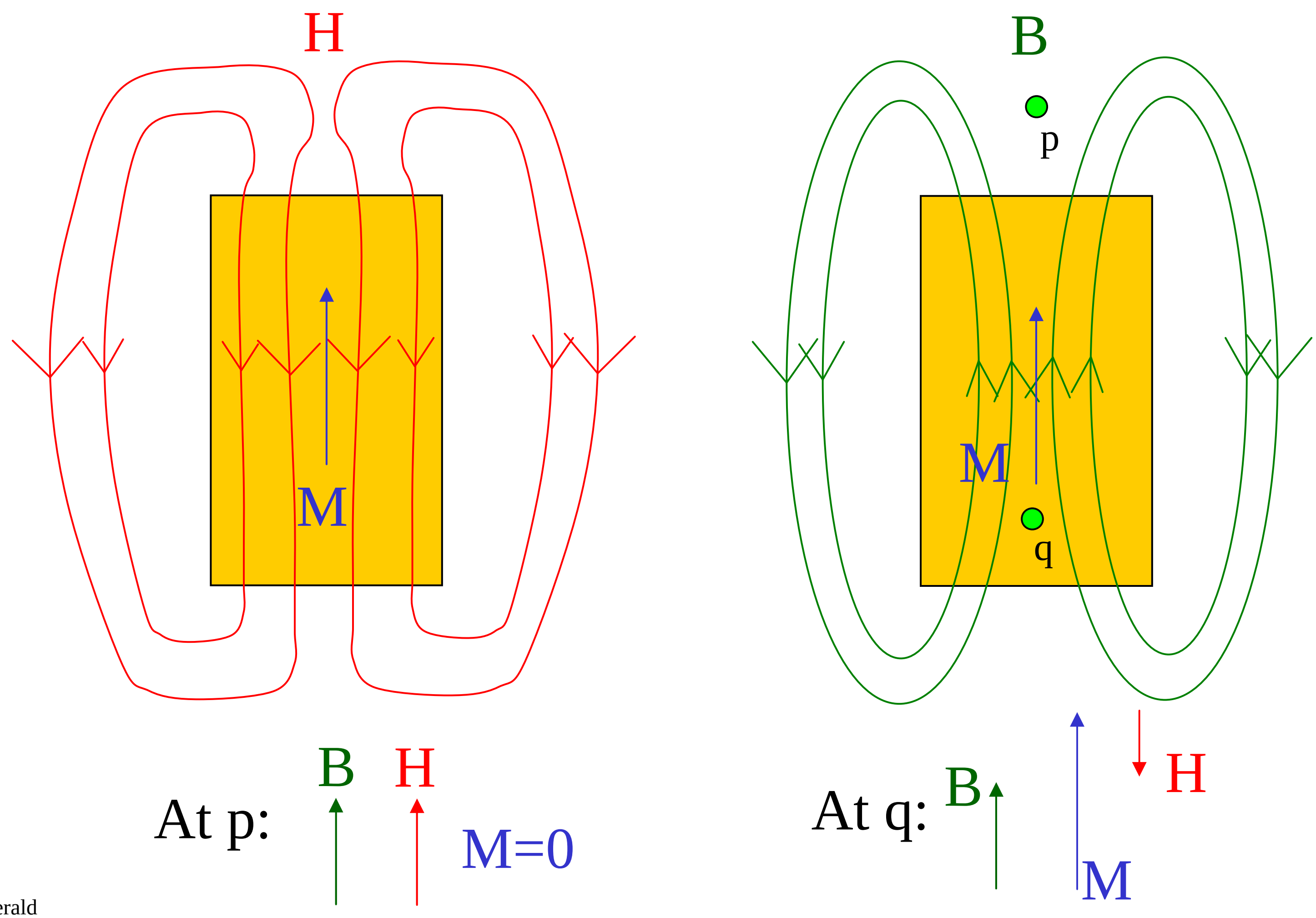
$$M = \chi H \quad \frac{\partial M}{\partial H} = \chi \quad \mu = 1 + 4\pi\chi$$

$$B = 4\pi M + 1 \quad B = \mu H$$

B magnetic induction
χ magnetic susceptibility
H magnetic field strength (applied field)
M magnetization

