Session #3: Homework Solutions

Problem #1

From a standard radio dial, determine the maximum and minimum wavelengths

(λ_{max} and λ_{min}) for broadcasts on the
(a) AM band
(b) FM band

Solution

$$c = \nu\lambda, \;\therefore\; \lambda_{min} = \frac{c}{\nu_{max}}; \; \lambda_{max} = \frac{c}{\nu_{min}}$$

AM $\lambda_{min} = \dfrac{3 \text{ x } 10^8 \text{m / s}}{1600 \text{ x } 10^3 \text{Hz}} = 188 \text{ m}$ $\lambda_{max} = \dfrac{3 \text{ x } 10^8}{530 \text{ x } 10^3} = 566 \text{ m}$

FM $\lambda_{min} = \dfrac{3 \text{ x } 10^8}{108 \text{ x } 10^6} = 2.78 \text{ m}$ $\lambda_{max} = \dfrac{3 \text{ x } 10^8}{88 \text{ x } 10^6} = 3.41 \text{ m}$

Problem #2

For light with a wavelength (λ) of 408 nm determine:

(a) the frequency
(b) the wave number
(c) the wavelength in Å
(d) the total energy (in Joules) associated with 1 mole of photons
(e) the "color"

Solution

To solve this problem we must know the following relationships:

$\nu\lambda = c$; $1/\lambda = \bar{\nu}$; 1 nm = 10^9 m = 10 Å

$E = h\nu$; $E_{molar} = h\nu \text{ x } N_A$ ($N_A = 6.02 \text{ x } 10^{23}$)

(a) ν (frequency) $= \dfrac{c}{\lambda} = \dfrac{3 \text{ x } 10^8 \text{ m/s}}{408 \text{ x } 10^{-9} \text{ m}} = 7.353 \text{ x } 10^{14} \text{s}^{-1}$

(b) $\bar{\nu}$ (wavenumber) $= \frac{1}{\lambda} = \frac{1}{408 \times 10^{-9}\text{m}} = 2.45 \times 10^{6}\text{m}^{-1}$

(c) $\lambda = 408 \times 10^{-9}\text{m} \times \frac{10^{10}\text{Å}}{\text{m}} = 4080\text{Å}$

(d) $E = h\nu \times N_A = 6.63 \times 10^{-34} \times 7.353 \times 10^{14} \times 6.02 \times 10^{23}$ J/mole

$= 2.93 \times 10^{5}$ J/mole = 293 kJ/mole

(e) visible spectrum: violet (500 nm) red (800 nm) 408 nm = UV

Problem #3

For “yellow radiation” (frequency, ν, = 5.09×10^{14} s^{-1}) emitted by activated sodium, determine:

(a) the wavelength (λ) in [m]
(b) the wave number ($\bar{\nu}$) in [cm^{-1}]
(c) the total energy (in kJ) associated with 1 mole of photons

Solution

(a) The equation relating ν and λ is $c = \nu\lambda$ where c is the speed of light = 3.00×10^{8} m.

$$\lambda = \frac{c}{\nu} = \frac{3.00 \times 10^{8}\ \text{m/s}}{5.09 \times 10^{14}\ \text{s}^{-1}} = 5.89 \times 10^{-7}\text{m}$$

(b) The wave number is 1/wavelength, but since the wavelength is in m, and the wave number should be in cm^{-1}, we first change the wavelength into cm:

$$\lambda = 5.89 \times 10^{-7}\ \text{m} \times 100\text{cm/m} = 5.89 \times 10^{-5}\ \text{cm}$$

Now we take the reciprocal of the wavelength to obtain the wave number:

$$\bar{\nu} = \frac{1}{\lambda} = \frac{1}{5.89 \times 10^{-5}\ \text{cm}} = 1.70 \times 10^{4}\ \text{cm}^{-1}$$

(c) The Einstein equation, $E = h\nu$, will give the energy associated with one photon since we know h, Planck’s constant, and ν. We need to multiply the energy obtained by Avogadro’s number to get the energy per mole of photons.

$h = 6.62 \times 10^{-34}$ J.s

$\nu = 5.09 \times 10^{14}$ s^{-1}

$E = h\nu = (6.62 \times 10^{-34}\ \text{J.s}) \times (5.09 \times 10^{14}\ \text{s}^{-1}) = 3.37 \times 10^{-19}$ J per photon

This is the energy in one photon. Multiplying by Avogadro's number:

$$E \cdot N_{Av} = (3.37 \times 10^{-19} \text{ J per photon})\left(\frac{6.023 \times 10^{23} \text{ photons}}{\text{mole}}\right)$$
$$= 2.03 \times 10^{5} \text{ J per mole of photons}$$

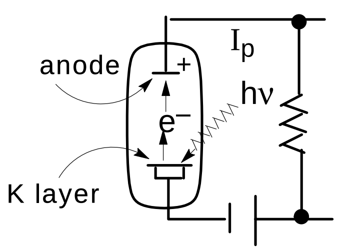
we get the energy per mole of photons. For the final step, the energy is converted into kJ:

$$2.03 \times 10^{5} \text{ J} \times \frac{1 \text{ kJ}}{1000 \text{ J}} = 2.03 \times 10^{2} \text{ kJ}$$

Problem #4

Potassium metal can be used as the active surface in a photodiode because electrons are relatively easily removed from a potassium surface. The energy needed is 2.15×10^{5} J per mole of electrons removed (1 mole $= 6.02 \times 10^{23}$ electrons). What is the longest wavelength light (in nm) with quanta of sufficient energy to eject electrons from a potassium photodiode surface?

