Materials Selection for Mechanical Design I

A Brief Overview of a Systematic Methodology

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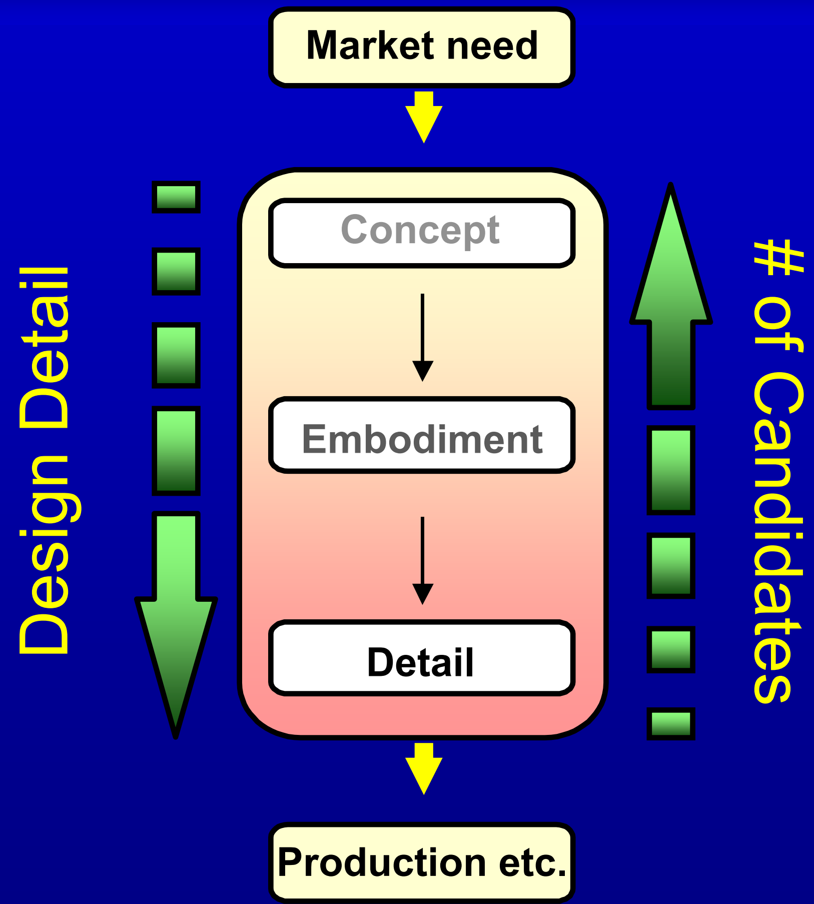
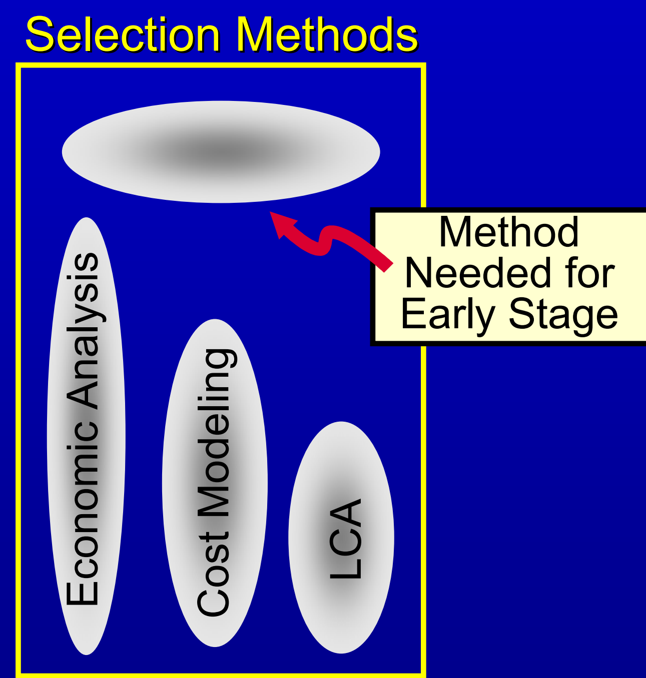
Relationship To Course

- A key concept throughout this course is how to select among technology choices
  - Economic Analysis
  - Cost Modeling
  - Life Cycle Assessment
- Focus has been on economic assessment of alternatives
- How does this fit into larger technology choice problem?

Approach Changes as Design Evolves





What parameters define material selection?