H and B

- H has the possibility of switching directions when leaving the material; B is always continuous



Maxwell and Magnetic Materials

- Ampere's law $\oint H \cdot d\ell = I = 0$
- For a permanent magnet, there is no real current flow; if we use B, there is a need for a fictitious current (magnetization current)
- Magnetic material inserted inside inductor increases inductance

$$\Phi_B = BA \sim 4\pi MA = 4\pi\chi HA = 4\pi\chi\left(\frac{4\pi}{c}In\right)A$$

$$L = \frac{N\Phi_B}{I} = \frac{(4\pi)^2}{c}n^2 lA\chi$$

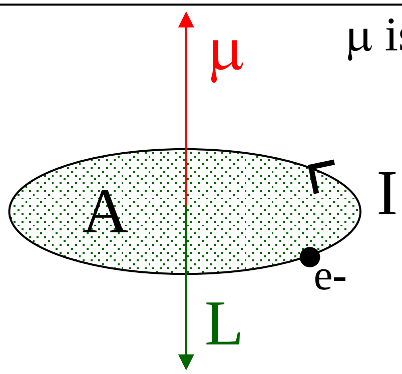
L increased by ~χ due to magnetic material

Material Type	χ
Paramagnetic	$+10^{-5}$-10^{-4}
Diamagnetic	-10^{-8}-10^{-5}
Ferromagnetic	$+10^{5}$

Microscopic Source of Magnetization

- No monopoles
- magnetic dipole comes from moving or spinning electrons

Orbital Angular Momentum



μ is the magnetic dipole moment

$$Energy = E = -\vec{\mu} \cdot \vec{H} = -|\mu||H|\cos\theta$$

What is μ? For θ=0, $E = -\mu H \approx -\Phi_B I$ since energy ~ LI^2 and for 1 loop $\mathrm{L} = \frac{\Phi_B}{I}$

$$\Phi_B = \iint H \cdot dS \sim HA$$

$$\therefore \mu H = \Phi_B I = HAI \quad \text{and} \quad \therefore \mu = IA$$

$$I = -\frac{e}{c}\frac{\omega}{2\pi} \quad A = \pi r^2$$

$$\mu = -\frac{e}{2c}\omega r^2$$

Microscopic Source of Magnetization

- Classical mechanics gives orbital angular momentum as:

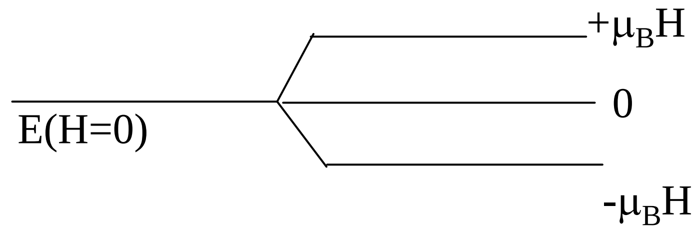
$$\vec{L} = \vec{r} \times \vec{p} = mr^2\omega$$

$$\mu_L = -\frac{e}{2mc} L_{QM} = -\frac{e\hbar}{2mc} L_Z = -\mu_B L_Z$$

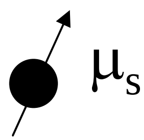
$$\left(\mu_B = \frac{e\hbar}{2mc} \right)$$

$$L_Z = m_\ell = -\ell, \ldots, 0, \ldots \ell$$

Example for l=1:

