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PROFESSOR: OK, settle down. Let's get started. One announcement, yesterday we had our first weekly I would say minor celebration. And by and large, it went well. Please make sure that you go to your assigned recitation. If you miss your recitation, you need to get down to see Hillary so that we make sure that we have enough copies of the weekly quizzes on hand. If you've joined the class, you have to check in with her. She's down the hall here in room 8-201. And you'll be assigned to a section.

What else do I have by way of announcements? Oh yes. Just reminding you, this was from 2003. See it doesn't change. It's the same s-block, p-block, and d-block elements. So that's coming up a week from tomorrow, two celebrations next. And of course the contests, the contests with hot prizes.

All right. Let's get down to business. Last day we looked at the Rutherford-Geiger-Marsden experiment. Oh there's one other one. If you look at the readings, there's this one section called the archives. My predecessor, Professor Wit, wrote a set of lecture notes. And they look something like this. And some students have said that they find these a little more expository in certain sections on certain topics than the book to be. And I have no preference.

But if you take a look at this it'll say LN1, lecture notes 1. If you go to this, read this. If you find that's helpful, good. If you don't find it helpful, than stick with the book. Just letting you know what that is.

All right, so last day we looked at the Rutherford-Geiger-Marsden experiment in which a high energy beam of alpha particles bombarded a thin, gold foil. And on the basis of the scattering results, namely most of the particles went through with minor scattering. And a tiny fraction of them were scattered through large angles.

The Thomson plum pudding model was rejected in favor of Rutherford's nuclear model of the atom. And then subsequently, Bohr came up with the quantitative representation off the Rutherford nuclear model. And we were partway through the treatment of Bohr last day when we adjourned. So let's right pick up the thread from where we left off.

And so just to remind you, the Bohr model is for a 1-electron atom gas phase. So this is either atomic hydrogen, it could be helium plus lithium 2 plus roentgenium 110 plus Its doesn't matter how many protons. there's always only want electron. And it's a planetary model. The positive charge concentrated in the nucleus,  $Z$  is the proton number. And at a distance  $r$  from the nucleus is a circular orbit in which resides 1 electron. It has a charge of

minus  $E$ . And I just designated them  $q_1$  and  $q_2$ . You could have done it the other way. But I had to choose something.

So we went through and looked at the constitutive equations here. So first of all, the energy of the system-- and this is only going to be the energy of the electron. Because we assume that the nucleus is far more massive. And so we don't have to get into things like reduced mass or anything like that. So just measure the energy of the electron.  $\frac{1}{2}mv^2$  is the newtonian component. And then coulombic energy that's stored is  $z$  times  $e$  squared over  $4\pi\epsilon_0 r$ , where  $\epsilon_0$  is the permittivity of vacuum. And it's the factor, the  $4\pi\epsilon_0$ , is the factor that allows us to take electrostatic energies and put them on the same plane as mechanical energies. When we run through this, we always end up in joules.

Then there's a force balanced to make sure that the electron neither falls into the nucleus nor flees and breaks free of the atom. And the force balance is if you put a ball at the end of a string, and you whip it around on a tether, you have a centrifugal force that's trying to get the ball to break away. And then the string is pulling in. So the pull in, in this case, is the coulombic force. And the force that makes the ball want to flee is this  $mv^2$  over  $r$ . And that must be net zero. Otherwise we're going to have a shift in orbit.

And then lastly we have the quantum condition. And this was the breakthrough of Bohr where he enunciated that the quantum condition is going to give us this energy level quantization. And this was a big departure from what had been in the past. The only antecedent idea of this nature was the work by Planck who said that light is quantized. But as I told you last day, who knows what light really is. The Newtonian notion of a ball orbiting was very compelling here. And the notion that the movement of the electron could in some way be discontinuous was quite a major departure.

So I left you last day with three equations and three unknowns. And what I'm not going to do right now is solve the equations. Because I've been lecturing long enough to know that's the way to kill interest, quench a lecture. And so if you really want to go through the algebra, be my guest. You're smart enough to do that. Instead I'm going to show you the results. But those are the three equations that you need.

So we have an equation in  $r$ . We have an equation at  $v$ , and an equation in  $e$ . So let's go after them. If you first look at the solution for  $r$ . This is the radius of the orbit of the electron, the orbit of the electron. If you go through and solve, you'll end up with this,  $\epsilon_0$  times the square of the Planck constant divided by  $\pi$  times  $m$ . It's always the electron. So this is the radius of the electron orbit. This is the mass of the electron times the square of the elementary charge-- that whole thing I'm going to group-- times the square of the quantum number divided by  $z$ , the proton number.