Example: SUV Liftgate

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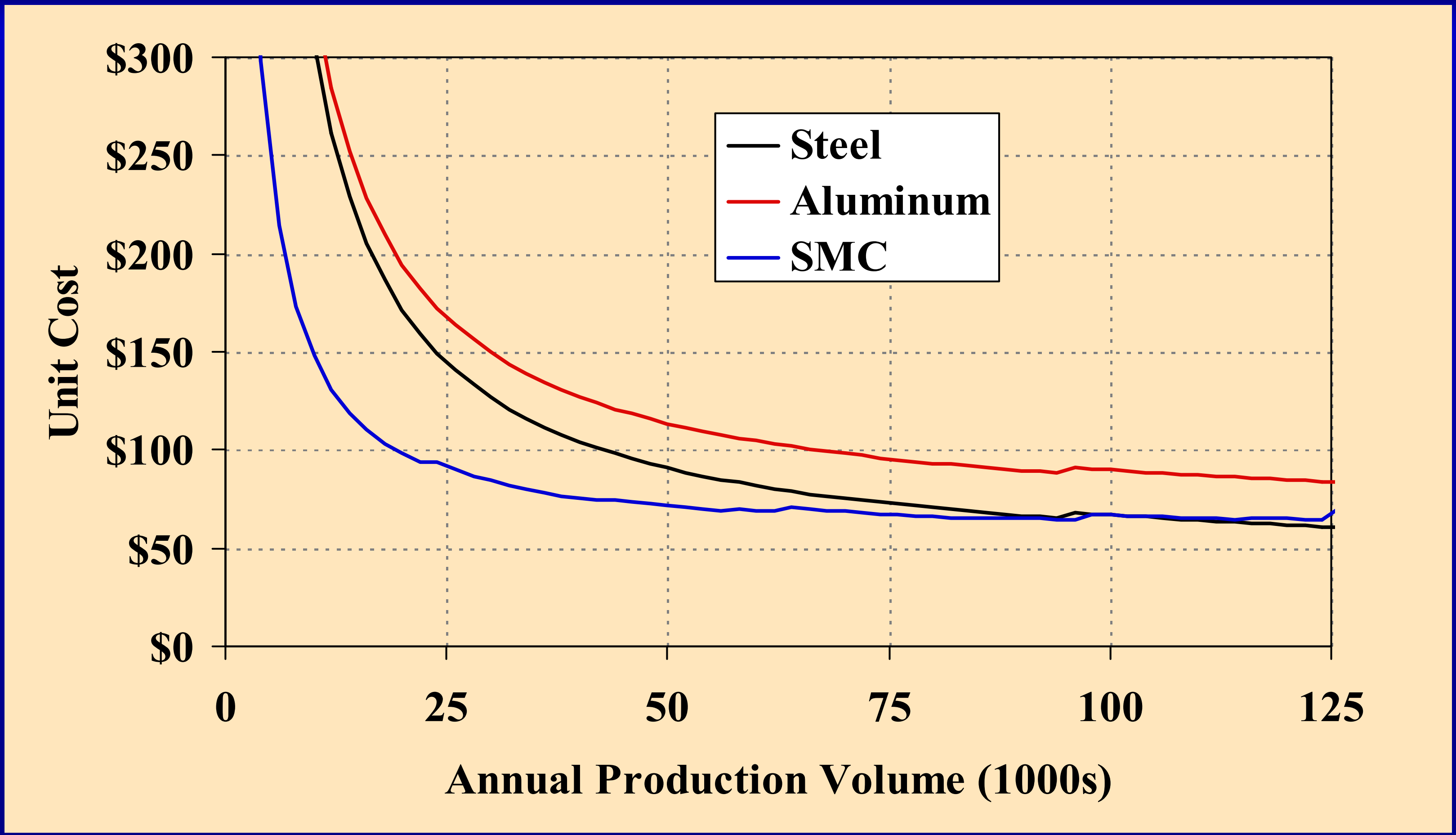
Schematic of components in an SUV liftgate (rear door).

Attractive Options
May Be Found Outside of Expertise



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Need Method for Early Material Selection: Ashby Methodology*

Four basic steps

1. **Translation:** ***express design requirements as constraints & objectives***
2. **Screening:** ***eliminate materials that cannot do the job***
3. **Ranking:** ***find the materials that do the job best***
4. **Supporting information:** ***explore pedigrees of top-ranked candidates***

M.F. Ashby, *Materials Selection in Mechanical Design, 3rd Ed.*, Elsevier, 2005

First Step: Translation

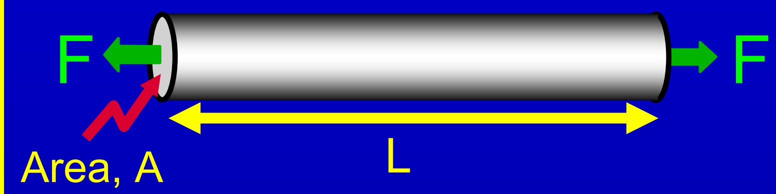
"Express design requirements as constraints and objectives"

Using design requirements, analyze four items:

- **Function: What does the component do?**
  - *Do not limit options by specifying implementation w/in function*
- **Objective: What essential conditions must be met?**
  - *In what manner should implementation excel?*
- **Constraints: What is to be maximized or minimized?**
  - *Differentiate between binding and soft constraints*
- **Free variables: Which design variables are free?**
  - *Which can be modified?*
  - *Which are desirable?*

Identifying Desirable Characteristics
Example: Materials for a Light, Strong Tie

- **Function:**
  - **Support a tension load**
- **Objective:**
  - **Minimize mass**
- **Constraints:**
  - **Length specified**
  - **Carry load F, w/o failure**
- **Free variables:**
  - **Cross-section area**
  - **Material**



- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $F / A < \sigma_y$

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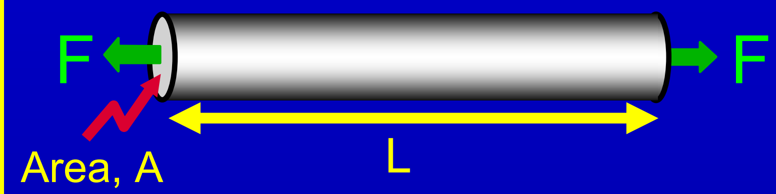
Identifying Desirable Characteristics
Example: Materials for a Light, Strong Tie

- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $F / A < \sigma_y$
- **Rearrange to eliminate free variable**

$$m \geq (F)(L)\left(\frac{\rho}{\sigma_y}\right)$$

- **Minimize weight by minimizing** $\left(\frac{\rho}{\sigma_y}\right)$



Material Index

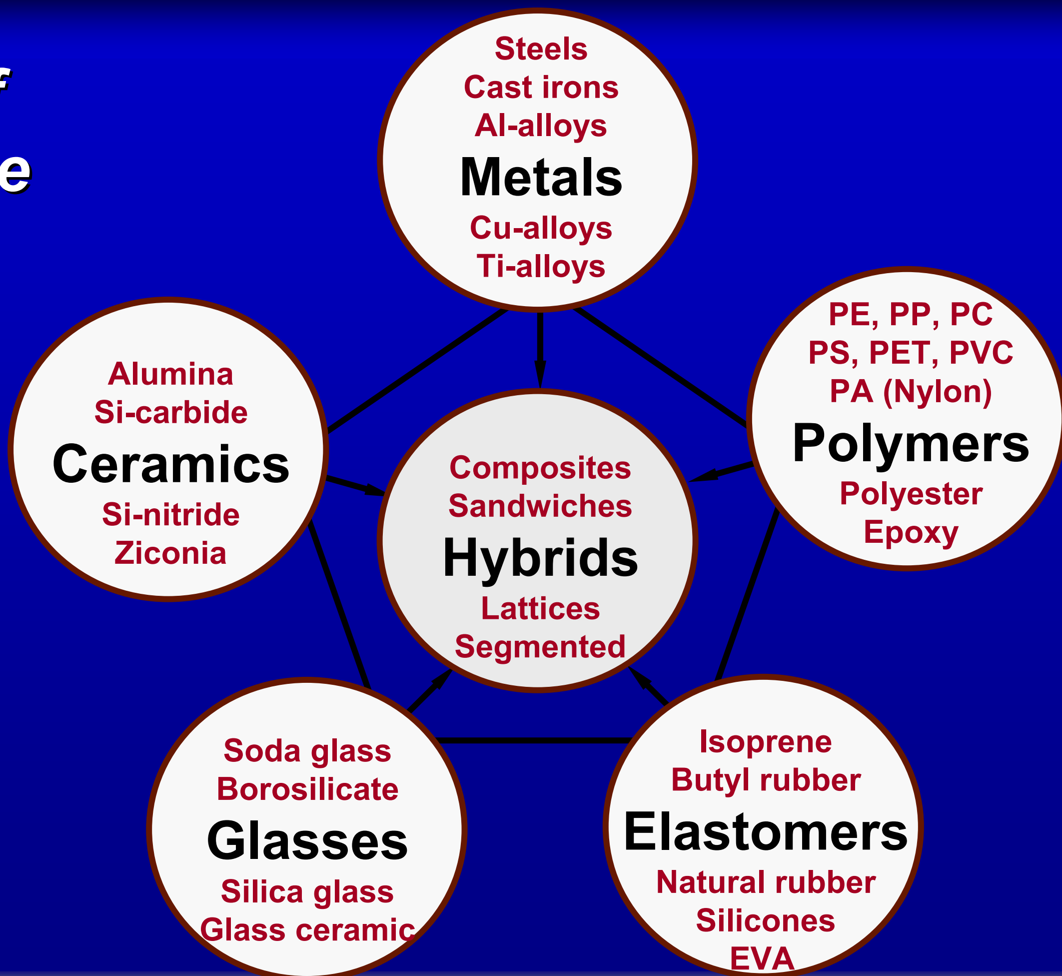
$$\left(\frac{\sigma_y}{\rho}\right)$$

or maximize

Second Step: Screening

"Eliminate materials that cannot do the job"

