3.225: Electrical and Mechanical Properties of Materials
Test 1
October 24, 2006

1. A single crystal (cubic) experiences the following stress state :

$$\begin{pmatrix} 13 & 2 & -3 \\ 2 & 19 & 7 \\ -3 & 7 & 24 \end{pmatrix} \text{MPa}$$

Determine the engineering strains and the strain matrix for the specimen given the following material constants:

$E_{11} = 45\ \text{GPa}$

$G_{12} = 20\ \text{GPa}$

$\nu_{12} = 0.43$

Also determine the strain energy density associated with this stress state.

Solution:

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{44} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}$$

$$S_{11} = \frac{1}{E_{11}} = \frac{1}{45\ \text{GPa}} = 2.222 * 10^{-11}\ \text{Pa}^{-1}$$

$$S_{12} = \frac{-\nu_{12}}{E_{11}} = \frac{0.43}{45\ \text{GPa}} = -9.555 * 10^{-12}\ \text{Pa}^{-1}$$

$$S_{44} = \frac{1}{G_{12}} = \frac{1}{20\ \text{GPa}} = 5.000 * 10^{-11}\ \text{Pa}^{-1}$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix} = \begin{pmatrix} 2.222*10^{-11} & -9.555*10^{-12} & -9.555*10^{-12} & 0 & 0 & 0 \\ -9.555*10^{-12} & 2.222*10^{-11} & -9.555*10^{-12} & 0 & 0 & 0 \\ -9.555*10^{-12} & -9.555*10^{-12} & 2.222*10^{-11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.000*10^{-11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.000*10^{-11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.000*10^{-11} \end{pmatrix} \begin{pmatrix} 13*10^6 \\ 19*10^6 \\ 24*10^6 \\ 7*10^6 \\ -3*10^6 \\ 2*10^6 \end{pmatrix}$$

The engineering strains are:

$$\begin{pmatrix} -1.220 * 10^{-4} \\ 6.865 * 10^{-5} \\ 2.275 * 10^{-4} \\ 3.500 * 10^{-4} \\ -1.500 * 10^{-4} \\ 1.000 * 10^{-4} \end{pmatrix}$$

The strain matrix is:

$$\begin{pmatrix} -1.220 * 10^{-4} & 5.000 * 10^{-5} & -7.500 * 10^{-5} \\ 5.000 * 10^{-5} & 6.865 * 10^{-5} & 1.750 * 10^{-4} \\ -7.500 * 10^{-5} & 1.750 * 10^{-4} & 2.275 * 10^{-4} \end{pmatrix}$$

The strain energy per unit volume is given by:

$$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \sum_{i=1}^{6} \sigma_i \epsilon_i = 4.139 \text{ KJ.m}^{-3}$$

2.(a) The stress history for a polyethylene (cross-section: 0.2" * 0.5") is given in the following table:

Load (lbs)	Duration (hrs)
20	200
10	50
40	100
5	100
50	50

Calculate the strain at 150 hours and 425 hours given the following creep compliance data for polyethylene:

Time (hrs)	J(t) $\left(\frac{10^{-4}}{\text{psi}}\right)$
0	0.5
100	0.55
200	0.57
300	0.60
400	0.65
500	0.67

Calculate the strain at 150 hours and 425 hours.

Solution:

$$\Delta\epsilon\ (t) = \Delta\sigma\ (\tau_1).J\ (t - \tau_1) + \Delta\sigma\ (\tau_2).J\ (t - \tau_2) + \ldots.\ t \geqslant \tau_1,\ \tau_2 \ldots.$$

For t=150 hours:

$$\epsilon\ (150\ hrs) = \Delta\sigma\ (0).J\ (150 - 0) = \frac{20}{(0.5)\ (0.2)}\ [0.56 * 10^{-4}] = 0.0112$$

For t = 425 hours:

$$\epsilon\ (425) = \Delta\sigma\ (0).J\ (425 - 0) + \Delta\sigma\ (200).J\ (425 - 200) + \Delta\sigma\ (250).J\ (425 - 250) + \Delta\sigma\ (350).J\ (425 - 350)$$

$$\epsilon\ (425) = \Delta\sigma\ (0).J\ (425) + \Delta\sigma\ (200).J\ (225) + \Delta\sigma\ (250).J\ (175) + \Delta\sigma\ (350).J\ (75)$$

$$\epsilon\ (425) =$$

$$\frac{20}{(0.5)\ (0.2)}\ [0.655 * 10^{-4}] + \frac{-10}{(0.5)\ (0.2)}\ [0.578 * 10^{-4}] + \frac{30}{(0.5)\ (0.2)}\ [0.565 * 10^{-4}] +$$

$$\frac{-35}{(0.5)\ (0.2)}\ [0.538 * 10^{-4}] = 5.44 * 10^{-3}$$

2.(b) Plot relaxation modulus v/s time for an amorphous polymer and explain the concept of shift factor.