So what we see here? Well, everything inside the parentheses is constant. These are all constant. Pi obviously is geometric constant. And the rest of these are constants you could look up in your table of constants. And we noticed that there is a set of solutions to this. The radius of the electron can occupy various discrete values defined by  $n$ . So we say that the radius takes on a plurality of values a function of  $n$ .

And furthermore, the functionality goes as the square of  $n$ . It's  $n$  squared times a constant, where that constant is inside those parentheses. And we notice that because the  $r$  goes as  $n$  squared it's nonlinear. It's nonlinear.

This is so important I'm going to write it down one more time. So the radius of the electron orbit takes multiple values. It takes multiple values. And they're discrete. The physicists like to use a different term. When something is discretized, the physicists say it is quantized. So these values are quantized. You cannot continuously vary the radius and nonlinear values, multiple values.

So let's plug in. Because I want to get a sense of scale. So let's look at the simplest one. The most primitive 1-electron atom would be hydrogen. In which case,  $Z$  equals 1. So I've just got a proton orbited by an electron. So look at atomic hydrogen. So in that case,  $Z$  equals 1. And I'm going to look at  $n$  equals 1, which is the lowest number here, right?  $R$  scales as  $n$  squared. So the lowest value or  $r$  is obtained when  $n$  equals 1. And this is termed the ground state. The ground state.

So I want to ask what is the radius of the electron orbit ground state in atomic hydrogen? And if I plug in these values, I'll call this  $r_{sub\ 1}$ . It turns out to be 5.29 times 10 to the minus 11 meters, or 0.529 angstroms. I love the angstrom. It's a great unit. It's a great unit. It's not an SI unit. But I like the angstrom. I'll show you why.

If you try to express this in SI units, well there's 10 to the minus 11 meters. The SI units go in units of clusters of 1,000. So for example, you've got the meter. You've got the kilometer. You've got the micrometer. You've got 10 to the minus 9 meters, which is the nanometer.

This is a Goldilocks problem. This one's too big. And then the next one down here is 10 to the minus 12 meters, which is the picometer. So this is either 52.9 picometers, or a 0.0529 nanometers. And that's no good. I want numbers like 3, 7, simple to remember. So 0.529, this is about 1/2 angstrom. It's good to know.

But you try to publish, you know what happens in a scientific literature today? The Literary Lions that control the journals, they'll circle that and say you have to convert to SI units. And so they have to right some goofy nanometer thing or something. I know you think I'm crazy, but I love the angstrom. So there.

Anyway, so here it is. Once you know that this is 0.529, this is on your table of constants. It's right on your table of constants. So you don't have to go and calculate all this stuff. Which means if you do your homework with your table of constants, you will know where those numbers lie, as opposed to opening this thing up for the first time on

the first celebration of learning on October the 7th, and with 47 entries and they're tiny, tiny font. And you're wondering where is that thing. Just a word to wise.

So now we know what this is. We know this is 0.529 angstroms. So now I can write an equation for the radius of a 1-electron atom anywhere, anytime.  $r$  of  $n$  is going to be equal to a naught which is this value here. And it is termed the Bohr radius. So you can write it as a function of the Bohr radius, times the square of  $n$  divided by  $Z$ . So that's for all 1-electron atoms, gas phase.

And you can see that as  $Z$  goes up, the  $r$  goes down, which makes sense. So suppose instead of hydrogen, we talk helium plus. What's the only difference? Helium plus has 2 protons in the nucleus. Which means that the coulombic force of attraction between the same 1-electron and now a doubly charged nucleus is going to be stronger.

So the first orbit is going to get pulled in. And all the other orbits are going to get pulled in. By how much are they going to get pulled in? By that much. So this is the functional representation of all of that physics. All right, there's three equations, three unknowns. Let's look at energy.

So if you go through and solve for energy, you get this one. Minus this big monstrosity, mass of the electron to the fourth power of the elementary charge times 8 times the square of the permittivity of vacuum times the square of the Planck constant, all times the square of the Proton number divided by the square of the quantum number. And I just to make sure everybody is with me here. I always want to write  $n$  here.  $n$  equals 1, 2, 3, takes on integer values. And we'll say it again here.  $n$  equals 1, 2, 3, et cetera, et cetera.

