3.185 Problem Set 8

Drag Force, Advanced Heat and Mass Transfer

Due Friday December 5, 2003

1. Settling of magnesia particles in water (17)

   Small and relatively uniform magnesium oxide particles can be made by precipitation from a supersaturated aqueous solution. Here we will study how quickly they sink through stagnant solution so they can be collected from the bottom of the container.

   (a) Derive an expression for the terminal rising/sinking velocity of a sphere in a fluid, starting from the friction factor definition for flow past a sphere, the weight and buoyancy force, and the relation between f and Re in Stokes flow. (7)

   (b) Consider a uniform suspension of approximate spheres of magnesia ($\rho = 3850\frac{\text{kg}}{\text{m}^3}$) in water. If the water is 10 cm deep, above what diameter do all of the spheres reach the bottom within one minute? (That is, above what diameter is the terminal velocity at least 10 cm/minute?) (7)

   (c) Check the Reynolds numbers at your critical velocities in parts 1b to ensure Stokes flow is a valid assumption. (3)

2. Drag force on a flat plate (18)

   A flat plate measuring 1m $\times \frac{1}{4}$m is held in the air such that two edges (the "sides") are parallel to the wind, and two (the leading and trailing edges) are perpendicular to it.

   Data: air $\rho \simeq 1.9\frac{\text{kg}}{\text{m}^3}$, $\eta \simeq 10^{-5}\frac{\text{N}\cdot\text{s}}{\text{m}^2}$.

   (a) Based on what you know about drag on a flat plate, is the drag higher when the longer (1m) edges are parallel to the wind, or perpendicular to it? (5)

   (b) For the lower-drag case in part 2a, calculate the total drag force on the plate (*i.e.* on both sides of it) for a (very low) wind velocity of $1\frac{\text{cm}}{\text{sec}}$. (6)

   (c) Calculate the drag force for the same plate in a $10\frac{m}{s}$ wind. (7)

3. Turbulence and Mixing in a Tube (36)

   (a) Sketch the (time-smoothed) velocity profiles for laminar and turbulent flow in a tube. (4)

   (b) Show that the pressure drop (pressure difference between the entrance and exit) required to drive water flow at a rate of 0.001 m^3/s through a tube 30 m long and 1 cm (0.01 m) diameter with inner surface roughness of approximately 10 microns (10^{-5} m) is approximately 5.3 MPa ($5.3 \times 10^6\frac{\text{N}}{\text{m}^2}$). (8)

   (c) Using the Hägen-Poiseuille equation, estimate the effective turbulent viscosity, that is, the viscosity which would give the pressure drop from part 3b for this flow rate if flow were laminar. (5)

   (d) A (water-soluble) macromoleculular substance is introduced into the flow near the entrance of the tube. Using a turbulent Prandtl number of one, estimate the turbulent diffusivity of the macromolecule from the turbulent viscosity in part 3c. How much time does it take to achieve uniform mixing of the substance across the tube? (5)

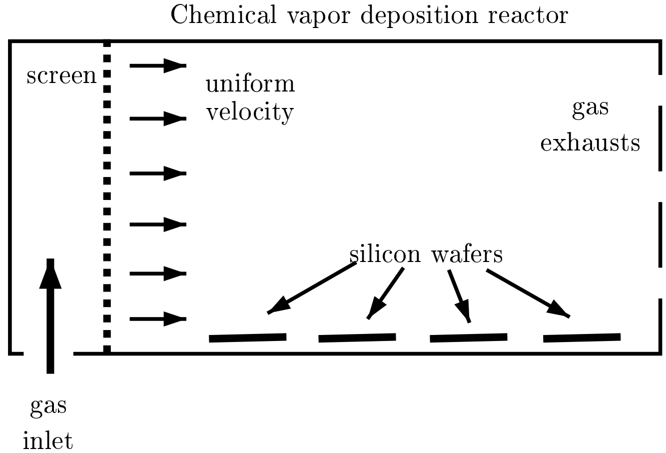
(e) Given that power required to drive this flow is flow rate times pressure drop (power = $Q\Delta P$), estimate the average energy dissipation rate per unit volume in the tube under conditions described in part 3b. (4)

(f) Estimate the turbulent microscale, the size of the smallest eddies, in this tube. (5)

(g) If the diffusivity of the macromoleculular substance in water is $D = 10^{-6}\text{cm}^2/\text{s}$ ($= 10^{-10}\text{m}^2/\text{s}$), estimate the time required for total homogenization of the mixture. (5)

Data: water density $\rho = 1000\frac{\text{kg}}{\text{m}^3}$, viscosity $\eta = 0.001\frac{\text{kg}}{\text{m}\cdot\text{s}}$.

4. Chemical Vapor Deposition (29)



A CVD reactor, pictured above, pases a dilute mixture of silane gas (SiH_4) in argon over heated substrates to deposit a layer of silicon. The screen at the entrance distributes the flow evenly over the entrance, so the velocity profile there is uniform. Assume the deposition is diffusion-limited, so the equilibrium SiH_4 concentration at the substrates is zero, and ignore exit conditions and natural convection instabilities (due to the placement of hot substrates below the cooler gas).

Data:

- chamber dimensions: 0.5m high × 2m wide × 2m long
- argon viscosity $\eta = 3 \times 10^{-5}\frac{\text{N}\cdot\text{s}}{\text{m}^2}$
- silane diffusivity in argon $D_{\text{SiH}_4} = 2 \times 10^{-4}\frac{\text{m}^2}{\text{s}}$
- silicon density $\rho_{\text{Si}} = 2500\frac{\text{kg}}{\text{m}^3}$

(a) Calculate the mass transfer prandtl number in the (mostly argon) gas at an operating temperature of 500K and pressure of 0.1 atm (you may need the ideal gas law for the argon density). (3)

(b) Given a flow rate of $0.2\frac{\text{m}^3}{\text{s}}$, calculate the Reynolds number Re_H. Is the flow likely to be laminar or turbulent? (4)

(c) Sketch the velocity and concentration boundary layers over the substrates as accurately as you can. (4)

(d) Using a concentration of 1 mol% silane in the gas, calculate the diffusive flux of silane to the substrates at distances of 10 cm and 30 cm from the entrance. (9)

(e) Calculate the deposition rate in $\frac{\text{nm}}{\text{sec}}$ at a location 10 cm from the entrance, and another 30 cm from the entrance. If a wafer is placed near the entrance, how uniform will the deposited layer be (qualitatively)? (5)

(f) How would you change the design to make the deposited layer more uniform? (4)