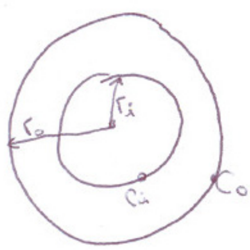
Problem Set 8 Solutions

Problem 1



$$\vec{J}(r) = -D\nabla c$$

The system is at steady state, therefore $\vec{J}(r) \cdot r =$constant.

$$\frac{dc}{dt} = 0 = \nabla \cdot \vec{J}$$

In cylindrical coordinates, $D = D(c)$, $C = C(r)$.
In 1-D:

$$\int 0 dr = \int \frac{-d}{dr}\left(rD\frac{dc}{dr}\right)dr$$

$$B = rD\frac{dc}{dr} \Rightarrow \frac{B}{D}\ln r = c - A$$

$$C = A + B\ln r \text{Form of concentration profile}$$

Use Boundary Conditions to determine A and B.

$$C_u = A + B\ln r_i \qquad C_o = A + B\ln r_o$$

$$C_o - C_i = B\ln r_o - B\ln r_i = B\ln\frac{r_o}{r_i}$$

$$B = \frac{C_o - C_i}{\ln\frac{r_o}{r_i}}$$

$$C_i = A + \frac{C_o - C_i}{\ln\frac{r_o}{r_i}}\ln r_i$$

$$A = C_i - \frac{C_o - C_i}{\ln\frac{r_o}{r_i}}\ln r_i$$

$$C = C_i - \frac{C_o - C_i}{\ln \frac{r_o}{r_i}} \ln r_i + \frac{C_o - C_i}{\ln \frac{r_o}{r_i}} \ln r$$

$$\frac{C - C_i}{C_o - C_i} = \frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}$$

b.

$$C = A + B \ln r \text{from part (a)}$$

This means the graph of concentration vs. -log r should be a straight line. The curve shows a non-linear profile, so data does *not* support assumption.

$D_{Carbon in Fe} \approx 5\text{cm}^2/\text{sec}$ @ $1000°C$.

Problem 2

a.

$$C = A + B\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right)$$

$$C(0,t) = 10^18\mathrm{atoms/cm}^3 = A$$

$$C(\alpha,t) = 0 = A + B \Rightarrow B = -10^18\mathrm{atoms/cm}^3$$

$$C = 10^18 - 10^18\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right)$$

After 30 minutes

$$C = 10^16 atoms/cm^3 = 10^18 - 10^18\mathrm{erf}\left(\frac{x}{\sqrt{4(10^{-11})(1800)}}\right)$$

$$\mathrm{erf}[-] = 0.99$$

$$\frac{x}{\sqrt{4(10^{-11})(1800)}} = 1.83$$

$$x \approx 4.9 \times 10^{-4} cm$$

b.

$$\begin{aligned}\int_0^{1800} J(x=0,t)dt &= \int_0^{1800} (C_s - C_c)\sqrt{\frac{D}{\pi t}}dt \\ &= \int_0^{1800} (10^{18})\sqrt{\frac{10^{-11}}{\pi}}\left(\frac{1}{t^{\frac{1}{2}}}\right)dt \\ &= 1.78 \times 10^{12}[2t^{\frac{1}{2}}]_0^{1800} = 1.5 \times 10^{14}\mathrm{atoms/cm}^2\end{aligned}$$

3. 2.6 from Porter, David A., and K. E. Easterling. *Phase Transformations in Metals and Alloys*. 2nd ed. New York, NY: Chapman & Hall, 1992. ISBN: 0412450305.