Solution



I_p, the photocurrent, is proportional to the intensity of incident radiation, i.e. the number of incident photons capable of generating a photoelectron.

This device should be called a phototube rather than a photodiode – a solar cell is a photodiode.

Required: $1\text{eV} = 1.6 \times 10^{-19}$ J
$E_{rad} = h\nu = (hc)/\lambda$

The question is: below what threshold energy ($h\nu$) will a photon no longer be able to generate a photoelectron?

$$2.15 \times 10^{5} \text{ J/mole photoelectrons} \times \frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ photoelectrons}}$$

$$= 3.57 \times 10^{-19} \text{ J/photoelectron}$$

$$\lambda_{threshold} = \frac{hc}{3.57 \times 10^{-19}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{3.57 \times 10^{-19}} = 5.6 \times 10^{-7}\text{m} = 560 \text{ nm}$$

Problem #5

For red light of wavelength (λ) 6.7102 x 10^{-5} cm, emitted by excited lithium atoms, calculate:

(a) the frequency (ν) in s–1;
(b) the wave number ($\bar{\nu}$) in cm^{-1};
(c) the wavelength (λ) in nm;
(d) the total energy (in Joules) associated with 1 mole photons of the indicated wavelength.

Solution

(a) $c = \lambda\nu$ and $\nu = c/\lambda$ where ν is the frequency of radiation (number of waves/s).

For: $\lambda = 6.7102 \times 10^{-5}\text{cm} = 6.7102 \times 10^{-7}\text{m}$

$$\nu = \frac{2.9979 \times 10^{8}\ \text{ms}^{-1}}{6.7102 \times 10^{-7}\ \text{m}} = 4.4677 \times 10^{14}\text{s}^{-1} = 4.4677\ \text{Hz}$$

(b) $$\bar{\nu} = \frac{1}{\lambda} = \frac{1}{6.7102 \times 10^{-7}\ \text{m}} = 1.4903 \times 10^{6}\ \text{m}^{-1} = 1.4903 \times 10^{4}\ \text{cm}^{-1}$$

(c) $$\lambda = 6.7102 \times 10^{-5}\ \text{cm} \times \frac{1\ \text{nm}}{10^{-7}\ \text{cm}} = 671.02\ \text{cm}$$

(d) $$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34}\ \text{Js} \times 2.9979 \times 10^{8}\ \text{ms}^{-1}}{6.7102 \times 10^{-7}\ \text{m}}$$

$$= 2.96 \times 10^{-19}\ \text{J/photon} = 1.78 \times 10^{5}\ \text{J/mole photons}$$

Problem #6

Calculate the “Bohr radius” for He^{+}.

Solution

In its most general form, the Bohr theory considers the attractive force (Coulombic) between the nucleus and an electron being given by:

$$F_c = \frac{Ze^2}{4\pi\varepsilon_o r^2}$$

where Z is the charge of the nucleus (1 for H, 2 for He, etc.). Correspondingly, the electron energy (E_{el}) is given as:

$$E_{el} = -\frac{Z^2}{n^2}\,\frac{me^4}{8h^2\varepsilon_o^{\,2}}$$

and the electronic orbit (r_n):

$$r_n = \frac{n^2}{Z}\,\frac{h^2\varepsilon_o}{\pi me^2}$$

$$r_n = \frac{n^2}{Z}a_o$$

$$\text{For He}^+ \ (Z=2),\ r_1 = \frac{1}{2}a_o = \frac{0.529}{2} \times 10^{-10}\ \text{m} = 0.264\text{Å}$$

Problem #7

(a) Determine the atomic weight of He^{++} from the values of its constituents.
(b) Compare the value obtained in (a) with the value listed in your Periodic Table and explain any discrepancy if such is observed. (There is only one natural ^4_2He isotope.)

Solution

(All relevant data are in the P/T and T/C.)

(a) The mass of the constituents (2p + 2n) is given as:

$$\begin{aligned} 2p &= 2 \times 1.6726485 \times 10^{-24}\ \text{g} \\ \underline{2n} &= \underline{2 \times 16749543 \times 10^{-24}}\ \text{g} \\ (2p + 2n) &= 6.6952056 \times 10^{-24}\ \text{g} \end{aligned}$$

The atomic weight (calculated) in amu is given as:

$$\frac{6.6952056 \times 10^{-24}\ \text{g}}{1.660565 \times 10^{-24}\text{g}} / \text{amu}$$

$$\text{He} = 4.03188\ \text{amu}$$

(b) The listed atomic weight for He is 4.00260 (amu). The data indicate a mass defect of

2.92841×10^{-2} amu, corresponding to 4.8628×10^{-26} g/atom.

This mass defect appears as nuclear bond energy:

$$\Delta E = 4.8628 \times 10^{-29} \text{ kg} \times 9 \times 10^{16} \text{ m}^2/\text{s}^2 = 4.3765 \times 10^{-12} \text{ J/atom}$$

$$= 2.6356 \times 10^{12} \text{ J/mole}$$

$$\Delta m = \frac{\Delta E}{c^2} = 2.928 \times 10^{-5} \text{ kg/mole} = 0.02928 \text{ g/mole}$$