So again we say we see that  $e$  is a function of  $n$ . It's discretized. It's quantized. Why? Because once you impose the quantum condition here on angular momentum, it propagates through the entire model. So radius this quantized, energy is quantized, you're going to see velocity is quantized. Because the quantum condition is pervasive. So  $e$  to the  $n$ , and I'm going to take this whole quantity in parentheses and just call it giant  $K$ .

These are all positive quantities. Mass is positive. And squares and fourth powers of numbers must be positive. So this is  $K$  times  $Z$  squared over  $n$  squared That's good.

And we can go and evaluate  $K$ . And when we evaluate  $L$  in SI units, we get 2.18 times 10 to the minus 18 joules. This is joules per atom. Or you can multiply this by Avogadro's number. If you multiply it by Avogadro's number, then that will give you 1.312 megajoules per mol.

So that's the energy of the electron in the ground state of atomic hydrogen. And then we can mediate that with  $Z$  and  $n$ , and go to electrons that are outside the ground state, above the ground state, or ground state electrons in

atoms that have more than 1 proton, or both.

And so let's take a look at the graphical representation of that. So instead of Cartesian coordinates, because this is spatial distribution, I'm going to go to energy coordinates and give you an energy level diagram. So again, not to scale. Because this thing goes is  $1$  over the square. So that's going to be messy. So let's start here. And on the left side I'm going to designate the energy. And on the right side I'm going to designate the quantum number. I'm going to start down here. That's the lowest energy.

You see these are all negative values, first of all. They're all negative values. Because  $Z$  is a square,  $n$  is a square, and  $K$  is a positive quantity. So  $n$  equals  $1$  is the lowest state. It's the ground state. It has a value of minus  $K$ . So I'm going to do this one just for atomic hydrogen. So I'm going to write atomic H. So now  $Z$  equals  $1$ . You can do it later for  $Z$  equals  $2$ ,  $3$ , whatever.

So this is atomic hydrogen. So ground state energy is minus  $K$ . What happens if we go to  $n$  equals  $2$ ?  $n$  equals  $2$  it becomes  $K$  divided by  $2$  squared  $4$ . So it should be  $3/4$  of the way up. I'm not going to go quite  $3/4$  of the way. Because I want to leave room for some fine structure. That's why it's not to scale.

All right so this is minus  $L$  over  $4$ . What if we go to  $n$  equals  $3$ ? Well it's not symmetric here. It's nonlinear. But this should be really what? Minus  $K$  over  $3$  squared is  $9$ . You get the picture.  $3$ , you can go  $4$ ,  $5$ , and so on until  $n$  equals infinity. What happens when  $n$  equals infinity? I've got minus  $K$  over infinity, which is vanishingly small, zero. Where is the electron when  $n$  equals infinity?  $r$  is  $n$  squared times the Bohr radius. That's a great, great distance away.

What does it mean? Physically it means that the electron is so far away that it is no longer bound. It's no longer part of the atom. And when it's no longer part of the atom, and the potential energy that's stored is a result of the charges coming together from infinity. I'm starting to talk like someone out of 802. What's the energy if I take  $2$  charged particles at infinite separation? I bring them into some finite separation. Voila. There it is. So when they're in infinite separation there's no energy stored. Hence, you are at that point.

So  $n$  equals infinity means  $r$  equals infinity, which means  $E$  equal zero. There's no stored energy. So this means the electron is no longer bound. And therefore, if it's no longer bound, we have a term for it. It's called free. It's a free electron. And if the electron is free, then the atom is electron deficient. So if the electron is free, that means the atom is now an ion. Because it's lost an electron. It's no longer net neutral. Or we say an atom hasn't turned into an ion. Or the electron has been ionized.

What's the energy for that? We can calculate what that energy is. It would be called the ionization energy. So if I started with an electron down in here, and I sent it all the way to infinity. See that's an energy space. Which is the

equivalent in Cartesian space to go from here to infinity, same idea.

Do you see the models? This is Cartesian. It's like a. Map This is energy coordinates. It's different. And you're going to be able to think from one model to another. What's the energy consequences? What are the Cartesian consequences? Until we get to the point where there is no Cartesian representation. Because the abstraction level is too high, we'll have to content ourselves with this. So get comfortable moving from there to there. And then some day we're going to say it's too complicated. There's no Cartesian thing. We'll be comfortable by then with this.

