LJG

3.225 Electronic and Mechanical Properties of Materials
Professor Lorna Gibson
Test 1: Elasticity, Viscoelasticity and Plasticity
October 21, 2003

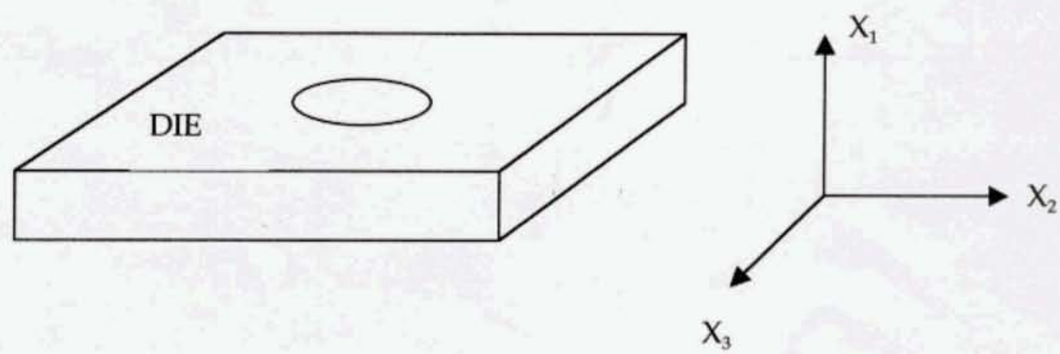
1. (a) Prove the reciprocal relation:

$$\nu_{12}E_2 = \nu_{21}E_1$$

taking the Poisson's ratio as:

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$ under a uniaxial stress in the x_1 direction

(b) A rigid die is designed to take a cylindrical specimen. A transversely isotropic specimen is placed into the die such that the isotropic plane (X_2-X_3) is normal to the axial direction of the cylinder, X_1. The specimen is loaded in the axial direction (X_1) and remains elastic throughout the loading. The elastic constants of the material of the specimen give a hydrostatic stress state in the specimen. How are the elastic constants E_1, E_2, ν_{12} and ν_{32} related?



(c) What gives rise to the linear relationship between stress and strain for crystalline materials at small strains?

8

2. The relaxation modulus of polymethylmethacrylate (PMMA, T_g = 100C) is given in the figure below as a function of time at a temperature of 115C. Plot the relaxation modulus as a function of time for a temperature of 120C. on the same plot. Don't forget to hand in this page with your solutions.

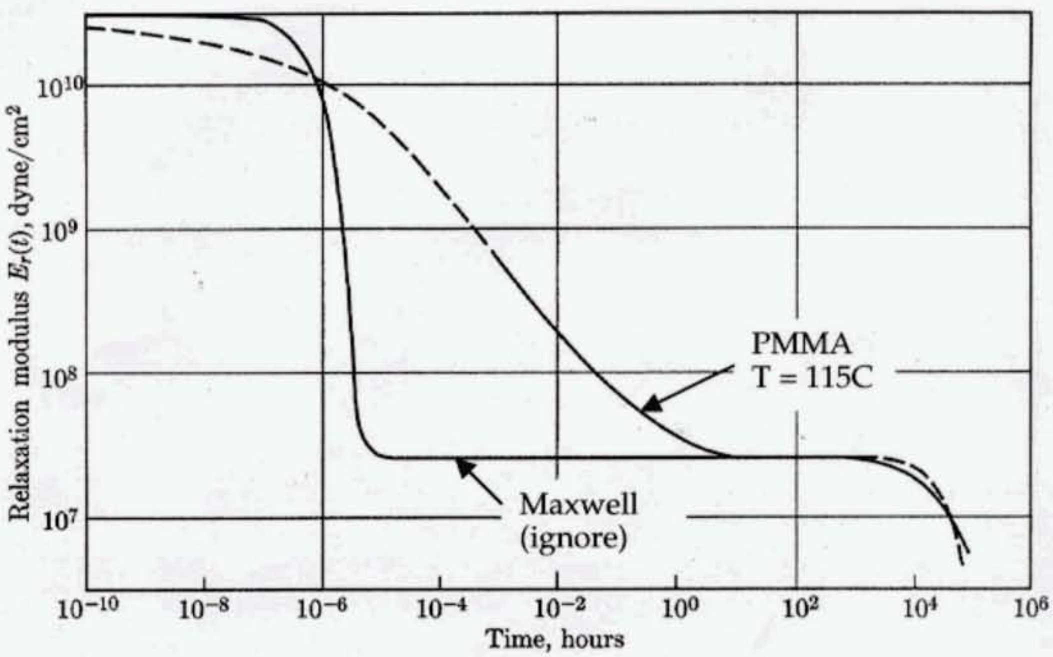


Fig. 6.25. Comparison of double Maxwell model with relaxation behavior of PMMA at 115°C, from Fig. 6.24.

