2003

Given: $e, m_0, c, \varepsilon_0, k_b, h, \hbar, a, Z_{Na}^{+}, Z_{Cl}^{-}, n\ (n^2 = \varepsilon_\infty), \varepsilon_r = \varepsilon_s$

PROBLEM# 1

(a)

$$n = \frac{4\text{ Na atoms}\times Z_{Na} + 4\text{ Cl atoms}\times Z_{Cl}}{a^3} = \frac{4\times 1 + 4\times 7}{a^3} = \frac{32}{a^3} = 2.56\times 10^{29}$$

$$\sigma = \frac{ne^2\tau}{m_0} = \frac{32e^2\tau}{a^3 m_0} \rightarrow \text{calculate using } \tau = 10^{-14}s$$

(b)

$$v_{max} = v_{th} = \sqrt{\frac{3k_b T}{m_0}} \rightarrow \text{calculate at different T}$$

$$\text{if no field applied then: } v_{avg} = \langle v_{th} \rangle = 0 \rightarrow \text{as summed over all directions}$$

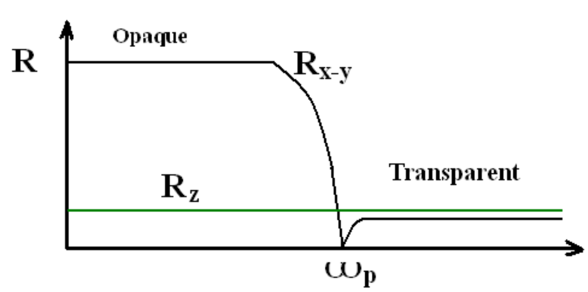
$$\text{if field, E applied then: } v_{avg} = v_{Drift} = \frac{e\tau}{m_0}E$$

(c)

In the plane of the NaCl material (x and y directions) we'll have ω_p concept:

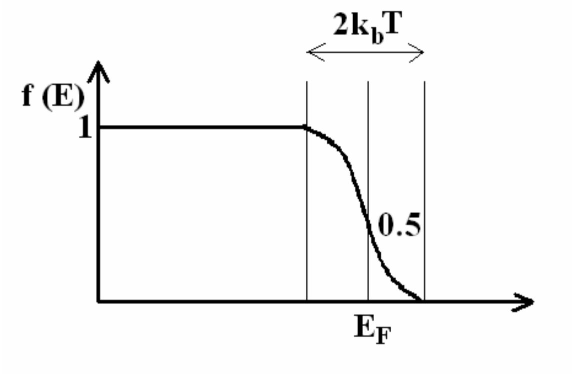
$$\omega_p^{x\text{-}y} = \frac{ne^2}{m_0\varepsilon_0} = \frac{32e^2}{a^3 m_0 \varepsilon_0}$$

However, in the direction perpendicular to the plane (z-direction), L = a & therefore light doesn't see much electrons density in this direction. Hence light will pass through z-direction no mater what the frequency is.



PROBLEM# 2

(a)



Number of electrons per volume in fermi surface those contribute to the electronic conduction are only at ($E_F \pm k_b T$), let say we represent that by n_F

$$n_F = g(E_F)\times f(E_F)\times 2k_bT = k_bT\times g(E_F) = \frac{0.7k_bTm^*}{\hbar^2 a} = 4.68\times10^{26}$$

$$\sigma = \frac{n_F e^2 \tau_F}{m^*} = \frac{k_b e^2}{m^*}\times g(E_F)\times\tau_F T$$

To calculate τ_F,

$$k_F^{\,3\text{-}d} = (3\pi^2 n)^{1/3} = \frac{6.71}{a}, n = \frac{32}{a^3} \Rightarrow g^{3\text{-}d}(E_F) = \frac{m^* k_F}{\pi^2\hbar^2} = g(E_F) = \frac{0.7m^*}{\hbar^2 a}$$

Plug-in these values into σ and equate it to $\frac{ne^2\tau}{m_0}$

$\tau_F = 540\ \tau$

(b)