OK so now we're taking an electron from here up to here. While we ask, what is the ionization energy? The ionization energy must equal the  $\Delta E$  of the transition. So what's that? The  $\Delta E$  of the transition is going to equal always  $E_{\text{final}} - E_{\text{initial}}$ .  $E_{\text{final}} - E_{\text{initial}}$ . Which is equal to  $E_{\infty} - E_1$ , the ground state. Well  $E_{\infty}$ , we just said, is zero. And the ground state energy is equal to  $-K$ . So  $0 - (-K)$  is  $K$ .

So you also get the energy. From infinity down to ground state is the ionization energy. So we can define the ionization energy in terms of this transition. Define ionization energy as the minimum energy to remove an electron from the ground state of an atom in a gas phase. So that means there's no solids, no liquids. There's no work function here. There's no lattice energy, and so on. So there's a definition of the ionization energy.

And we can be a little bit more elaborate. Even though right now I'm just going to do a little break here. I don't want to mislead people. But just an aside. I'm nonlinear. I can have multiple conversations at once. And you are capable of stacking. So we're going to break now. We're not going to talk about 1-electron atom. We're going to follow the thread of ionization energy.

I'm going to take lithium. Lithium in its normal state has 3 protons, 3 electrons. So I'm going to take lithium gas. And I'm going to ionize it and make lithium plus. So this is a lithium plus ion in the gas phase. It's still got 2 electrons. So this isn't Bohr model stuff. But anyway, here we are. So the energy for this action would be called the ionization energy. Because I took a neutral atom, and I pulled an electron out of the ground state, and so on. So this is an ionization energy.

But now I can continue this process. And I can take Lithium plus. And I can lose an electron from that, which will than give me lithium 2 plus. And this is called also an ionization energy. This is called the second ionization energy. So this is the first ionization energy. But just as when you write an equation, when the coefficient is 1, you don't write the 1. I don't write 1 lithium here. I know it's 1. This is the second ionization energy. And then I can keep stripping away electrons. And I can take lithium 2 plus in the gas phase, and take away that electron leaving me with just the lithium nucleus, lithium 3 plus, plus electron. And that's called the third ionization energy.

OK, now what can we say? What's the relationship here between any of this except the definition and the Bohr model? well? The Bohr model applies only to 1-electron atoms. Are there any 1-electron atoms on this board? So we can calculate the energy, the third ionization energy. We can get that from the Bohr model. You can do it in your head. You can do it in your head right? It's just  $K$  times  $Z$  squared right here. It's going from 1. So if this is 2.18, it's going to be 9 times that trivially.

OK, so this is just 3 squared times  $K$ . And this when you have to get from the literature. So you have to go to primary sources. Which is why you're going to learn how to use the proper database. And this one here also you get from the literature. But this is on your periodic table. Your periodic table, one of the data points it gives is the first ionization energy of all of the elements. And so even though lithium normally is a solid at room temperature, the ionization energy for lithium as given on your periodic table is for this reaction. It's for the gas. Anyway. So that's little aside.

All right, the last quantity that we could get from the Bohr model is  $v$ , the velocity. And I solve for the velocity. We don't talk about this very much. But we're going to do so once today. And here it is. So I went through the algebra. And you get  $nh$  over  $2\pi mr$ , where  $r$  is the radius. There's the quantum number. This is quantized as well. So I regrouped this. And I already have a nice, cool expression for  $r$  in terms of the Bohr radius. So I use that because that's on the table. So this is  $2\pi$  times the mass of the electron times the Bohr radius,  $1/2$  angstrom, times  $Z$  proton number divided by  $n$ , where  $n$  equals 1, 2, 3, and so on.

So let's again get a sense of scale. So let's try for sense of scale. Let's do velocity of the ground state electron in atomic hydrogen. So that means  $Z$  equals 1,  $n$  equals 1. So I plug in the numbers. And I get  $v_1$  for hydrogen, atomic hydrogen, gives me 2.18 times  $10$  to the 6 meters for second.

I don't know. Is that fast? Is that slow? I don't know. But I do know this much. I know that the speed of light is equal to 3 times  $10$  to the 8 meters per second. So  $10$  to the 8 divided by  $10$  to the 6. 2 and 3, that's roughly 1, speaking as an engineer. Who cares? So this is about 1% of the speed of light. That's pretty good. That gives me something I can hang on to. I would say that if this thing is zipping around at 1% of the speed of light, I would say that's relatively fast. One more time, 1% of the speed of light. That's relatively fast.

Remember last day I told you that Bohr simply dismissed the concept of the use of classical electrodynamics down to atomic dimensions. Well here's another example of why a lot of these assumptions aren't going to work so well. We're talking about the ground state electron velocity in this putative planetary model of a 1-electron atom. You're already getting into relativistic effects. So, just another example.

