

Session #4: Homework Solutions

Problem #1

A photon with a wavelength (λ) of $3.091 \times 10^{-7} \text{ m}$ strikes an atom of hydrogen. Determine the velocity of an electron ejected from the excited state, $n = 3$.

Solution

$$E_{\text{incident photon}} = E_{\text{binding}} + E_{\text{scattered } e^-}$$

$$E_{\text{binding}} = -K \left(\frac{1}{3^2} \right) \qquad E_{\text{scattered}} = \frac{1}{2} m v^2$$

$$E_{\text{incident photon}} = \frac{hc}{\lambda} \qquad \therefore \frac{hc}{\lambda} = \frac{K}{9} + \frac{1}{2} m v^2 \qquad \therefore \left[\left(\frac{hc}{\lambda} - \frac{K}{9} \right) \frac{2}{m} \right]^{\frac{1}{2}} = v$$

$$\left[\left(\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3.091 \times 10^{-7}} - \frac{2.18 \times 10^{-18}}{9} \right) \frac{2}{9.11 \times 10^{-31}} \right]^{\frac{1}{2}} = v$$

$$\therefore v = 9.35 \times 10^5 \text{ m/s}$$

Problem #2

Determine the minimum potential that must be applied to an α -particle so that on interaction with a hydrogen atom, a ground state electron will be excited to $n = 6$.

Solution

$$\Delta E_{1 \rightarrow 6} = qV \qquad \therefore V = \frac{\Delta E_{1 \rightarrow 6}}{q}$$

$$\Delta E_{1 \rightarrow 6} = -K \left(\frac{1}{1^2} - \frac{1}{6^2} \right) = \frac{35}{36} K$$

$$q = +2e$$

$$\therefore V = \frac{35}{36} \times \frac{2.18 \times 10^{-18}}{2 \times 1.6 \times 10^{-19}} = 6.62 \text{ V}$$

Problem #3

Determine if an electron travelling at a velocity of 7.2×10^6 km/hr is capable of ionizing a hydrogen atom with its orbiting electron in the ground state.

Solution

The energy of the ground state electron in hydrogen is $-K = -2.18 \times 10^{-18}$ J (I.E. = 2.18×10^{-18} J/atom); the kinetic energy of an electron travelling at $v = 7.2 \times 10^6$ km/hr is

$$E_{\text{kin}} = \frac{mv^2}{2} = \frac{9.1 \times 10^{-31} \text{ kg} \times (2 \times 10^6 \text{ m/s})^2}{2}$$

$$E_{\text{kin}} = 1.8 \times 10^{-18} \text{ J}$$

$E_{\text{kin}} < \text{I.E. (H)}$ - the electron cannot ionize the hydrogen atom.

Problem #4

Determine for hydrogen the velocity of an electron in an $n=4$ state.

Solution

This problem may be solved in a variety of ways, the simplest of which makes use of the Bohr quantization of the angular momentum:

$$mvr = n \times \frac{h}{2\pi} \quad (r = r_0 n^2)$$

$$mvr_0 n^2 = n \times \frac{h}{2\pi}$$

$$v = \frac{h}{2\pi m r_0 n} = 5.47 \times 10^5 \text{ ms}^{-1}$$

(A numerically correct result is obtained by taking:

$$E_{\text{el}} = -\frac{1}{n^2} K = \frac{mv^2}{2}$$

The negative sign reflects the E_{pot} term, which happens to be $-2E_{\text{kin}}$.)

Problem #5

Determine the wavelength of radiation emitted by hydrogen atoms upon electron transitions from $n=6$ to $n=2$.

Solution

From the Rydberg relationship we obtain:

$$\frac{1}{\lambda} = \bar{\nu} = R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = 1.097 \times 10^7 \left(\frac{1}{36} - \frac{1}{4} \right) = (-)2.44 \times 10^6$$

$$\lambda = \frac{1}{\bar{\nu}} = \frac{1}{2.44 \times 10^6} = 4.1 \times 10^{-7} \text{ m} = 0.41 \text{ } \mu\text{m} = 4100 \text{ \AA}$$

Problem #6

Calculate the minimum potential (V) which must be applied to a free electron so that it has enough energy to excite, upon impact, the electron in a hydrogen atom from its ground state to a state of $n=5$.