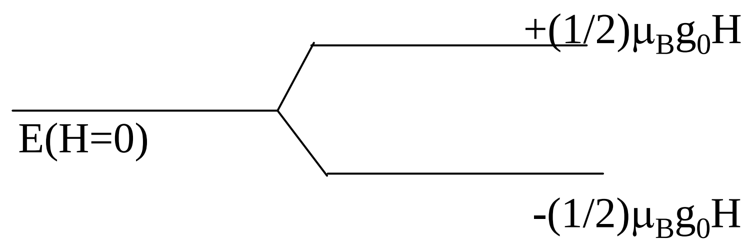


Spin Moment



$$\mu_s = -\frac{e}{mc} S \overset{Q.M.}{=} -g_0 \frac{e\hbar}{2mc} S_z = -g_0 \mu_B S_Z$$

$$S_Z = m_S = \pm\frac{1}{2} \quad g_0 = 2 \text{ for electron spin}$$



Total Energy Change for Bound Electron in Magnetic Field

- Simple addition of energies if spin-orbital coupling did *not* exist

$$E = -\vec{\mu} \cdot \vec{H} = \mu_B \left(L_Z + g_0 S_Z \right) H = \mu_T H$$

But spin-orbit coupling changes things such that:

$$\mu_T \neq \mu_B \left(L_Z + g_0 S_Z \right) = \mu_B J_Z$$

QM definitions:

$$L = \hbar L_Z = \hbar m_\ell$$

$$S = \hbar S_Z = \hbar m_s$$

$$J = L + S$$

$$J = \hbar J_Z$$

$$\mu_T = g \mu_B J_Z$$

$$g = \frac{3}{2} + \frac{1}{2} \left[\frac{S(S+1) - L(L+1)}{J(J+1)} \right]$$

$$E = -\vec{\mu}_T \cdot \vec{H} = g \mu_B \vec{J} \cdot \vec{H} = g \mu_B J_Z H$$

Total Energy Change for Bound Electron in Magnetic Field

- Kinetic energy from Lorentz force has not been included

$$p_H = -\frac{e}{2c}\underbrace{\vec{r}\times\vec{H}}$$ → Lorentz for circular orbit

$$\text{Energy change} = \frac{p^2}{2m} = \frac{e^2}{8mc^2}\left(\vec{r}\times\vec{H}\right)\cdot\left(\vec{r}\times\vec{H}\right)$$

For the plane perpendicular to H and assuming circular orbit:

$$\text{Energy change} = \frac{e^2}{8mc^2}r^2H^2 = \frac{e^2}{8mc^2}\left(x^2+y^2\right)H^2$$

$$\therefore \boxed{\Delta E_{TOT} = \underbrace{g\mu_B HJ_Z}_{} + \underbrace{\frac{e^2}{8mc^2}H^2\left(x^2+y^2\right)}_{}}$$

Numbers:

μ_BH for H=10^{-4} Gauss =10^{-4} eV	for H=10^{-4} Gauss =10^{-9} eV

The Lorentz effect is minimal with respect to magnetic moment interaction, if it exists

Atoms with Filled Shells

- J=0 (L=0, S=0)
- Only Lorentz contribution $\Delta E = \frac{e^2}{8mc^2} H^2 \left(x^2 + y^2\right)$
- Leads to *diamagnetism*

Need to sum over all e- in atom:

$$\Delta E_{atom} = \frac{e^2}{12mc^2} H^2 \frac{2}{3} \sum_i r_i^2 \quad \text{(for a spherical shells)}$$

$$\chi = -\frac{\mathrm{N}}{\mathrm{V}} \frac{\partial^2 E}{\partial H^2}$$

$$\left(M = -\frac{1}{V} \frac{\partial E}{\partial H} \qquad \chi = \frac{\partial M}{\partial H} \right)$$

$$\chi = -\frac{e^2}{6mc^2} \frac{N}{V} \sum_i r_i^2 = -\frac{e^2}{6mc^2} \frac{N}{V} \left\langle r^2 \right\rangle Z_i$$

$$\boxed{\chi = -\frac{e^2}{6mc^2} \frac{N}{V} \left\langle r^2 \right\rangle Z_i} \qquad \sim -10^{-5}$$

Atoms with Partially Filled Shells

- J not zero
- Need Hund's rules from QM
  - Fill levels with same ms to maximize spin
  - maximize L (first e- goes in largest l)
  - J=|L-S| for n<=(2l+1), J=|L+S| for n>(2l+1)
- Conventional notation: $^{(2S+1)}X_J$