shift twice

115 → 100

100 → 120

3 3. (a) A wire drawing die is used to reduce the diameter of a wire from 0.2" to 0.18". The wire is fed into the die (at the 0.2" diameter end) at a rate of 1" per minute. What is the rate at which the wire exits the die? $\Delta V = 0$

4 (b) A circular bilayer plate has a thin film of one material, with a Young's modulus of E_1, a yield strength σ_{y1} and a coefficient of thermal expansion of α_1, on a thick substrate (with properties E_2, σ_{y2} and α_2) is heated by ΔT above the temperature at which the bilayer is stress-free. Find the ΔT to initiate yielding in the thin film using the von Mises criterion. $\epsilon = (\alpha_2 - \alpha_1)\Delta T$ $\sigma = \frac{E}{1-\nu}(\alpha_2-\alpha_1)\Delta T$

3 (c) Given the stress field around a screw dislocation is $\sigma_{\theta z} = \frac{Gb}{2\pi r}$, calculate the elastic strain energy per unit length of dislocation line.

deriv in class

3.225 TEST.

1. (a) Prove $\nu_{12} E_2 = \nu_{21} E_1$

$S_{12} = S_{21}$

apply σ_1 $\quad \epsilon_1 = S_{11} \sigma_1$

$\epsilon_2 = S_{21} \sigma_1$

$$\nu_{12} = -\frac{\epsilon_2}{\epsilon_1} = \frac{-S_{21}}{S_{11}}$$

$$S_{21} = \frac{-\nu_{12}}{E_1}$$

apply σ_2 $\quad \epsilon_2 = S_{22} \sigma_2$

$\epsilon_1 = S_{12} \sigma_2$

$$\nu_{21} = -\frac{\epsilon_1}{\epsilon_2} = -\frac{S_{12}}{S_{22}}$$

$$S_{12} = -\frac{\nu_{21}}{E_2}$$

$$S_{12} = S_{21}$$

$$\therefore \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$\nu_{12} E_2 = \nu_{21} E_1$$

1(b) RIGID DIE $\Rightarrow$ $\epsilon_2 = \epsilon_3 = 0$; $\sigma_2 = \sigma_3$

$$\epsilon_2 = S_{21}\sigma_1 + S_{22}\sigma_2 + S_{23}\sigma_3 = 0.$$

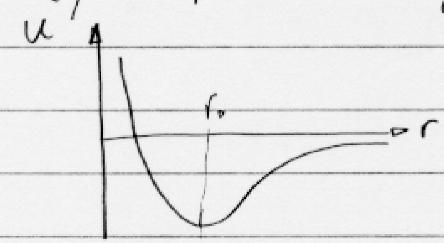
$$-\frac{\nu_{12}}{E_1}\sigma_1 + \frac{1}{E_2}\sigma_2 - \frac{\nu_{32}}{E_2}\sigma_2 = 0$$

$$\therefore \quad \sigma_1 \frac{\nu_{12}}{E_1} = \frac{\sigma_2}{E_2}(1-\nu_{32})$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}\frac{(1-\nu_{32})}{\nu_{12}}$$

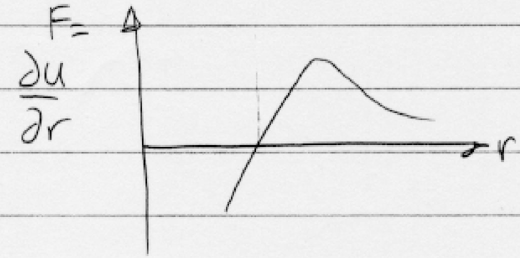
$\Rightarrow \sigma_1 = \sigma_2$ $(= \sigma_3)$ FOR $E_1(1-\nu_{32}) = E_2\nu_{12}$.

(c) energy - separation diagram $u(r)$



minimum energy @ $r = r_0$

force - separation diagram



at small strains $F \propto r$

$\Rightarrow \sigma \propto \epsilon$

$\Rightarrow$ Hooke's Law.