Need effective way of evaluating large range of material classes and properties



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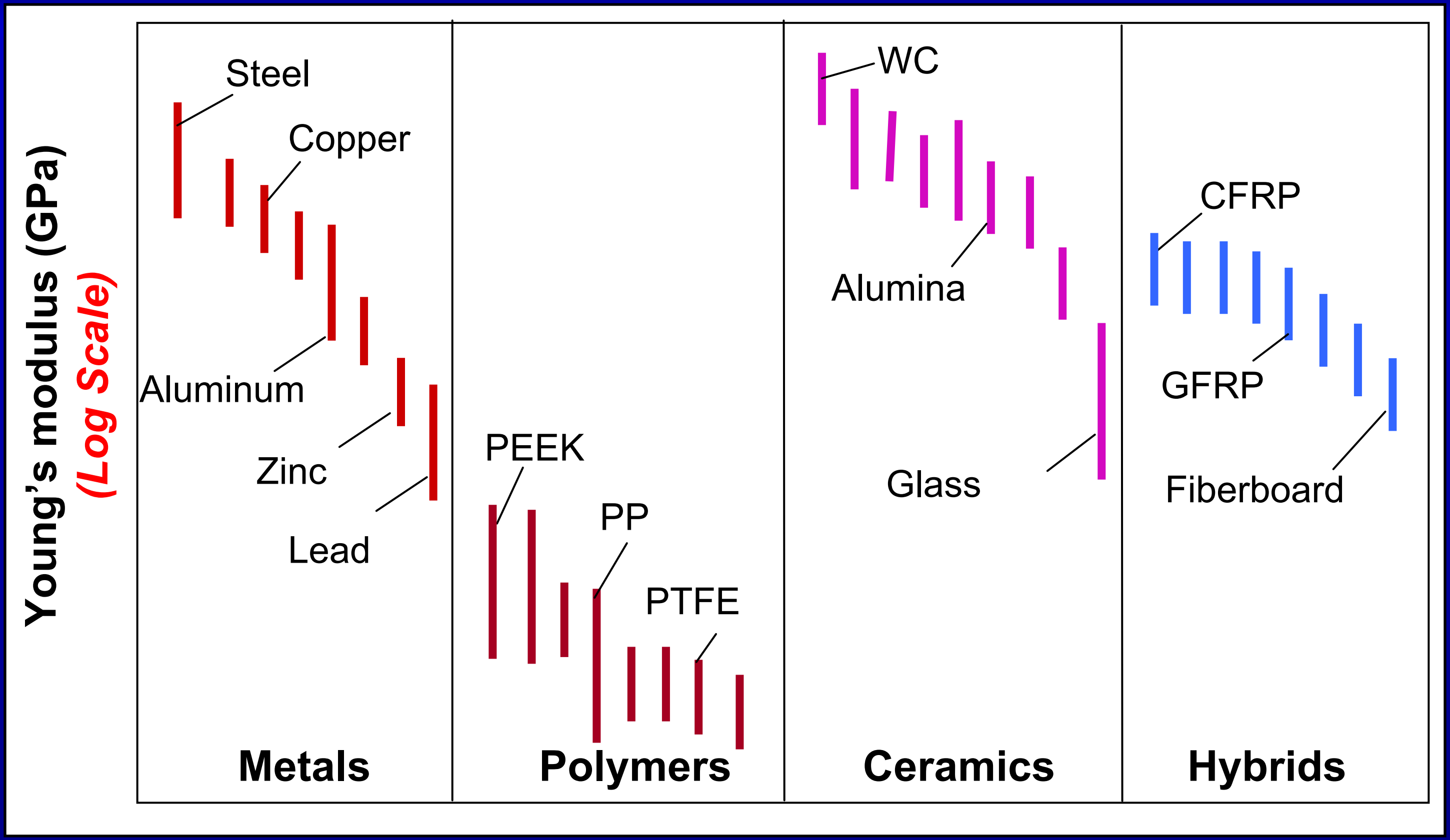
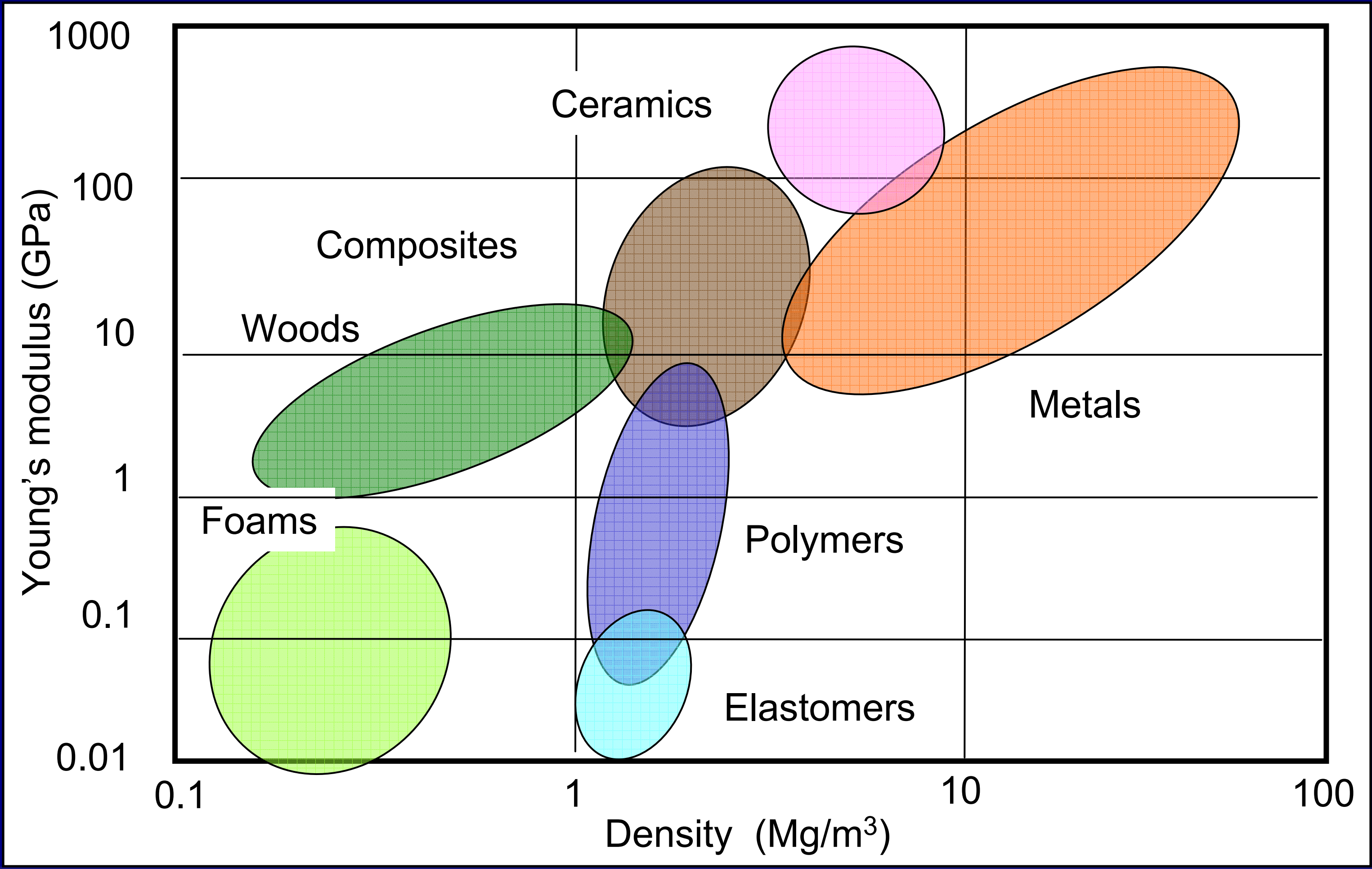
Comparing Material Properties: *Material Bar Charts*