L	0	1	2	3	4	5	6
X	S	P	D	F	G	H	I

- J=0 when L and S are not zero is a special case
  - 2nd order effect--> perturbation theory
- Partially filled shells give atoms *paramagnetic* behavior

$$\Delta E = g\mu_B H J_Z$$

($+10^{-2}$-10^{-3}>10^{-9} eV for diamagnetic component)

d-shell (l = 2)												
n	$l_x = 2,$	$1,$	$0,$	$-1,$	$-2,$	S	$L = \|\Sigma l_x\|$	J		*Symbol*	
1	↓					1/2	2	3/2		$^2D_{3/2}$	Sc
2	↓	↓				1	3	2		$^3F_2$	Ti
3	↓	↓	↓			3/2	3	3/2	$J=\|L-S\|$	$^4F_{3/2}$	V
4	↓	↓	↓	↓		2	2	0		$^5D_4$	Cr
5	↓	↓	↓	↓	↓	5/2	0	5/2		$^6S_{5/2}$	Mn
6	↓↑	↑	↑	↑	↑	2	2	4		$^5D_4$	Fe
7	↓↑	↓↑	↑	↑	↑	3/2	3	9/2	$J=L+S$	$^4F_{9/2}$	Co
8	↓↑	↓↑	↓↑	↑	↑	1	3	4		$^3F_4$	Ni
9	↓↑	↓↑	↓↑	↓↑	↑	1/2	2	5/2		$^2D_{5/2}$	Cu
10	↓↑	↓↑	↓↑	↓↑	↓↑	0	0	0		$^1S_4$	Zn

f-shell (l = 2)														
n	$l_z = 3,$	$2,$	$1,$	$0,$	$-1,$	$-2,$	$-2,$	S	$L = \|\Sigma l_z\|$	J			
1	↓							1/2	3	5/2		$^2F_{5/2}$	
2	↓	↓						1	5	4		$^3H_4$	Ce
3	↓	↓	↓					3/2	6	9/2	$J=\|L-S\|$	$^4I_{9/2}$	Pr
4	↓	↓	↓	↓				2	6	4		$^5I_4$	Nd
5	↓	↓	↓	↓	↓			5/2	5	5/2		$^6H_{5/1}$	Pm
6	↓	↓	↓	↓	↓	↓		3	3	0		$^7F_6$	Sm
7	↓	↓	↓	↓	↓	↓	↓	7/2	0	7/2		$^8S_{7/1}$	Eu
8	↓↑	↑	↑	↑	↑	↑	↑	3	3	6		$^7F_6$	Gd
9	↓↑	↓↑	↑	↑	↑	↑	↑	5/2	5	15/2		$^6H_{15/2}$	Tb
10	↓↑	↓↑	↓↑	↑	↑	↑	↑	2	6	8		$^5I_8$	Dy
11	↓↑	↓↑	↓↑	↓↑	↑	↑	↑	3/2	6	15/2	$J=L+S$	$^4I_{15/1}$	Ho
12	↓↑	↓↑	↓↑	↓↑	↓↑	↑	↑	1	5	6		$^3H_6$	Er
13	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	↑	1/2	3	7/2		$^2F_{7/2}$	Tm
14	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	0	0	0		$^1S_0$	Yb

Figure by MIT OpenCourseWare.

Temperature Dependence of Paramagnetism

- Temperature dependence determined by thermal energy vs. magnetic alignment energy (same derivation as for molecular polarizability in the case of electric dipoles)

$$f = e^{\frac{-U}{k_b T}} = e^{\frac{pE\cos\theta}{k_b T}}$$ for electric dipoles; $$f = e^{\frac{-U}{k_b T}} = e^{\frac{\mu H\cos\theta}{k_b T}}$$ for magnetic dipoles;

$$\overline{\mu}_Z = \frac{\int \mu_Z f d\Omega}{\int f d\Omega}$$

For low H fields and/or low T,

$$\overline{\mu}_Z = \frac{\mu^2 H}{3k_b T} = \frac{\mu_B^2 J^2 H}{3k_b T}$$

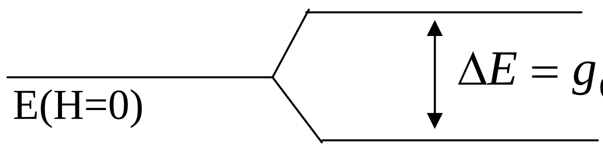
$$M = \frac{N}{V}\frac{\mu_B^2 J^2 H}{3k_b T}$$