Solution

We can picture this problem more clearly: an electron is accelerated by a potential, V_x , and thus acquires the kinetic energy $e \times V_x$ $[=(mv^2)/2]$ which is to be exactly the energy required to excite an electron in hydrogen from $n=1$ to $n=5$.

$$e \cdot V_x = -K \left(\frac{1}{25} - \frac{1}{1} \right)$$

$$V_x = \frac{K}{e} \times \frac{24}{25} = \frac{2.18 \times 10^{-18}}{1.6 \times 10^{-19}} \times \frac{24}{25} = 13.1 \text{ Volt}$$

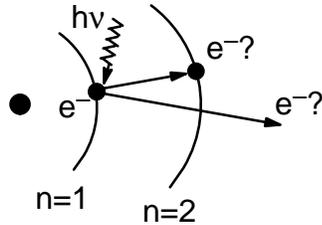
$$[13.1 \text{ eV} = 13.1 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{\text{eV}} = 2.08 \times 10^{-18} \text{ J} = -K \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)]$$

Problem #7

Light of wavelength $\lambda = 4.28 \times 10^{-7} \text{ m}$ interacts with a "motionless" hydrogen atom. During this interaction it transfers all its energy to the orbiting electron of the hydrogen. What is the velocity of this electron after interaction?

Solution

Required: First of all, a sketch:



possibly to $n = \infty$ (ionization),
depending on the magnitude of $E(h\nu)$

let us see: $E(h\nu) = (hc)/\lambda = 4.6 \times 10^{-19} \text{ J}$

To move the electron from $n=1$ to $n=2$ (minimum energy required for absorption of the photon), we have:

$$\begin{aligned}\Delta E &= \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) K = \frac{3}{4} K \\ &= \frac{3}{4} \times 2.18 \times 10^{-18} \text{ J} = 1.6 \times 10^{-18} \text{ J}\end{aligned}$$

We recognize that the photon energy is less than the ΔE_{min} (for $n=1 \rightarrow n=2$). This means that no interaction can take place – the photon will “pass by” and the electron will continue to orbit in its 1s state! Its orbiting velocity can be obtained from:

$$\begin{aligned}mvr &= n \left(\frac{h}{2\pi} \right) \\ v &= n \left(\frac{h}{2\pi mr} \right) = 2.19 \times 10^6 \text{ m/s}\end{aligned}$$

(You will get full credit if you simply state that no interaction is possible, or something equivalent. You will get “Brownie points” if you go beyond this statement – for example, assume an excited electron in motionless hydrogen, an unlikely occurrence, which could absorb the photon and enter a different excited state.)

Problem #8

What is the *energy gap* (in eV) between the electronic states $n=3$ and $n=8$ in a hydrogen atom?

Solution

$$\text{Required: } \Delta E_{\text{el}} = \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) K ; \quad K = 2.18 \times 10^{-18}$$

$$\text{Or } \bar{\nu} = \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) R ; \quad R = 1.097 \times 10^7 \text{ m}^{-1}$$

(Since only the energy gap is asked, we are not concerned about the sign.)

$$\Delta E = (1/9 - 1/65) K = 0.0955 \times 2.18 \times 10^{-18} \text{ J}$$

$$\Delta E = 2.08 \times 10^{-19} \text{ J} = 1.3 \text{ eV}$$

Problem #9

- (a) From information provided in your Periodic Table of the Elements, determine the *first ionization energies* (in Joules) for the horizontal columns (1) Na to Ar and (2) Ca to Cu.
- (b) On a graph, plot the values obtained as a function of atomic number and attempt to explain the apparent difference in the change of ionization energy with increasing atomic number for the two series of atoms.

Solution

- (a) The required data can be obtained by multiplying the ionization potentials (listed in the Periodic Table) with the electronic charge ($e^- = 1.6 \times 10^{-19}$ C).

<u>Element</u>	E.I. $\times 10^{18}$ (J)	<u>Element</u>	E.I. $\times 10^{18}$ (J)
Na	0.822	Ca	0.978
Ng	1.22	Sc	1.05
Al	0.958	Ti	1.09
Si	1.30	V	1.08
P	1.68	Cr	1.08
S	1.66	Mn	1.19
Cl	2.07	Fe	1.26
Ar	2.52	Co	1.26
		Ni	1.22
		Cu	1.24

- (b) [The purpose of this question is to provide you with an opportunity for graphic data presentation. By scaling, the E.I. dependence on the atomic number can be accentuated or suppressed.]