Good for elementary selection (e.g., find materials with large modulus)

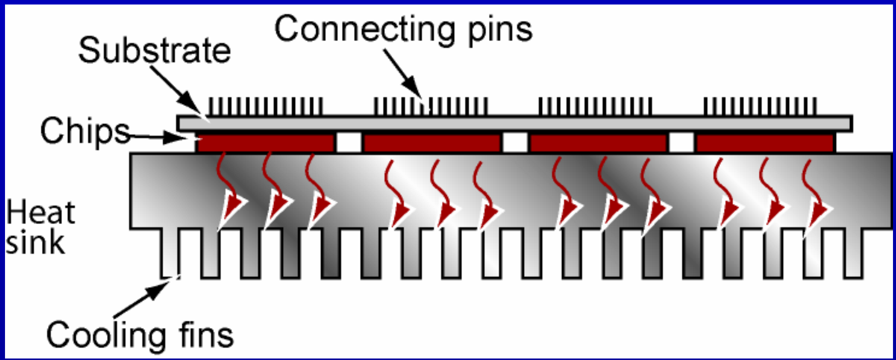
Comparing Material Properties: *Material Property Charts*



Screening Example: Heat Sink for Power Electronics

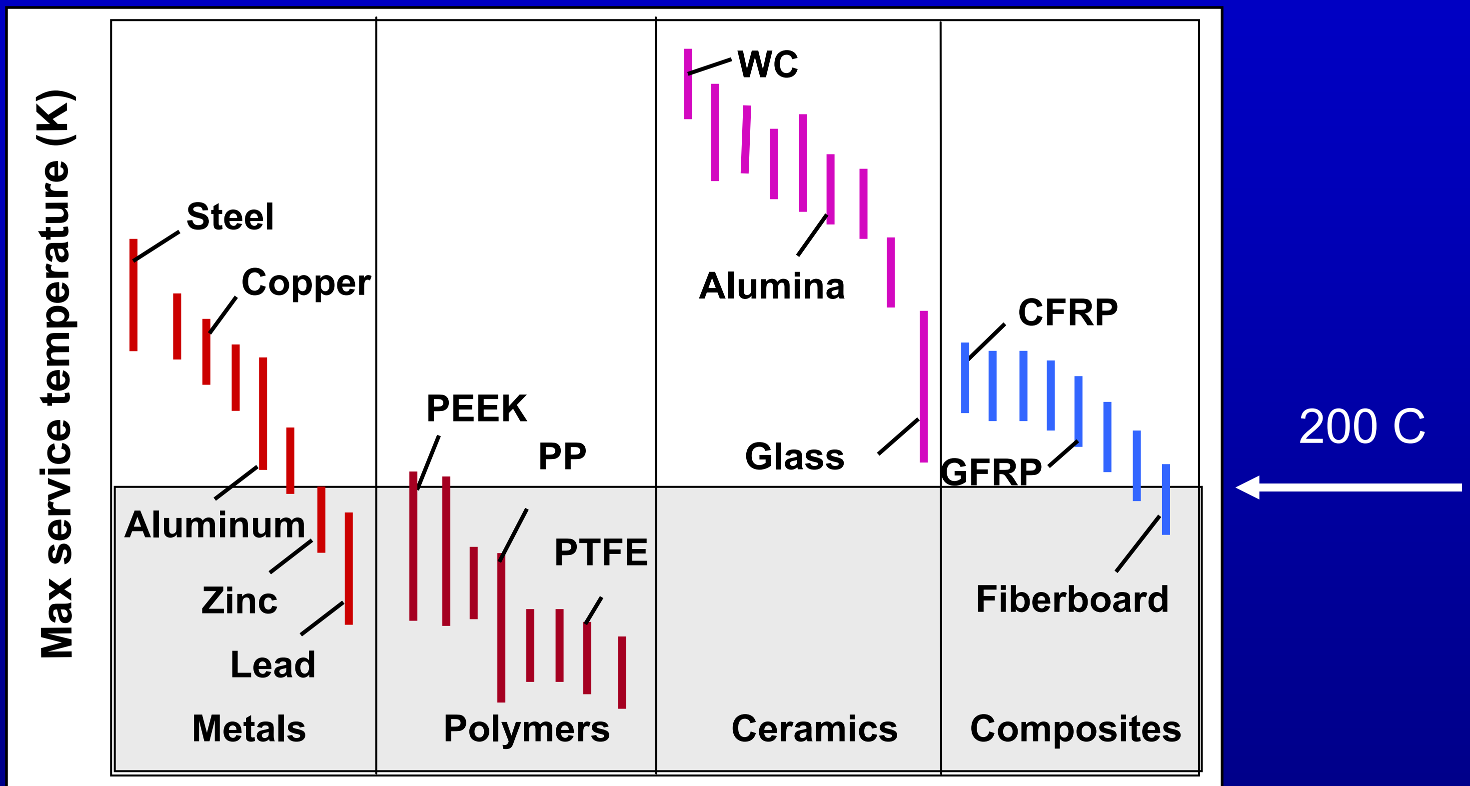
- **Function:**
  - **Heat Sink**
- **Constraints:**

1. **Max service temp > 200 C**
2. **Electrical insulator → R > 10^{20} μohm cm**
3. **Thermal conductor → T-conduct. λ > 100 W/m K**
4. **Not heavy → Density < 3 Mg/m^3**