$$\chi = \frac{N}{V}\frac{\mu_B^2 J^2}{3k_b T}$$

$$\chi_{QM} = \frac{1}{3}\frac{N}{V}\frac{\mu_B^2 g^2 J(J+1)}{k_b T} \qquad \chi \propto \frac{1}{T}$$ Curie's Law

Effect of De-localized electrons on Magnetic Properties

- Pauli Paramagnetism
  - dues to the reaction of free e- to magnetic field

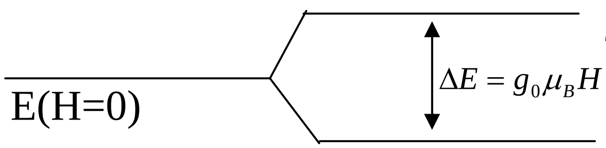


$$\Delta E = g_0 \mu_B H \qquad \mu = -g_0 \mu_B S$$

$$M = -\mu_B \left(n_+ - n_- \right)$$ (n[+] is the density of free electrons parallel to the H field)

$$g_+(E) = \frac{1}{2} g(E)$$

$$g_-(E) = \frac{1}{2} g(E)$$



$$\Delta E = g_0 \mu_B H$$

$$g_+(E) = \frac{1}{2} g\left(E + \mu_B H\right)$$

$$g_-(E) = \frac{1}{2} g\left(E - \mu_B H\right)$$

For exact solution, need to expand about E_f for n+ and n-

Only e- near Fermi surface matter:

$$\Delta E = g_0 \mu_B H$$

$$\left(n_+ - n_- \right) = \Delta n \approx g\left(E_F\right) \frac{\Delta E}{2}$$

$$\Delta n \approx g\left(E_F\right) \mu_B H$$

$$M = \mu_B^2 H g\left(E_F\right), \quad \chi = \mu_B^2 g\left(E_F\right)$$

Note: Pauli paramagnetism has no T dependence, whereas Curie paramagnetism has 1/T dependence

Effect of De-localized electrons on Magnetic Properties

- Landau paramagnetism
  - Effect of bands/Fermi surface on Pauli paramagnetism
  - F=qvxB for orbits
  - orbit not completed under normal circumstances
  - however, average effect is not zero

$$\chi_{Land} = -\frac{1}{3}\chi_{Pauli}$$

Ferromagnetism

- Most important but not common among elements
- Net magnetization exists without an applied magnetic field
- To get $\chi \sim 10^4$-10^5 as we see in ferromagnetism, most moments in material must be aligned!
- There must be a missing driving force

NOT dipole-dipole interaction: too small $E_{dipole} = \frac{1}{r^3}\left[\vec{\mu}_1 \cdot \vec{\mu}_2 - 3(\vec{\mu}_1 \cdot \hat{r})(\vec{\mu}_2 \cdot \hat{r})\right] \sim 10^{-4} eV$

Spin Hamiltonian and Exchange

$$H^{spin} = -\sum J_{ij} S_i S_j \qquad J_{ij} \equiv \text{exchange constant}$$

Assuming spin is dominating magnetization,

$$H = -\frac{1}{2}\sum_{\vec{R},\vec{R}'} \vec{S}(\vec{R}) \cdot \vec{S}(\vec{R}') J(\vec{R} - \vec{R}') - g\mu_B H \sum_{\vec{R}} S(\vec{R})$$

Exchange

$E \sim -JS_1S_2$

J negative, $E \sim +S_1S_2$--> Energy $\downarrow$ if $\downarrow \uparrow$

J positive, $E \sim -S_1S_2$--> Energy $\downarrow$ if $\uparrow \uparrow$

Fe, Ni, Co ---> J positive!