By the way, why do we use the letter  $c$  for speed of light? It comes from the latin word *celeritas*, which means swiftness. And we get the modern word acceleration, deceleration from that.

OK. All right. So the Bohr model, we've now rolled it all out. We have the energy portrait. We have the radii, discrete. We have quantization. And we have velocities if we ever want to look at those again. Now what's the next thing we do in science? We compare the predictions of the Bohr model with data. Are there any data to support this?

Because remember, all Rutherford said was plum pudding doesn't make sense. Instead I'm going to concentrate the positive mass in the center. And then Bohr came along and said, not only is it going to be a planetary model, I'm going to have circular orbits. So now we've gone a long way from Geiger-Marsden. So is there any data? Well there were data in 1853. Remember, Bohr published this in 1913. In 1853 there was a spectroscopist by the name-- I'm going to tell you his name-- in Uppsala, Sweden. And his name was Angstrom.

Angstrom was doing experiments on hydrogen in gas discharge tubes. So he measured emissions from gas discharge tube. And it was filled with various gases including atomic hydrogen. And in order to take his data, what he used was this device here-- there's the Bohr radius, just an example. He used the prism spectrograph. So here's a gas discharge tube. And I'm going to show you the physics of that in a second.

Basically you've got a pair of electrodes. You've got gas in the tube. And as this cartoon shows, you apply a potential across the electrodes. And beyond a certain threshold potential, the tube begins to glow. And the glow goes in all directions. And blinds you when you're in the lab. So what you do, is you cover this up a bit. You have a narrow slit. And then you force the light to come through in a thin ribbon, and then expose it to a prism.

What the prism does, is it takes the light and breaks it into its components, sort of rainbow-like, and magnifies the difference. As refraction goes, different wavelengths will refract different amounts. And then you shoot this across the room. There's two counters. One is a scintillation screen and an army of graduate students who sit there in the dark. But they're no good because you can't stick them into the publication. You need to have data that people will rely upon.

So instead you use a photographic plate. And if you put even a tiny, tiny angle of separation across a great enough distance, you start to get enough line splitting that you can see. And then you go backwards. And you know the geometry here. And you can figure out what the wavelength must have been to go this distance, et cetera, et cetera. And these are all color coded, not because they had color film in those days, but just to let you know that 656 nanometers, if you were the graduate student sitting there, you'd see a red line, a green line, a blue line, and a violet line.

So that's how he made the measurements. And he published those measurements. And so they lay.

And then the story gets a little thicker. In 1885, there's a Swiss high school math teacher. I mean, I can't make this stuff. This is true story. There's a Swiss high school math teacher by the name of J. J. Balmer. We had J. J. Thomson. Now we've got J. J. Balmer. And J. J. Balmer, he loved to play with numbers. And he was studying this set of lines. And he was trying to come up with a pattern. Can you see a pattern there? 410 434 486, 656, do they go squares? Are they primes? What's the pattern there?

So Balmer puzzled over this for awhile. And he finally came up with the equation to represent those lines. So he studied Angstrom's data found the pattern. And here's the pattern that he found. He said that those are wavelengths. If I take instead wave number,  $\bar{\nu}$  is called wave number, which is the reciprocal of the wavelength.

So if I take the reciprocal of the wavelength, I end up with those 4 lines conforming to a series that goes like this.  $\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$ , where  $n$  equals 3, 4, 5, 6. And there's a constant here which we're going to designate  $R$ . And the value of  $R$ -- I'm going to put that on the next board-- the value of  $R$  as expressed in SI units today, would be  $1.1 \times 10^7$  reciprocal meters. Wavelength is a meter. Reciprocal wavelength or wave number must be reciprocal meters.

So how this all of this support the Bohr model? Well in order to explain it, I've got a first tell you what the physics of the gas discharge tube are. So let's go inside the gas discharge tube and understand those physics. So here's the gas discharge tube. It's made of borosilicate glass. And we fill it with gas. And in this case, the gas is going to contain among other things hydrogen. So this is a hydrogen gas phase atom. And this is probably at low pressure.

And then I said I need electrode. I'll get the electrodes inside. So I've got to have a really good glass blower who can make a glass to metal seal, and have a feedthrough to an electrode. So this is still a vacuum seal. How do they get the gas in the first place? We don't show you this in the books. I'll tell you because I did this in my Ph.D. What you do is you have a little side tube here. You evacuate. This goes to a vacuum pump. And then over here you have a gas source. You evacuate. Put in the gas to whatever pressure you want. And then the glass blower disconnects. This pulls this down to a reduced pressure, and seals it. But the books don't show you that. That's a secret.

