

# 3.185 Problem Set 7 Adjusted

## Fluid Dynamics

Due Friday November 14, 2003

### 1. Power-law non-Newtonian fluid behavior (20)

A polymer melt is forced through a flat channel of thickness  $\delta$  between two fixed horizontal plates by a pressure difference  $\Delta P$  (inlet pressure minus outlet pressure). This channel length  $L$  is much longer than its width  $W$ , and it is a lot longer and wider than it is thick ( $\delta$ ), so you may assume fully-developed flow and neglect edge effects. Take the  $x$  direction to be the direction of flow, parallel to the plates, and the  $y$  direction to be straight up, perpendicular to the plates.

This polymer is a pseudoplastic non-Newtonian fluid, whose behavior can be modeled using a “power law”:

$$\tau_{yx} = -\mu_0 \left| \frac{\partial u_x}{\partial y} \right|^{n-1} \frac{\partial u_x}{\partial y},$$

which is to say, shear stress is proportional to shear strain rate to the  $n$  power, with the extra  $\partial u_x / \partial y$  there to get the sign right. The differential equation describing  $x$ -momentum equation for this laminar 1-D flow (where  $u_y = u_z = 0$  and  $d\bar{u}/dx = d\bar{u}/dz = 0$ ) can be written with the above power law substituted for  $\tau_{yx}$ :

$$\rho \frac{\partial u_x}{\partial t} = -\frac{\partial P}{\partial x} + \mu_0 \frac{\partial}{\partial y} \left( \left| \frac{\partial u_x}{\partial y} \right|^{n-1} \frac{\partial u_x}{\partial y} \right) + F_x.$$

The general solution to this equation for steady-state flow driven only by pressure gradient is:

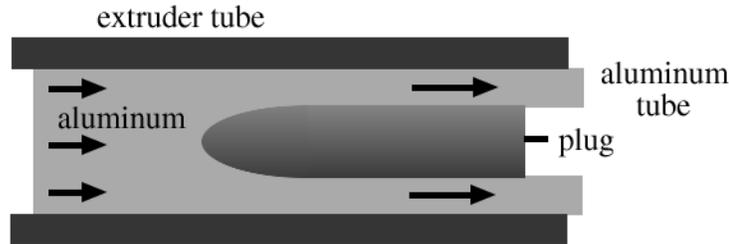
$$u_x = -\frac{\mu_0 L}{\Delta P} \frac{n}{n+1} \left( -\frac{\Delta P}{\mu_0 L} y + C_1' \right)^{\frac{n+1}{n}} + C_2.$$

Note that this is only valid for negative  $y$ ; the positive  $y$  solution is symmetric.

- If the polymer is pseudoplastic, is  $n$  greater or less than one? (3)
- Determine the specific solution which fits the above general solution to the no-slip boundary conditions at the two stationary plates, and/or the symmetry plane boundary condition halfway between them (you need just two of these three conditions). Note: it might help to set  $y = 0$  halfway between the plates. (8)
- Sketch this velocity profile for an appropriate pseudoplastic  $n$  value of your choosing. (5)
- For channel flow of this kind, a Newtonian fluid’s average velocity will be 2/3 of its maximum velocity. For this pseudoplastic fluid, will the average velocity be more or less than 2/3 of its maximum? (4)

2. Extrusion of a tube (16)

A seamless polymer or metal (*e.g.* an aluminum or magnesium alloy) tube is made by extruding the solid through the cylindrical die pictured in cross-section below. (The central plug is held in place by a “spider” whose details are not significant for this problem.)



You may assume “flow” of the solid is steady-state and incompressible, that the plug and tube are long enough to establish fully-developed flow, and that gravity is *not* an important driver for the flow. For the first two parts, you may also assume Newtonian flow with constant viscosity.

- (a) Using the handout *Solving Fluid Dynamics Problems*, write the assumptions which you can make for flow through the annulus between the plug and the tube. (4)
- (b) Write (or xerox) the appropriate form of the Navier-Stokes equations for this process, and based on your assumptions from part 2a, cancel the terms you can in the Navier-Stokes equations. (6)
- (c) On a single (large) graph, sketch the flow profiles you would expect to see in the annulus (with the same maximum velocity) assuming the solid behaves like a:
  - i. Newtonian fluid (2)
  - ii. Pseudoplastic fluid (2)
  - iii. Bingham plastic (2)

3. Torsional viscometer (40)

A device for measuring viscosity of thermoplastic polymer resins consists of a tall cylindrical cup of inner radius  $R_2$  which holds the resin, and a cylindrical rod of radius  $R_1$  inserted downward through most of the length of the container, which is far enough from the bottom that shear below the rod is negligible. The rod is attached to a motor which turns it at a constant rate, and the cup is attached to a torque meter. The device measures torque for several rotational speeds of the rod.

Assume for your calculations that the resin is Newtonian with uniform density and viscosity and that flow in the viscometer is laminar, and focus on the velocity between the rod and cup away from the bottom.

- (a) Sketch the device and write (or xerox) the relevant form of the Navier-Stokes equations. (3)
- (b) Write assumptions which allow you to simplify that horrible mess of equations, and cancel the appropriate terms in the equations above. Write the resulting simplified equations. (7)
- (c) Determine the general solution of the  $\theta$ -velocity in your simplified equations. (You need not calculate the variation of pressure with respect to  $r$  or  $z$ , but you will need to show that it doesn't vary with  $\theta$  to get rid of the  $\partial p/\partial\theta$ .) (10)
- (d) Apply the boundary conditions associated with the rod and cup, and determine the velocity profile in the viscometer, and the torque at  $r = R_2$ , as a function of  $R_1$ ,  $R_2$  and the rotational speed of the rod. Is torque a function of  $r$ ? (8)
- (e) Suppose the cup is 20 cm long and 8 cm in diameter, and the rod is 2 cm in diameter. At a constant rotational speed of 120 rpm, the torque is measured at  $2.0 \times 10^{-3} \text{N} \cdot \text{m}$ . What is the viscosity of the molten polymer? (8)

- (f) At a rotational speed of 300 rpm, the torque is twice as high. Would you say this is a Newtonian fluid? If not, is it pseudoplastic or dilatant? (4)

4. Fluid flow in blood vessels (24)

You are designing a material for use in artificial blood vessels, and would like to know the shear forces and pressures which these blood vessels must sustain. You decide to treat blood vessels as cylindrical tubes with constant flow rate to calculate those quantities for vessels of various sizes, from the aorta to capillaries.

For a very rough estimate, you may use the properties of water:  $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ ,  $\eta = 10^{-3} \frac{\text{kg}}{\text{m}\cdot\text{s}}$ .

- (a) Use an approximate volume of a heart ventricle and pulse to estimate the average flow rate through the aorta. (4)
- (b) Based on this flow rate, and aorta diameter of 0.7 cm, calculate the average velocity and Reynolds number of flow in the aorta. Is the flow likely to be laminar or turbulent? (4)
- (c) Use this Reynolds number and, if flow will be turbulent, the “smooth pipe” curve on p. 188 of  $W^3R$  to calculate the friction factor and approximate shear stress on the walls of the aorta. (6)
- (d) Suppose a capillary is 1mm long and  $50\mu\text{m}$  in diameter. If the average flow velocity is  $1 \frac{\text{mm}}{\text{sec}}$ , what is the pressure drop (same as pressure difference) from one end to the other? (6)
- (e) Give at least two important reasons why the above analysis breaks down for blood flow in real arteries and capillaries. (4)