\# 2. PMMA $T_g = 100°C$

given $E_r(t)$ @ $T = 115°C$

$E_r(t)$ @ $T = 120°C$ $= ?$

shift factor: $\log a_T = \dfrac{C_1 (T - T_g)}{C_2 + T - T_g} = \log\left(\dfrac{t_T}{t_{T_g}}\right)$

$C_1 = -17.44$ $\quad C_2 = 51.6$

shift from $T = 115°C$ to $T_g = 100°C$

$= 388°K$ $\quad = 373°C$

$$\log\left(\frac{t_{115}}{t_{T_g}}\right) = \frac{-17.44\,(388 - 373)}{51.6 + 388 - 373} = \frac{-261.6}{66.6} = -3.93$$

$$\frac{t_{115}}{t_{T_g}} = 10^{-3.93} = 1.17 \times 10^{-4}$$

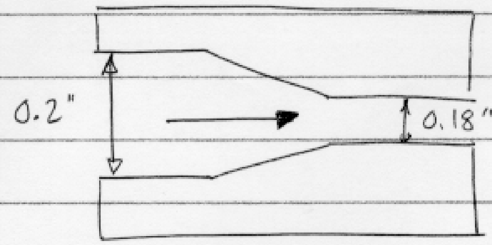
$$t_{115} = 1.17\times10^{-4} \times t_{T_g} \qquad t_{T_g} = \frac{t_{115}}{(1.17\times10^{-4})}$$

$$\log\left(\frac{t_{120}}{t_{T_g}}\right) = \frac{-17.44\cdot(393 - 373)}{51.6 + 393 - 373} = \frac{-348.8}{71.6} = -4.87$$

$$t_{120} = 1.34\times10^{-5}\, t_{T_g} = (1.34\times10^{-5})\,\frac{t_{115}}{(1.17\times10^{-4})}$$

$$\boxed{t_{120} = 0.11\; t_{115}}$$

#3(a)



$\vec{V}_{in} = 1''/\text{min}$ $\quad\quad$ $\vec{V}_{out} = ?$

VOLUME = CONSTANT

IN TIME Δt, A LENGTH Δx_{in} $(= V_{in}\,\Delta t)$ MOVES ALONG LHS.
THIS IS EQUIVALENT TO A VOLUME $\Delta x_{in} \dfrac{\pi (0.2)^2}{4}$.

IN SAME TIME Δt, A LENGTH Δx_{out} $(= V_{out}\,\Delta t)$ MOVES ALONG RHS. THIS IS EQUIVALENT TO A VOLUME $\Delta x_{out} \dfrac{\pi (0.18)^2}{4}$

$\because$ VOLUME = CONSTANT

$$\Delta x_{in} \frac{\cancel{\pi} (0.2)^2}{\cancel{4}} = \Delta x_{out} \frac{\cancel{\pi} (0.18)^2}{\cancel{4}}$$

$$V_{in} \cancel{\Delta t} (0.2)^2 = V_{out} \cancel{\Delta t} (0.18)^2$$

$$V_{out} = \left(\frac{1''}{\text{min}}\right)\left(\frac{0.2}{0.18}\right)^2 = 1.23 \text{ in/min.}$$

3b. BILAYER PLATE ; ΔT

$$\sigma_1 = \sigma_2 = \sigma \qquad (\sigma_3 = 0)$$

VON MISES : $\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]} = \sigma_y$

$$\sigma_e = \sqrt{\frac{1}{2}\left[(0) + \sigma^2 + \sigma^2\right]} = \sigma = \sigma_y$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}\sigma_2 = \frac{\sigma}{E}(1-\nu) = (\alpha_2 - \alpha_1)\,\Delta T$$

$$\sigma = (\alpha_2 - \alpha_1)\,\Delta T\,\frac{E}{1-\nu} = \sigma_y$$

$$\Delta T = \frac{\sigma_y}{E}\,\frac{(1-\nu)}{(\alpha_2 - \alpha_1)}$$

3(c) $\sigma_{\theta z} = \frac{Gb}{2\pi r}$ FOR SCREW DISLOCATION.

$\frac{u^{el}}{L} = ?$ $du^{el} = \frac{1}{2}\frac{\sigma_{\theta z}^2}{G} dV$

$dV = 2\pi r L\, dr.$

$$u^{el} = \int_{r_0}^{R} \frac{1}{\cancel{2}} \frac{\sigma_{\theta z}^2}{G} \cancel{2}\pi r L\, dr.$$

$$\frac{u^{el}}{L} = \int_{r_0}^{R} \frac{G^{\cancel{2}} b^2}{4\pi^{\cancel{2}} r^{\cancel{2}}} \frac{1}{\cancel{G}} \cancel{\pi}\cancel{r}\, dr$$

$$= \frac{Gb^2}{4\pi} \int_{r_0}^{R} \frac{dr}{r}$$

$$= \frac{Gb^2}{4\pi} \ln\left(R_0/r\right)$$

$$= \alpha G b^2. \qquad \alpha \sim 0.5 \text{ to } 1.0.$$