So we've got a gas at low pressure. And I've got an electrode over here and an electrode over here. And they're connected to a variable voltage power supply. So I'm going to put an arrow with the  $V$  meaning it's variable voltage. I can change the voltage. And this convention, this is the negative side. So this means the electrons leave the power supply and go like this. Which means this electrode will be negative. And this electrode will be positive. And thanks to Michael Faraday, we will call this one the cathode. And this one we will call the anode.

Now what happens? We start turning up the pressure, turning up the voltage rather. Low pressure here, but

electrical pressure is voltage. The voltage gets high enough, eventually the electrons will boil off the cathode. And this is a gas at low pressure. And they will accelerate from rest and go all the way across the tube and crash into the anode. And we complete the circuit. But if there's a gas in here, some of these electrons are going to hit the gas molecules. And when they hit the gas molecules, if they have enough energy to do so, they will cause electrons inside the gas molecules to be excited. And if they're excited enough, the electrons will jump up to a higher energy level.

But they can't be sustained. Because this is a ballistic collision. It's a one of. It's like a bowling alley. One ball, one pin, one impact. Now the pin is in the air. What happens to the pin? It falls back down. Why? Because gravity pulls it down. In this case, you've got the energetics pulling the electron back down. Now the electron goes from high energy to low energy. And when that happens, the energy difference is given off in the form of a photon. So I get photon emission when this falls back down. And this photon has a wavelength.

What I'm going to show you is that the set of lines that you get from exactly this configuration using this equation give you the Balmer series. So now you've got a model that Bohr postulated for atomic hydrogen on 1-electron atom that exactly predicts that set of 4 lines. Which were measured 50 years before. So let's go.

So first of all let's get the energy here. And I'm going to get the energy of this electron. This electron I'm going to call a ballistic electron. Why do I call it a ballistic electron? Because it's not bound. It's free. It boils off the cathode, flies through free space, and crashes into an anode. Clearly it's not part of an atom. But there's a second electron in this story. And it's the ground state electron in hydrogen. And it lives here.

So what's the energy of the ballistic electron? Well that's just  $\frac{1}{2} mv^2$  squared. And where did it get its energy from? It got its energy from the power supply. And what's the electrostatic energy? It's the product of the charge on the species times the voltage through which it was accelerated. So away we go. I know the charge on the electron is minus  $e$ . Whatever the voltage is there, 1 volt, 10 volts, 100 volts, whatever. Away we go.

By the way, I'm going to show you just one other thing in terms of order of magnitude. The kinds of voltages you see along here are 1 volt, 10 volts, that sort of thing. So suppose, to get an order of magnitude, suppose we had a species of charge  $e$ . So in other words, it's only 1 times the elementary charge. A species of charge  $e$  influenced by voltage of 1 volt.

So this is 1 in the voltage units, and 1 in the elementary charge units. How much energy would that be? That's equivalent to making 1 volt and accelerate an electron from rest across this gap. And the result would be, the energy then would simply equal  $1.6 \times 10^{-19}$  coulombs times 1 volt. And what's the energy going to be? Well I've got coulombs times volts. And I don't know how I convert one to the other. I don't have to. Why not? Because that's an SI unit. And that's an SI unit. This is an energy. So with impunity, I write  $1.6 \times 10^{-19}$  to the

minus 19 joules. That's the beauty of SI units.

So that's a good news. I know it's joules. The bad news is I hate this number. It's a stupid number,  $1.6 \times 10^{-19}$ . It's crazy. Why don't I come up with a number like 3, 7? So what I could do, is I could define. I could define a unit such that when the elementary charge is accelerated across the unit voltage, I would call that unit 1 electron volt. And so somebody thought of this before me. And hence, this is the unit of the electron volt. It takes these crazy things that we've been spewing here up until now, and rationalizes them into numbers that people can carry around in their heads.

So what's  $K$  now?  $K$  is  $2.18 \times 10^{-18}$  joules. Yuck! Let's convert that to electron volts. So I divide by  $1.6 \times 10^{-19}$ . And I got 13.6 electron volts. You'll remember that on your death bed, ionization energy of atomic hydrogen. Maybe we don't have to give out tables of constants. You just know this stuff. It's OK.