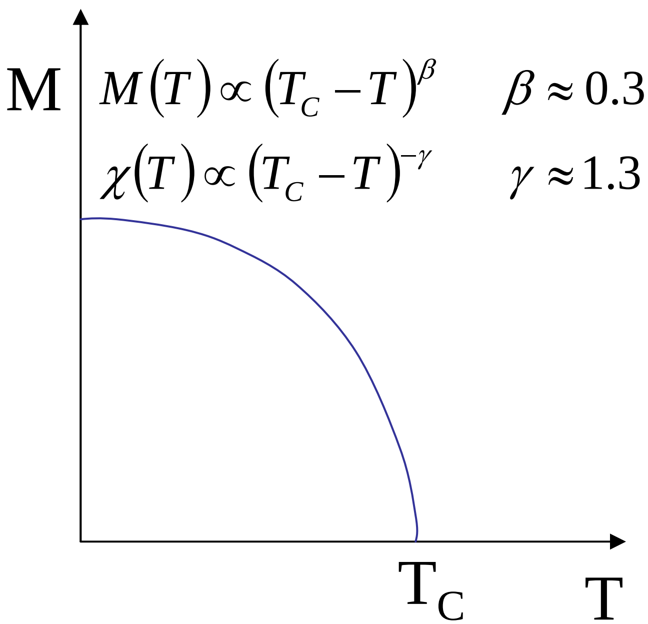
Other elements J is negative

Rule of Thumb:

$$\frac{r}{2r_a} \equiv \frac{\text{interatomic distance}}{2(\text{atomic radius})} > 1.5$$

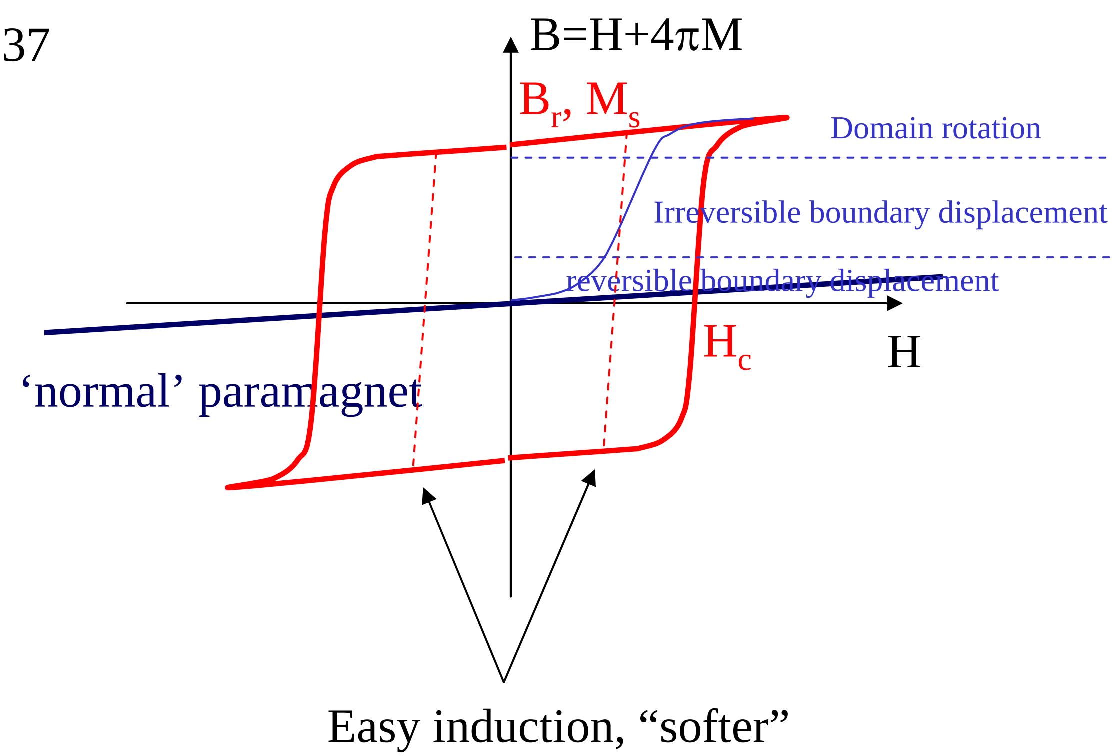
J is a function of distance!