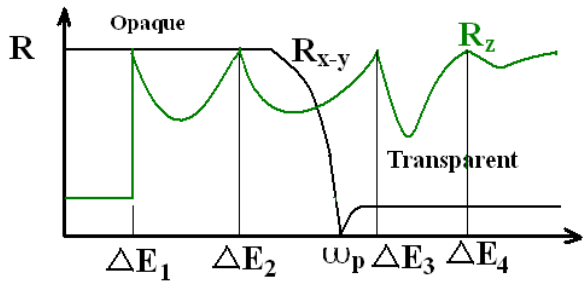
$$v_{max} = v_F = \frac{\hbar k_F}{m^*} = \frac{6.71\hbar}{am^*},\ E_F = \frac{\hbar^2 k_F^{\,2}}{2m^*} = \frac{22.5\hbar^2}{m^* a^2}$$

$$\text{at } T = 0\text{K},\ v_{avg} = \langle v\rangle = \frac{\int_0^{E_F} v\times g(E)\,dE}{\int_0^{E_F} g(E)\times dE},\ g(E)\sim\sqrt{E},\ v\sim\sqrt{E}$$

$$\text{at } T \neq 0\text{K},\ v_{avg} = \langle v\rangle = \frac{\int_0^{E_F-k_bT} v\times g(E)\,dE}{\int_0^{E_F-k_bT} g(E)\,dE} + \frac{\int_{E_F-k_bT}^{E_F+k_bT} v\times g(E)\times f(E)\,dE}{\int_{E_F-k_bT}^{E_F+k_bT} g(E)\times f(E)\,dE}$$

(c)
Here again calculate n_F as described above and calculate ω_p using n_F (instead of n as used in problem# 1). In x-y plane, we'll have ω_p concept. However, in z-direction, energy gets quantized (as $2\pi/L$ & $L = a$ here); therefore lights with right frequencies would get absorbed (if $h\nu = \Delta E$)

$$\omega_p^{x\text{-}y} = \frac{n_F e^2}{m_0 \varepsilon_0}$$



PROBLEM# 3

(a)
-Discussed in recitation in detail

Hint: In [100] & [010] directions it is quasi continuous as 2π/L is very small. In the [001] direction it is discontinuous due to very large 2π/L (as L = a).

(b)
It is descriptive. Large U → in all crystal directions means insulators (as e^- can't jump)

(c)
Again descriptive…give values of n_i ($\sim 10^{20}$), U for a good semiconductor to make the answer impressive. Usually U ~ 1 - 1.5 eV.

4
Calculate the polarizabilities at low frequencies (much before the critical frequency ω_T)
Given: $\varepsilon_s, \varepsilon_\infty, \varepsilon_0, a$

$$N = \text{Number of diapoles per volume} = \frac{4}{a^3}$$

$$\frac{\varepsilon_s - 1}{\varepsilon_s + 2} = \frac{4(\alpha_{ionic} + \alpha_{electronic})}{3\varepsilon_0 a^3} \rightarrow [1]$$

$$\frac{\varepsilon_\infty - 1}{\varepsilon_\infty + 2} = \frac{4(\alpha_{electronic})}{3\varepsilon_0 a^3} \rightarrow [2]$$

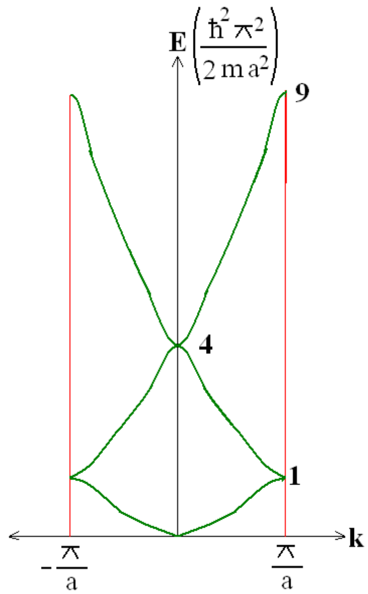
Solve equations [1] and [2] to obtain $\alpha_{electronic}$ and α_{ionic}.

2004

Given: e, m_0, c, ε_0, k_b, h, ħ, A

PROBLEM# 1

(a)



(b)

$$k_F^{\,1-d} = \frac{n\pi}{2}, n = \frac{Z}{a} \Rightarrow k_F^{\,1-d} = \frac{Z\pi}{2a}$$

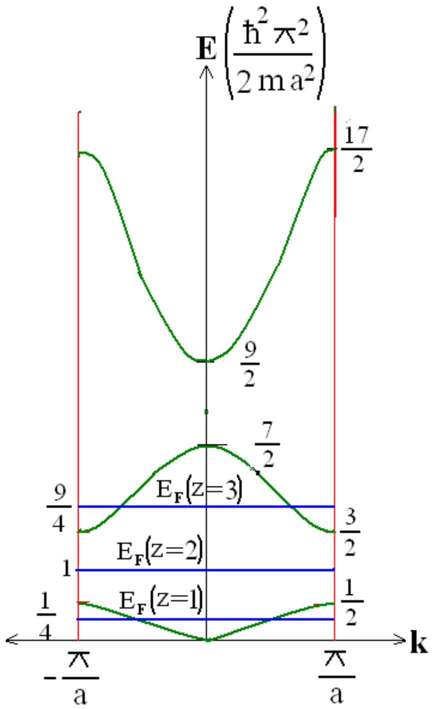
$$E_F^{\,1-d} = \frac{\hbar^2 k_F^{\,2}}{2m} = \frac{\pi^2\hbar^2}{2ma^2}\frac{Z^2}{4}$$