4. Thin Surface Layer (10^{-6}cm)

$$N_i = \text{Li concentration} = 10^{20}\text{atoms/cm}^3$$

$$t = \ ? \text{ at 1000K and } N_2 = 10^{19}\text{atoms/cm}^3$$

$$D_{Li@1000K} = 5 \times 10^{-12}\text{m}^2/\text{s} = 5 \times 10^{-8}\text{cm}^2/\text{s}$$

$$c(x,t) = \frac{N}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)$$

$$c(x,t) = \frac{N}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)$$

$$c(x,0) = 10^2 0\text{atoms/cm}^3$$

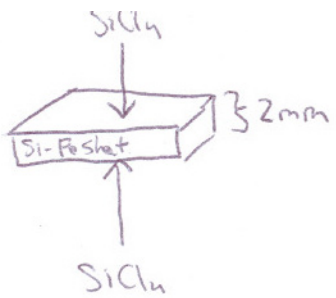
$$\text{Surface Concentration} = 10^2 0\text{atoms/cm}^3 \times 10^- 6\text{cm} = 10^1 4\text{atoms/cm}^2$$

$$\text{At surface,} x = 0 \Rightarrow c(x,t) = \frac{N}{\sqrt{4\pi Dt}}$$

$$10^{19}\text{atoms/cm}^3 = \frac{10^{14}\text{atoms/cm}^2}{\sqrt{4\pi(5 \times 10^{-8}\text{cm}^2/\text{s})t}} = 10^{19}\text{atoms/cm}^3 \times 7.927 \times 10^{-4} t^{\frac{1}{2}} = 10^{14}\text{atoms/cm}^2$$

$$t = 1.6 \times 10^{-4}\text{seconds}$$

Problem 5



$$D = 1.5 \times 10^{-12} m^2/s \text{ at } T = 1255K$$

Gassing Plate:

$$c(x,t) = \frac{4c_o}{\pi} \sin\frac{\pi x}{L} \exp\left(\frac{-\pi^2 Dt}{L^2}\right)$$

$$c_o = 0.03$$

$$c(x,t) = c(\frac{L}{2}, t) = c(1 \text{ mm}, t) = 0.025$$

$$c(1 \text{ mm}, t) = \frac{4(0.03)}{\pi} \sin\left(\frac{\pi(0.001\text{m})}{0.002\text{m}}\right) \exp\left(\frac{-\pi^2(1.5 \times 10^{-12} m^2/s)}{(0.002\text{m})^2} t\right)$$

$$0.6545 = \exp(-3.7 \times 10^{-6})$$

$$-0.4239 = -3.7 \times 10^{-6} t$$

$$t = 1.145 \times 10^5 \text{sec} \Rightarrow \approx 32 \text{ hours.}$$

Problem 6

2-D Lattice

$$a = 0.5\text{mm}$$

$$101 \times 101 = N = \Gamma_v = 10000\text{s}^{-1}$$

a.

$$D = \frac{\Gamma < r^2 >}{2d} f, \text{ random walk: } f = 1$$

$$D = \frac{10^4\text{s}^{-1}(0.5 \times 10^{-3})^2}{4} = 6.25 \times 10^{-4}\text{m}^2/\text{s}$$

$$x \approx \sqrt{4Dt}$$

$$[50.5 \times 0.5 \times 10^{-3}\text{m}] = \sqrt{4(6.25 \times 10^{-4})t}$$

$$\frac{6.376 \times 10^{-4}}{4(6.25 \times 10^{-4})} = t = 0.26\text{sec}$$

b.

$$\Gamma_{blue} = \Gamma_v X_v$$

$$X_v = \frac{1}{101^2} = \frac{1}{10201}$$

$$f \approx \frac{2-1}{2+1} = \frac{4-1}{4+1} = 0.6$$

$$\begin{aligned} D_{blue} &= \frac{\Gamma_{blue} < r^2 > f}{2d} = \frac{X_v \Gamma_v < r^2 > f}{4} = X_v D_v f \\ &= \left(\frac{1}{10201}\right)\left(6.25 \times 10^{-4}\frac{\text{m}^2}{s}\right)(0.6) = 3.67 \times 10^{-8}\frac{\text{m}^2}{s} \end{aligned}$$

$$t = 4336 \text{ sec}$$