Ferromagnetism



$$M(T) \propto (T_C - T)^{\beta} \qquad \beta \approx 0.33\text{-}0.37$$

$$\chi(T) \propto (T_C - T)^{-\gamma} \qquad \gamma \approx 1.3\text{-}1.4$$

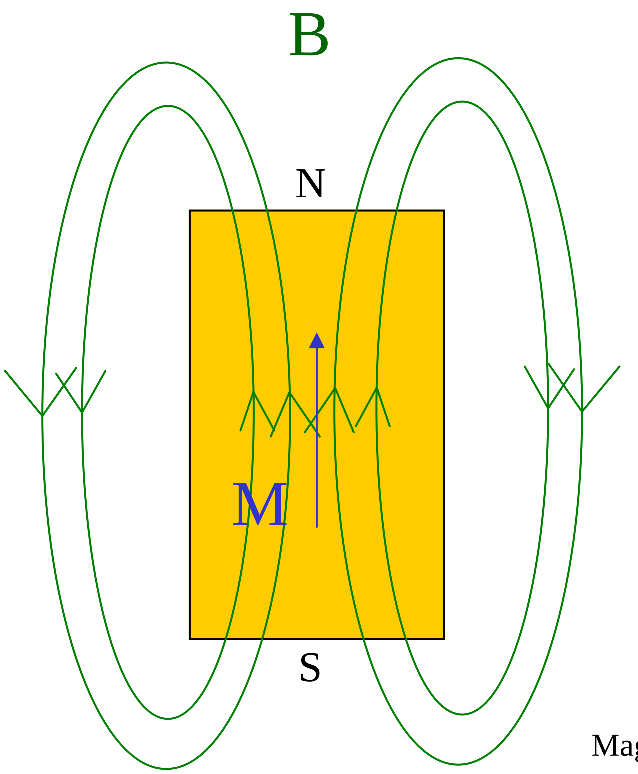


Magentic anisotropy

hardness of loop dependent on crystal direction

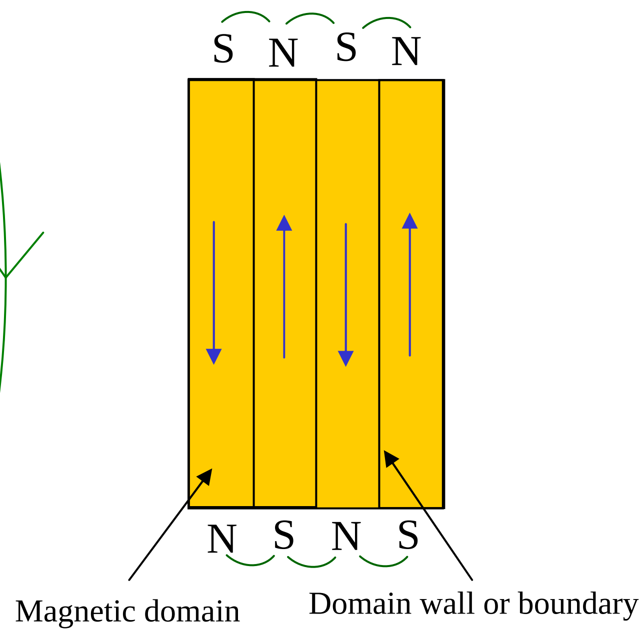
comes from spin interacting with bonding

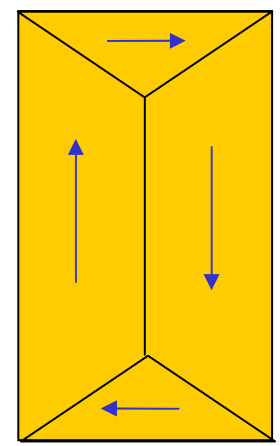
Domains in Ferromagnetic Materials



Magnetic energy

$$=\frac{1}{8}\int B^2 dV$$





Flux closure

No external field