Phase Transformations: Growth Phenomena

Today’s topics:

- **Diffusion-controlled vs. interface-controlled growth**
- **Diffusion-controlled growth: The Stefan Problem**
  - **melting of a pure material**
  - **interdiffusion with a moving boundary**
  - **alloy solidification**

Diffusion vs. interface control

- **In precipitation from supersaturated solution, precipitate growth requires long-range transport of solute to the growing particle and often particle growth kinetics can be modeled by solving a diffusion problem. The growth is said to be diffusion controlled.**

  This is a type of moving-boundary problem; diffusional growth is often parabolic in time.

Diffusion vs. interface control, cont'd

- **In processes like grain growth that do not involve a composition change but only interface motion, the boundary migration kinetics involve local atomic rearrangements as atoms jump from one grain to its neighbor. Such a process is said to be interface controlled.**

  Often, interface-controlled motion occurs at constant velocity and it is found to be linear in time.

Melting of a pure material

- **Melting is controlled by the rate of heat flow. We consider the motion of a planar solid/liquid interface via heat conduction:**

See Figure 20.1 in Balluffi, Robert W., Samuel M. Allen, and W. Craig Carter. *Kinetics of Materials*. Hoboken, NJ: J. Wiley & Sons, 2005. ISBN: 0471246891.

Want to model

•*T* profiles in liquid, solid

•Equation of motion of solid/liquid interface

Melting of a pure material, cont'd

- **The description:**

$$\frac{\partial T^L}{\partial t} = \kappa^L \frac{\partial^2 T^L}{\partial x^2}$$

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See Figure 20.1 in Balluffi, Robert W., Samuel M. Allen, and W. Craig Carter. *Kinetics of Materials*. Hoboken, NJ: J. Wiley & Sons, 2005. ISBN: 0471246891.

$$\frac{\partial T^S}{\partial t} = \kappa^S \frac{\partial^2 T^S}{\partial x^2}$$

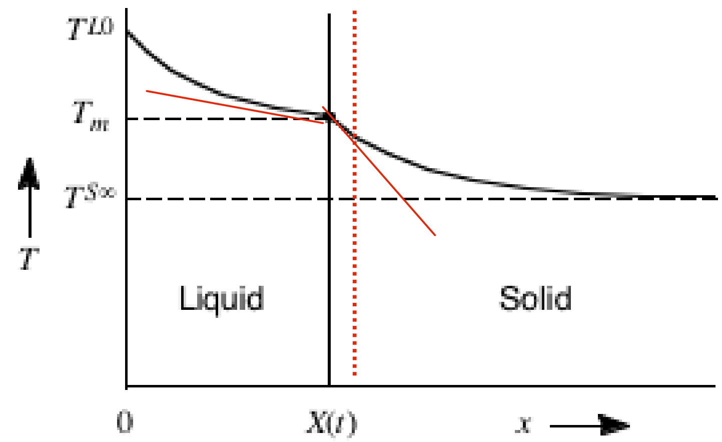
$$T^L(x=0,t) = T^{L0} \qquad T^L[X(t),t] = T_m$$

$$T^S(x=\infty,t) = T^{S\infty} \qquad T^S[X(t),t] = T_m$$

Equilibrium at S/L interface

Melting of a pure material, cont'd

Heat balance at S/L interface: difference in heat fluxes must be balanced by release of latent heat associated with melting.



$$\left(J^L - J^S\right)_{x=X} \delta t = -K^L \left(\frac{\partial T^L}{\partial x}\right)_{x=X} \delta t + K^S \left(\frac{\partial T^S}{\partial x}\right)_{x=X} \delta t = \rho H_m \delta X$$

$$\frac{dX}{dt} \rho H_m = -K^L \left(\frac{\partial T^L}{\partial x}\right)_{x=X} + K^S \left(\frac{\partial T^S}{\partial x}\right)_{x=X}$$

← **Stefan condition**

Melting of a pure material, cont'd

The solution gives a parabolic growth law

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$$X(t) = A\sqrt{t}$$

where

$$A = \frac{c_P^L \sqrt{4\kappa^L}\left(T^{L0} - T_m\right)\exp\left[-A^2/4\kappa^L\right]}{\sqrt{\pi}\rho H_m \operatorname{erf}\left(\frac{A}{\sqrt{4\kappa^L}}\right)} - \frac{c_P^S \sqrt{4\kappa^S}\left(T_m - T^{S\infty}\right)\exp\left[-A^2/4\kappa^S\right]}{\sqrt{\pi}\rho H_m \operatorname{erfc}\left(\frac{A}{\sqrt{4\kappa^S}}\right)}$$

T **profiles in S and L are error functions… (see *KoM* Eqs. 20.8–20.9).**

Planar growth controlled by diffusion

The solution method is analogous to that for heat-conduction controlled growth…

$$X = A\sqrt{t}$$

And the growth constant, *A*, is a function of the various c's, etc.

Solidification of a binary alloy in 1-D

A liquid alloy of concentration c_0 will reject solute ahead of the advancing S/L interface (note solute-rich boundary layer in liquid).

Modeling requires descriptions of both temperature and concentration fields…