All right. Now one last thing about this. So this has got gas in it. This is the cathode. And this beam of electrons, back in the 1800s, there was a popular term, it was called the ray. So instead of a beam of light, people refer to the ray of light. So then when they got to particle beams, they talk to them as rays. So this is now not an electron beam, it's an electron ray. And it comes off the cathode. And it's in a vacuum tube. So this could be called a cathode ray tube, a CRT.

Now see, I could flatten this. And I could spray it with phosphors. And then I could put some charge plates here. The electrons have a negative charge. So if I charge these plates, and I was clever about how I charge them and varied the charge, I could raster the electron beam like this about 30 times a second all up and down the screen. And then I could put some program signal in there. And I could sit here. And I could watch TV. It all started with the gas discharge tube. It says nothing about the content unfortunately. Very nice physics, but no content.

All right. So now we've got the electron. The electron is moving, the ballistic electron. Now I want to look at what happens when the ballistic electron smashes into one of those hydrogen atoms. So let's go back over here. So here's the incident particle. And it's going to be an electron in this case. So I'm going to designate this electron. This is my ballistic electron. So here's the incident electron. And this is ballistic, just to be clear. It's the ballistic incident electron.

Now this is a mixed metaphor here. Because I'm representing this in Cartesian space. But I've moved into energy space. So some people are going to get really upset. Because they're going to say, well this is Cartesian, but this isn't. It doesn't matter. It's my lecture. It's my model. It works. I'm the professor. So we've got this mixed metaphor here. But anyways, it helps a lot.

So what happens when this thing comes in? It depends on how much energy it has. Now if the incident energy, if

$E$  incident is tiny, nothing happens. This thing just zooms right on through. But if the incident energy, if  $E$  of the incident ballistic electron is greater than  $\Delta E$  for any transition that's feasible-- and in this case I'm going to assume I don't have thermal distribution of electrons. If I gave you Avogadro's number of hydrogen atoms because of the thermal distribution of energies-- and we'll come back to this later-- there might actually be, at any moment, some electrons that are thermally excited above the ground state. We're going to forget about that for now. We're going to spiral up the learning curve here. So first time we're going to assume all the electrons are in the ground state. If I don't enough energy to go from  $n$  equals 1 to  $n$  equals 2, nothing happens. If I have more than enough energy to go for  $n$  equals 1 to  $n$  equals 2, I will take that energy. And the electron will jump, steal that amount of energy, and then this thing moves on-- and I'm purposely making this vector shorter than the incident vector-- with that amount of energy raw. And this is called the scattered electron.

Go back to the bowling ball analogy. The bowling ball comes in with a certain energy, hits the pin, continues to roll. But you know that there's a loss of kinetic energy in the bowling ball. That's what we're seeing here. It's purely ballistic.

Now let's say we do have more than enough energy. Suppose I have enough energy to go from  $n$  equals 1 halfway between  $n$  equals 2 and  $n$  equals 3. There's only  $n$  equals 1,  $n$  equals 2. I can't take the electron up to  $n$  equals 2.3. It's unallowed. These are the only allowed states. So that differential amount of energy then resides with the electron that's ballistic. And it moves on here.

So we've got conservation of energy here. We can say that  $E$  incident will then equal the energy that's lost in the transition plus the energy that's still left with the scattered electron. And we can calculate what that transitional energy is. That transitional energy is going to equal  $\frac{1}{2} m v^2$  incident. This is the velocity, the incident electron. What's the energy to go from  $n$  equals 1 to  $n$  equals  $n$ ? Whatever it is, its minus  $K$  times  $Z$  squared. If it's hydrogen,  $Z$  is 1. It's going to be  $\frac{1}{n^2}$ . The final quantum number squared minus  $\frac{1}{n_i^2}$  over the square of the initial quantum number. And then, what's left over after this has been robbed from the incident ballistic energy is  $\frac{1}{2} m v^2$  of the scattered ballistic electron.

And the quantization dictates that only if  $E$  incident is greater than  $\Delta E$  going 1 up to  $n$ -- here I'm assuming everything is in its ground state. Later on we're going to be more sophisticated. But for first time through, all are ground state electrons. I have to have enough energy to go at least to  $n$  equals 2. I can go to  $n$  equals 3,  $n$  equals 4. In principle, if this thing had more than 13.6 electron volts, what would happen? It would kick this electron out. Gone! And you'd have 2 free electrons. So if it's greater than this, then the consequence is electron promotion.