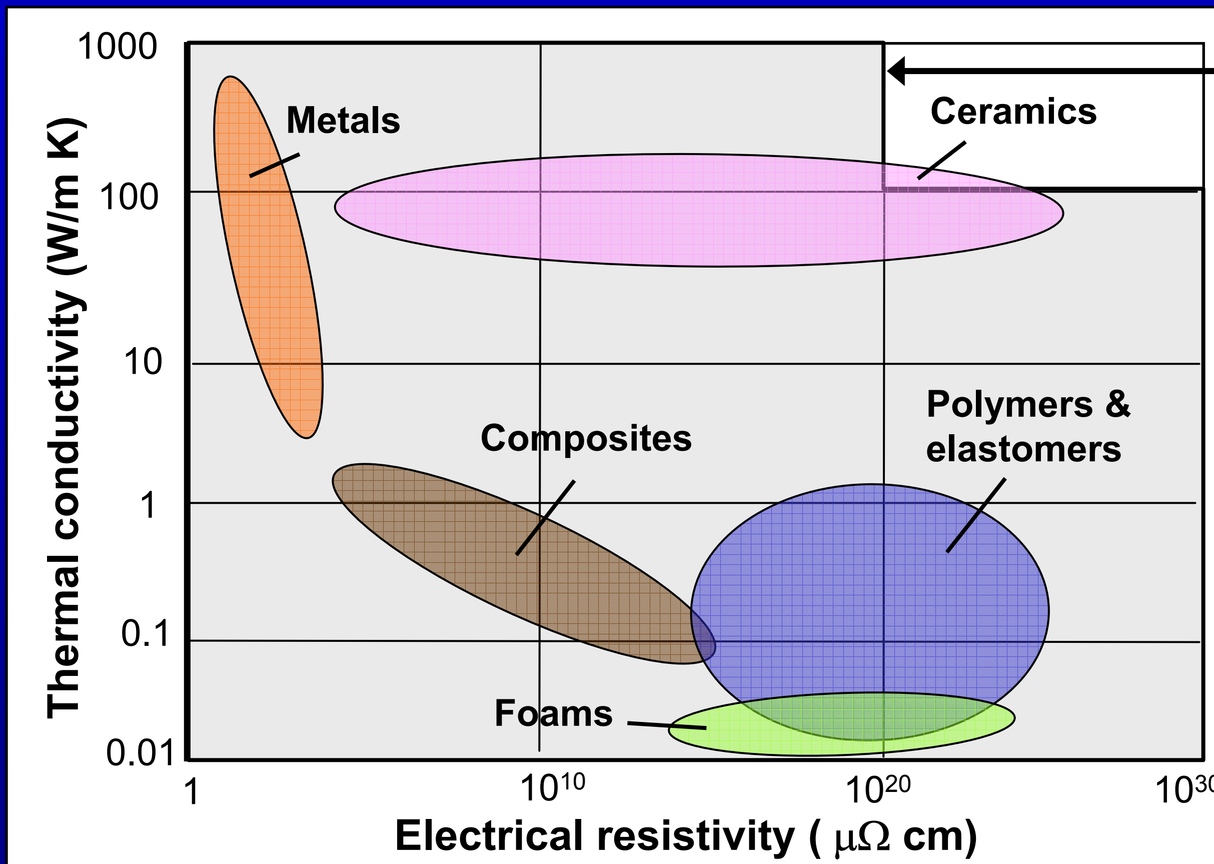


- **Free Variables:**
  - **Materials and Processes**

Heat Sink Screening: Bar Chart



Heat Sink Screening: Property Chart



R > 10^{20} μΩ cm

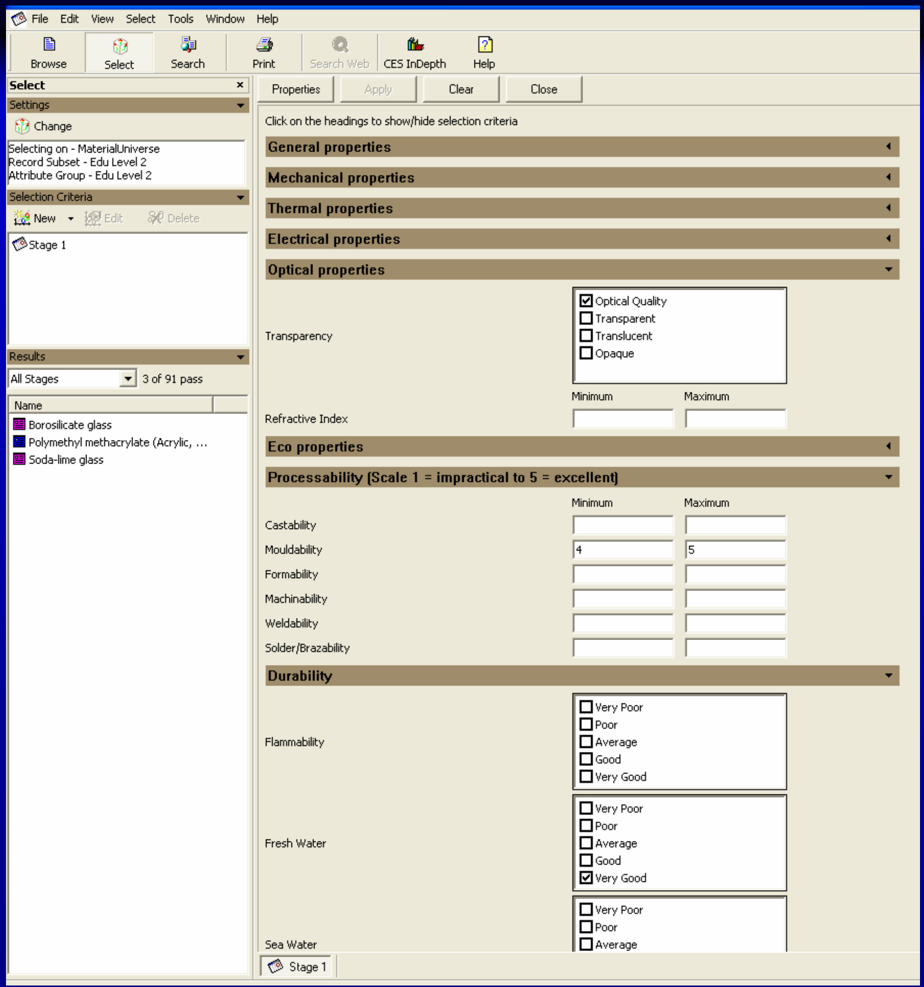
λ > 100 W/m K

Example using Granta Software: Automobile Headlight Lens

- **Function:**
  - **Protect bulb and lens; focus beam**
- **Objective:**
  - **Minimize cost**
- **Constraints:**
  - **Transparent w/ optical quality**
  - **Easily molded**
  - **Good resistance to fresh and salt water**
  - **Good resistance to UV light**