(c)

$$\sigma = \frac{k_b e^2}{m^*} \times g(E_F) \times \tau_F T,\ g^{1\text{-d}}(E_F) = \frac{4ma}{Z\hbar^2} \Rightarrow \sigma = \frac{4k_b e^2 a \tau_F T}{Z\hbar^2}$$

Use $\tau_F \sim 10^{-14}$s

(d)



(e)
Z = 2 (semimetal to semiconductor)

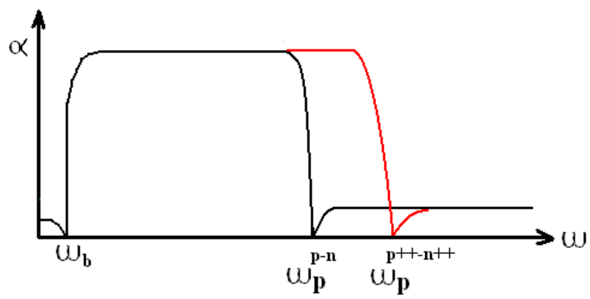
(f)
Discussed in the last recitation (energy gets quantized here)

PROBLEM# 2

(a)
Zn → p type → R_H +ve
Si → n type → R_H –ve

(b)
p++-n++ is heavily doped compared to the p-n diode. Absorption coefficient "α" defines how much light gets absorbed by the material ($I = I_0 \exp(-\alpha x)$). Photocurrent is generated in a diode when light gets absorbed. So in a way, absorption coefficient and photocurrent are analogues. This is also similar to plotting R vs w (reflectivity vs. frequency). There are few critical frequencies & few important points:

- Heavily doped material has higher carrier concentration → ω_p increases
- At ω_b (or 2πν) where hν > E_g → light starts getting absorbed → transparent to opaque: $\omega_b = \frac{E_g}{\hbar}$
- At ω_p opaque to transparent



PROBLEM# 3

(a)

Given: $\varepsilon_s, \varepsilon_\infty, \varepsilon_0, \rho,$ At.wt. of Si and O atoms, Avogadro # (A)

$\alpha_{electronic} = \alpha_{Si^{+4}} + 2\alpha_{O^{-2}}$

$$N = \text{Number of } SiO_2 \text{ molecules per volume} = \frac{\text{At.Wt. Si} + \text{At.Wt. O}}{\rho \times A}$$

$$\frac{\varepsilon_s - 1}{\varepsilon_s + 2} = \frac{N\left(2\alpha_{ionic} + \alpha_{electronic}\right)}{3\varepsilon_0} \rightarrow [1]$$

$$\frac{\varepsilon_\infty - 1}{\varepsilon_\infty + 2} = \frac{N\left(\alpha_{electronic}\right)}{3\varepsilon_0} \rightarrow [2]$$

Calculate $\alpha_{electronic}$ and α_{ionic} from these two equations.

(b)

Sketch ε_r vs. w with the critical frequencies ω_T and ω_{oe} (in class notes)

2005

Given: e, m_0, c, ε_0, k_b, h, ħ, A

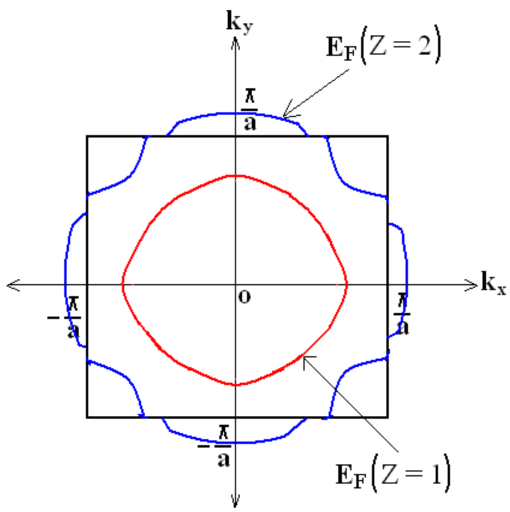
PROBLEM# 1

(a)

$$k_F^{2-d} = \sqrt{2n\pi}, n = \frac{Z}{a^2} \Rightarrow k_F^{2-d} = \frac{\sqrt{2\pi Z}}{a}$$

$$E_F^{2-d} = \frac{\hbar^2 k_F^2}{2m} = Z\frac{\pi\hbar^2}{ma^2}$$

(b)



