3.225: Electrical and Mechanical Properties of Materials
Test - 1
Tuesday, October 23, 2007
2:30 PM – 4:00 PM
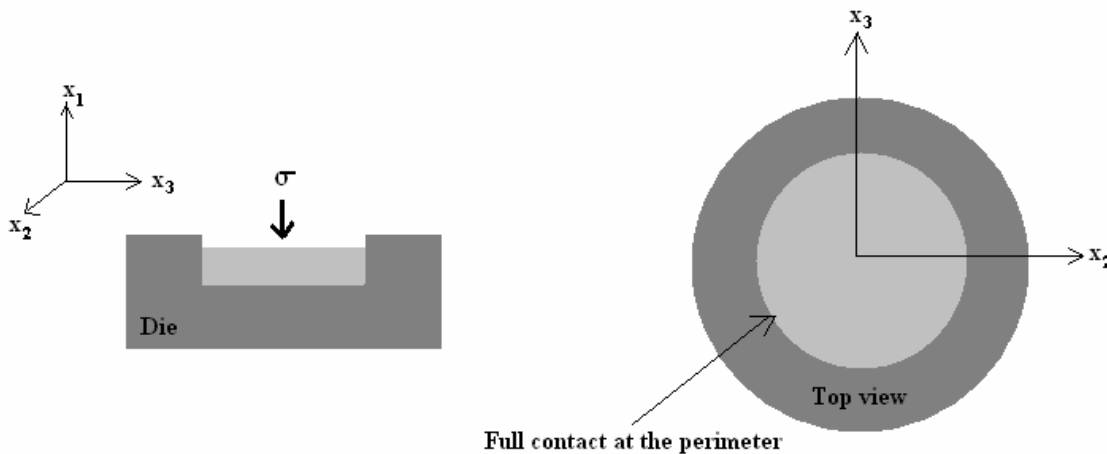
Problem 1.
(a) For an anisotropic material, prove that

     i.   compliance matrix is symmetric (i.e., $S_{ij} = S_{ji}$)
     ii. $v_{12}E_2 = v_{21}E_1$

(b) Derive the expressions of the elastic constant S_{11}, S_{22}, S_{33}, S_{12}, S_{13} and S_{23} in terms of the properties E_1, E_2, E_3, v_{21}, v_{31} and v_{32} for an anisotropic material.

(c) An anisotropic material is stamped out in a rigid cylindrical die as shown in the picture below. Assume the die walls are frictionless, and that the anisotropic material is in an elastic stress state.

Full contact at the perimeter

Applied normal stress is σ (i.e., $\sigma_1 = \sigma$). The elastic constants of the material give a hydrostatic stress state in the specimen. Prove that

     i. $\dfrac{E_2}{E_3} = \dfrac{1-v_{21}}{v_{32}}$

     ii.   At what applied stress, σ, the material would start yielding (use von Mises criterion)?
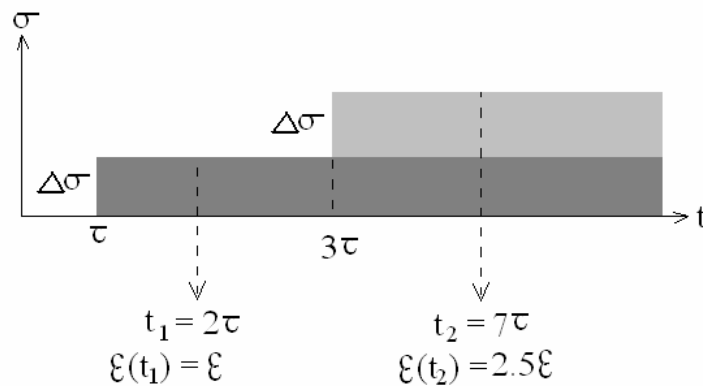
Problem 2.
A beam-like laminate with two isotropic layers of equal thickness is subjected to a temperature increase. The top layer has a Young's modulus E_1 and a coefficient of thermal expansion α_1. The bottom layer has a Young's modulus $E_2 = 0.2E_1$ and a coefficient of thermal expansion $\alpha_2 = 10\alpha_1$.

(a) Calculate the minimum and maximum stress in each layer in terms of the radius of curvature, ρ, E_1 and α_1.

(b) Sketch your result, showing the stress profile through the thickness of the laminate.

(c) Calculate the ratio of yield strengths of the two layers (i.e., σ_{y2}/σ_{y1}) in order for both the layers to yield at the same increase in temperature.

Problem 3.
(a) For an amorphous polymer, describe the changes with temperature in the relaxation modulus at a constant time ($E_r(t)$). Also plot E_r vs. T showing the changes in E_r for an amorphous polymer and a crystalline polymer on the same plot.

(b) The creep compliance, $J(t)$ of a polymer is represented as: $J(t) = (a\sqrt{t} + b)$. The applied stress is represented in figure below. The developed strains, $\varepsilon(t)$, at two times t_1 and t_2 are represented as below. Find out the constants 'a' and 'b'.

(c) What are the limitations of Maxwell and Voigt models?

Problem 4.
(a) Plot Tresca and von Mises yield boundaries in 2-dimensions. Write the equations for the boundaries. Assume that $\sigma_2 = 0$ and the axis labels are in terms of (σ_1/σ_y) and (σ_3/σ_y).

(b) For a material, the yield strength becomes 5/6 times that of the intrinsic lattice resistance at a temperature T and a strain rate $\dot{\gamma}$. Find the ratio of the plastic zone size of the same material, r_p, at two different temperatures T and 1.2T for the same strain rate, $\dot{\gamma}$. Assume that Q_b for the material is ≈ 60 KT. Explain ductile-brittle transition phenomena from this.

- Good luck ☺ -