  - **Good abrasion resistance (high hardness)**
- **Free variables:**
  - **Material choice**

Photo of headlight removed for copyright reasons.

Selection Criteria – Limit Stage



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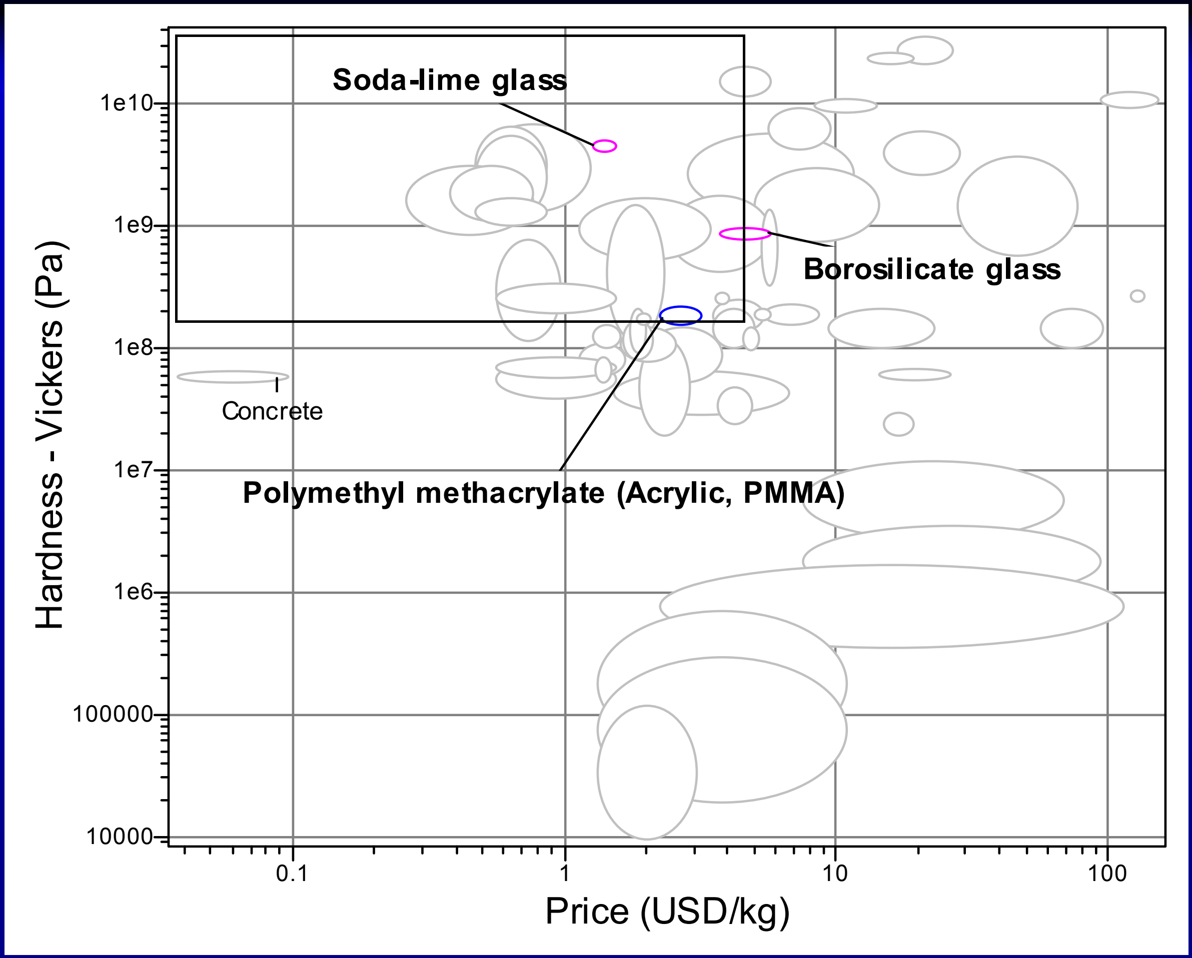
Property Chart



- Cheapest, hardest material is soda-lime glass – used in car headlights
- For plastics, cheapest is PMMA – used in car tail lights

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Third Step: Ranking
"Find the materials that do the job best"

What if multiple materials are selected after screening?

