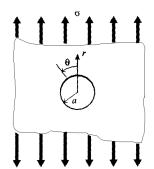
Stresses Around a Circular Hole

Uniaxial tension, hole of radius a:



Boundary conditions:

$$\sigma_{r} = \frac{\sigma}{2} (1 + \cos 2\theta)$$

$$\sigma_{\theta} = \frac{\sigma}{2} (1 - \cos 2\theta)$$

$$\tau_{r\theta} = \frac{\sigma}{2} \sin 2\theta$$

$$\sigma_{r} = \tau_{r\theta} = 0, r = 0$$

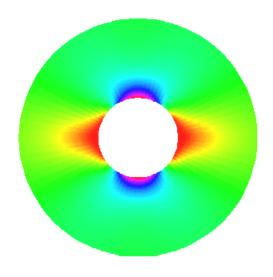
Assumed Airy stress function: $\phi = f(r)\cos 2\theta$

$$\nabla^4 \phi(r,\theta) = 0 \to f(r) = Ar^2 + Br^4 + C\frac{1}{r^2} + D$$

Evaluate constants from boundary conditions, then

$$\sigma_{\theta} = \frac{\partial^2 \phi}{\partial r^2} = \frac{\sigma}{2} \left(1 + \frac{a^2}{r^2} \right) - \frac{\sigma}{2} \left(1 + \frac{3a^4}{r^4} \right) \cos 2\theta$$

$$\sigma_{\theta, \text{max}} = 3\sigma @ r = a, \theta = \pi/2$$



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