(c)

Z = 1: Behaves as a metal

At T = 0K, $\sigma = ne^2\tau/m$

At T > 0K, $\sigma = \frac{k_b e^2}{m^*} \times g(E_F) \times \tau_F T$, $g(E_F) = \frac{m^*}{\pi\hbar^2}$ =constant

Z = 2: Behaves as a semimetal

At T = 0K, $\sigma << ne^2\tau/m$ (poor metal)

At T > 0K, $\sigma = \frac{k_b e^2}{m^*} \times g(E_F) \times \tau_F T$, $g(E_F) = \frac{m^*}{\pi\hbar^2}$ (same as Z =1)

(d)

$$k = \sqrt{2\pi n},\ n = \frac{Z}{a^2}$$

Fermi surface $\Rightarrow\ k^2 \ \le\ \dfrac{2\pi Z}{a^2}$

$$\Rightarrow\ k_x^{\ 2} + k_y^{\ 2}\ \le\ \frac{2\pi Z}{a^2}$$

Devide by $\left(\dfrac{2\pi}{L}\right)^2$, $(L = 8a)\ \Rightarrow\ \left(\dfrac{\pi}{4a}\right)^2$

$$\Rightarrow\ x^2 + y^2\ \le\ \frac{32Z}{\pi}$$

Where (x, y) is the coordinate of different energy states on the Fermi surface.

PROBLEM# 2

(a)

$$\sigma = \frac{ne^2\tau}{m_0}, n \sim 10^{22}\,cm^{-3}, \tau \sim 10^{-14}\,s$$

$$l = v_{th} \times \tau = \sqrt{\frac{3k_bT}{m_0}} \times \tau, \omega_p = \frac{ne^2}{m_0\varepsilon_0}$$

Draw R vs. λ (=2πc/ω)

(b)

a_{Al} ~ 0.4 nm, compare it to calculated *l*.

Use $\tau = \dfrac{1}{v_{th}\sigma N}$ (σ is scattering cross section and N is scatterers/volume). σ ~ a (lattice parameter). Calculate N for $\tau = 10^{-15}$ s. Describe if that kind of density (which I assume is pretty high) is possible physically or not.

(c)

$\sigma = \dfrac{ne^2\tau}{m_0}, n \sim 10^{22}\,cm^{-3}, \tau \sim 10^{-14}\,s$ => Similar to Al σ

However, semiconductors don't behave like metals due to band gaps…so wouldn't match with experiment.

PROBLEM# 3

(a)
Not sure

(b)
Use equation for width of the depletion region. Instead of V_{bi}, use ($V_{bi} + V_R$).

PROBLEM# 4

n > 1 due to bound charges
Transparent because band gaps in all directions with large U.

2006

Given: e, m_0, c, ε_0, k_b, h, ħ, A

PROBLEM# 1

(a)

$$J=\sigma E, v_D=\mu E, \sigma=\frac{Ze^2\tau}{a^3 m_0}$$

Find σ from the J-E plot & find μ from the v_D-E plot. Determine τ (=mμ/e). Calculate Z

(b)

J changes (as σ = neμ and n is changing here), but v_D remains the same

PROBLEM# 2

Descriptive (follow the lecture notes)

PROBLEM# 3

(a)

Find out ω_b (=2πν) where hν = E_g. If the yellow light ω range (3.15×10^{15} -3.27×10^{15} Hz) falls before ω_b, we can increase ω_p without any limit. Therefore the electrode can be a n type semiconductor. Not that a higher n is better than a higher p as the electron mobilities are 10 times higher than the hole mobilities.

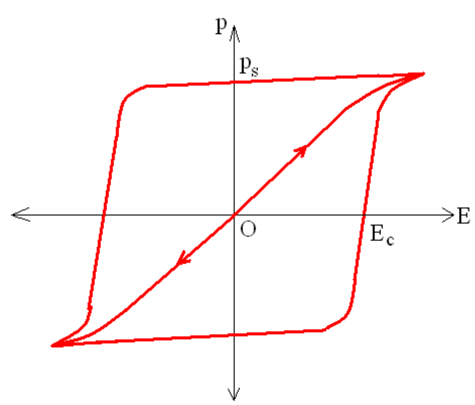
If the yellow light ω range falls after ω_b, then equate ω_p (= $ne^2/m_0\varepsilon$) to the minimum yellow light ω and find n. We can find n_i at RT (=298 K) from the given data. If n > n_i → n-type or else p-type. So dope accordingly if n- or p-type. Example: Si for n-type and Zn for p-type.

(b)

Descriptive

PROBLEM# 4

(a)



(b)

Descrptive