Acceptable Answer:

In the series of elements with Z 11 to 18, the 3s and 3p subshells are being successively filled. The E.I. for this series increases from 8.22×10^{-19} J (Na) to 2.52×10^{-18} J (Ar). This pronounced increase can be attributed to the effect of increasing nuclear charge (increasing attraction) on the electrons in the M shell.

In the series Ca to Cu, the 3d subshell is successively filled. We again observe an increase in E.I. However, this increase is much less pronounced than in the series Na to Ar because of effective shielding of the nuclear charge by the electrons in the K and L shells as well as by those in 3s and 3p sublevels.

[For the inquisitive, let us analyze the behavior in more detail. The increase in E.I. from Na to Mg (increased suitability) can be attributed to the formation of a filled subshell and the ensuing drop (Al) to first occupancy of the p subshell. The regularity (a slight E.I. drop) from P to S is the result of the first double occupancy of the p orbitals.

The question is: why should the 3d level, which is successively filled from Sc to Cu, be so much less sensitive to increasing nuclear charge? The answer to this question is provided by wave-mechanics. s and p orbitals are "penetrating". This means they "spend part of their time" in close proximity of the nucleus and, to a significant extent, "benefit" from increasing nuclear charge. (Remember: we saw that 2s levels also have a finite probability of occupying 1s areas, for example.) d levels and, even more so, f levels are virtually "non-penetrating". Therefore, since the screening effect of underlying electrons is pronounced and the increase in the "effective nuclear charge" from Z 21 to Z 29 is small, the ionization energy does not change significantly.]

Problem #10

Determine the energy gap (in eV) between the electronic states $n=7$ and $n=8$ in hydrogen.

Solution

Here we need to know the "basis" of the Rydberg equation [$E_{el} = -(1/n^2)K$] and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$:

$$\Delta E_{el} = K \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = 2.18 \times 10^{-18} \left(\frac{1}{49} - \frac{1}{64} \right) = 1.043 \times 10^{-20} \text{ J}$$

$$\Delta E_{el} = 1.043 \times 10^{-20} \text{ J} \times \frac{1 \text{ eV}}{(1.6 \times 10^{-19} \text{ J})} = 6.5 \times 10^{-2} \text{ eV}$$

Problem #11

Determine the frequency of radiation capable of generating, in atomic hydrogen, free electrons which have a velocity of $1.3 \times 10^6 \text{ ms}^{-1}$.

Solution

Remember the ground state electron energy in hydrogen ($K = -2.18 \times 10^{-18} \text{ J}$). The radiation in question will impart to the removed electron a velocity of $1.3 \times 10^6 \text{ ms}^{-1}$, which corresponds to:

$$E_{\text{Kin}} = \frac{mv^2}{2} = \frac{9.1 \times 10^{-31} \times (1.3 \times 10^6)^2}{2} \text{ Joules} = 7.69 \times 10^{-19} \text{ J}$$

$$E_{\text{rad}} = E_{\text{Kin}} + E_{\text{ioniz}} = 7.69 \times 10^{-19} + 2.18 \times 10^{-18} = 2.95 \times 10^{-18} \text{ J}$$

$$E_{\text{rad}} = hv; \quad v = \frac{E}{h} = \frac{2.95 \times 10^{-18}}{6.63 \times 10^{-34}} = 4.45 \times 10^{15} \text{ s}^{-1}$$

Problem #12

- (a) Determine if an energy level of $-1.362 \times 10^{-19} \text{ J}$ is an allowed electron energy state in atomic hydrogen.
(b) If your answer is yes, determine its principal quantum number (n). If your answer is no, determine n for the "nearest allowed state".

Solution

$$(a) \quad E_{\text{el}} = -\frac{1}{n^2} K$$

$$-1.362 \times 10^{-19} \text{ J} = -\frac{1}{n^2} \times 2.18 \times 10^{-18} \text{ J}$$

$$n = \sqrt{\frac{2.18 \times 10^{-18}}{1.362 \times 10^{-19}}} = 4.00 \quad \text{The answer is YES.}$$

$$(b) \quad n = 4.0$$

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