Solution:

Draw plot showing glassy, viscoelastic and Rubbery regimes. The plot of Relaxation Modulus v/s time for different temperatures gives a family of similar curves. Viscoelastic behavior at one temperature can be found from that at another by a shift in the time scale. The shift factor thus gives the displacement of the curve for different temperatures and is given by:

$$\text{Log}[a_T] = \text{Log}\left[\frac{t_T}{t_{T_0}}\right] = \frac{c_1\ (T - T_0)}{c_2 + T - T_0}$$

3.(a) When loads that act on the hub of a flywheel reach their working values, the nonzero stress components at the critical point in the hub where yield is initiated are:

$\sigma_{xx} = 100$ Mpa

$\sigma_{yy} = -14.0$ Mpa

$\sigma_{xy} = 50.0$ Mpa

The flywheel material has a yield stress Y = 300 Mpa

1. Assuming the material is a Tresca material, determine the factor of safety against yield.

2. Assuming the material is a Von Mises material, determine the factor of safety against yield.
3. Determine which criterion is more conservative.

Solution:

Tresca:

The calculated values of the principal stresses are:

$$\det\left[\begin{pmatrix} 100-\sigma & 50 \\ 50 & -14-\sigma \end{pmatrix}\right] = 0$$

$\sigma_1 = 118.8$ MPa

$\sigma_2 = -32.8$ MPa

According to the Tresca criteria:

$$\frac{Y}{2} = \frac{SF\,(\sigma_1 - \sigma_2)}{2}$$

Therefore:

SF = 1.98

Von Mises:

According to the Von Mises Criteria:

$$Y = \frac{SF}{2}\left[118.8^2 + 32.8^2 + (-32.8 - 118.8)^2\right]^{0.5}$$

SF = 2.17

Clearly the Tresca criteria is more *conservative* than the Von Mises.

3(b) The intrinsic lattice resistance at 0 K is given by:

$$\tau_0 = \frac{4E}{3Z}$$

Derive this equation.

Solution:

See classnotes.

3(c) How does dispersion hardening differ from solid solution hardening?

Solution:

Dispersion Hardening derives most of its strength from the energy stored in the elongation of the dislocation line bowing around impenetrable obstacles/dispersions. Solid solution hardening derives its strength from the lattice dilation due to the introduction of solute atoms resulting in an interaction with an interaction with the stress fields of preexisting dislocations. Dispersion hardening is typically much stronger than solid solution hardening.

4.(a) Explain Griffith's theory for crack propagation.

Solution:

Griffith observed that cracks cause stress concentrations but they cannot be allowed for by calculation of a linear elastic stress concentration K. According to the previous theory the stress concentration factor around a crack would go to infinity. This is ofcourse not observed in practice so he tried to explain how a stable crack could exist in a material. He postulated that a crack only becomes unstable if an increment of crack growth results in more strain energy being released than can be absorbed by the formation of the new surface (due to the crack). *Show simplified derivation.*

4.(b) A center-crack test specimen is loaded in tension. The width of the specimen is 35mm, the thickness is 6.5mm and the edge notch length is 7.5mm. The fracture toughness of the PMMA is 0.6 MPa.m^(1/2) and the yield strength is 80 MPa. Take the geometric factor Y=1.0

(i) What stress is required to break the specimen?
(ii) What is the plane strain plastic zone size for this material just prior to fracture?
(iii) Does the specimen meet the requirements of a plane strain fracture toughness test?

Solution:

(i)

$$\sigma_{break} = \frac{K_{IC}}{1.0\sqrt{\pi a}} = \sqrt{\frac{0.6*10^6}{\pi*3.75*10^{-3}}} = 5.527\ \text{MPa}$$

(ii)

$$r_p = \frac{1}{3\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2 = \frac{1}{3\pi}\left(\frac{0.6*10^6}{80*10^6}\right)^2 = 5.968\ \mu\text{m}$$

(iii) We require that:

$$B > 25\, r_p$$

6.5mm>25(5.968)=149.2 μm(OK)