So we're moving along. But this excited state is unstable. This excited state is unstable because it's like the

bowling pin that got thrown up. So what happens? The electron standing up there on  $n$  equals 2, for example, looking down to  $n$  equals 1. And it falls. When it falls it gives off radiation. And that radiation is conservation of energy there. And what we know is when it gives off an energy of the emitted photon, the energy of the emitted photon must equal  $\Delta E$  of the transition falling from 2 to 1. And we know how to calculate that. That's just that thing flipped around.

And this thing is equal to what? This is equal to  $h\nu$ . According to Planck it's  $hc/\lambda$  is equal to  $hc\nu$ . You know what this one is. This is minus  $kZ^2$  squared  $1/n^2$  -- in this case it's going to be --  $1/n_f^2$  over  $1/n_i^2$  minus  $1/n_i^2$ , in this case,  $1/2^2$  over  $1/n_f^2$ . And you can generalize this  $1/n_f^2$  over  $1/n_i^2$ .

So where am I going with this? Well I'm going to flip all of this around. And when I flip it all around, what I'm going to end up with is this equation here. And better than that, I'm going to end up with this equation. And I'm going to end up with this as the constant. And when I get that, we're going to say Bohr has done it. The data support the theory. So that's what we're going to do. But I think we're going to stop at this point today. So let me just jump here.

I mentioned to you that if you go here on your periodic table, there's the 13.6 electron volts. In the case of lithium, this is 5.4 electron volts. So you can see all the various values. This by the way, people no noise. No noise. It's 11:52. I'm holding court until 11:55. I'm simply changing topics. It's not, oh this is the part where I can talk to my neighbor. What are the rules? No talking. No food. No horseplay until 11:55. Then, still no horseplay, gentle talking, no food, no drink.

This is a sacred space. I'm not kidding you. Do you know why? In this secular America, this is sacred space because this is where people learn. The lecture hall is sacred space.

Now, here's the 13.6 electron volts. And there's the  $1.6 \times 10^{-19}$  joules. And if you multiply those 2, you'll get the 2.18 over here. All right.

Here's a cartoon showing the photon, higher energy orbit, lower energy orbit, electron emission transition, right out of your book. And there's a postulate 6. And we're going to finish this up at the beginning of the Friday lecture. So here's the whole series from  $n$  equals 3 to 2,  $n$  equals 4 to 2, and so on. See how that works.

OK. One of the things that we've learned here, is that it doesn't matter what the incident energy is here. The emission is characteristic of the energy levels inside the gas. Instead of using an incident electron, I could use an incident proton. I could use an incident alpha particle. I could use an incident neutron. Anything that has enough energy to kick this up from  $n$  equals 1 to  $n$  equals 2 will result in photon emission of this frequency.

That means the set of those lines is unique to the target. And this is the beginning of chemical analysis. I can use

this to characterize species, to it, stars. How do we analyze the composition a stars? Well, do we send a NASA spaceship out 25 light-years and grab some gas and bring it back to the lab? No. All we've got is the spectrograph. The star is hot. That means thermal excitation and cascading down with photon emission. And the lines we get are related to the energy levels within the stars.

So from a distance of a 100 light-years, I can tell you what the composition is. And if there's two gases there, what if there's hydrogen and helium? the helium lines will be there. And they'll be superimposed on the hydrogen lines unless they lie directly on top of one another. I'm going to be able to figure out what's there. That's how it works.

Now here's a story about an astronomer, Cecilia Payne. She's the first woman graduate student in astronomy at Harvard. She went on to chair the Faculty of Arts and Sciences, awarded tenure, but denied a professorship for 18 years because she was a woman. And in her thesis, she was the first person to figure out to the sun is dominantly hydrogen, not iron, which is what most astronomers thought. Why? Well, because the earth is made of iron. Meteorites are made of iron. The whole universe must be made of iron. Never mind the fact that the sun is glowing.

So here's how spectroscopy works. Look at this. See, what does this mean? This is an analogy. What could that mean? Well, they said iron again. I know this is misspelled. But maybe it's just a glitch in the instrumentation. So most people would look at that and say, the message is iron. But it's not about the word. It's about the pattern. It's about the pattern. And that's what's spectroscopy is.

By the way, if you somehow didn't catch this lecture. And you walked in, and all you saw was those four lines. You know those four lines. That set of lines is characteristic of atomic hydrogen, and nothing else. By the way, those lines are very faint. This is not to scale. And they're so faint, they're ghostlike. And what is the latin word for ghost? Specter. So what is a spectrum? It is a set of ghostlike lines. To this day, the term spectroscopy refers to the ability to study data that are so faint they're ghostlike. All right, we'll see you on Friday.