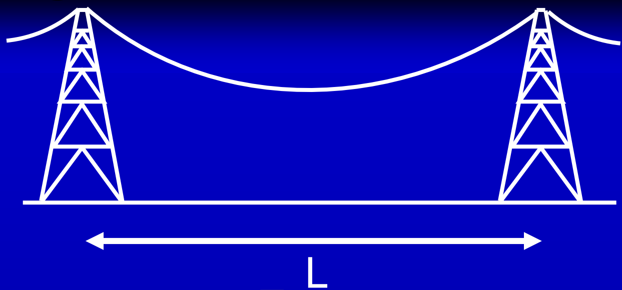
Which one is best?

What if there are multiple material parameters for evaluation?

Use Material Index

Single Property Ranking Example: Overhead Transmission Cable

- **Function:**
  - **Transmit electricity**
- **Objective:**
  - **Minimize electrical Resistance**
- **Constraints:**
  - **Length L and section A are specified**
  - **Must not fail under wind or ice-load → required tensile strength > 80 MPa**
- **Free variables:**
  - **Material choice**



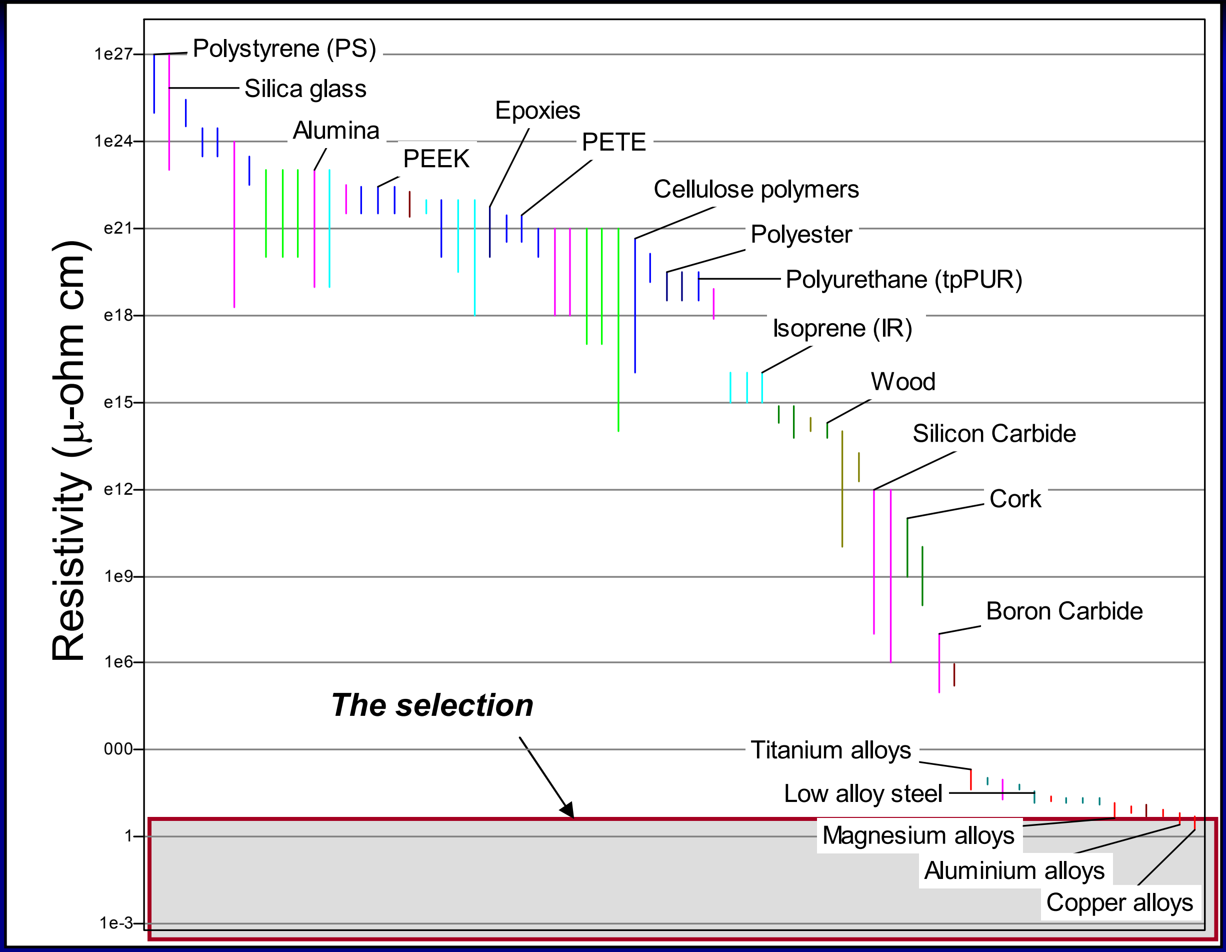
$$R = \rho_e \frac{L}{A}$$

Electrical resistivity

Screen on strength, rank on resistivity

Single Property Ranking Example: Overhead Transmission Cable

- Screening on strength eliminates polymers, some ceramics
- Ranking on resistivity selects *Al and Cu alloys*



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Advanced Ranking: The Material Index

The method

1. Identify function, constraints, objective and free variables
   - List simple constraints for screening
2. Write down equation for objective -- the "performance equation"
   - If objective involves a free variable (other than the material):
     - *Identify the constraint that limits it*
     - *Use this to eliminate the free variable in performance equation*
3. Read off the combination of material properties that maximizes performance -- the material index
4. Use this for ranking

The Performance Equation, *P*

$$P = \left[\begin{pmatrix} \text{Functional} \\ \text{requirements, } F \end{pmatrix}, \begin{pmatrix} \text{Geometric} \\ \text{parameters, } G \end{pmatrix}, \begin{pmatrix} \text{Material} \\ \text{properties, } M \end{pmatrix} \right]$$

or

$$P = f\left(F, G, M\right)$$

Use constraints to eliminate free variable

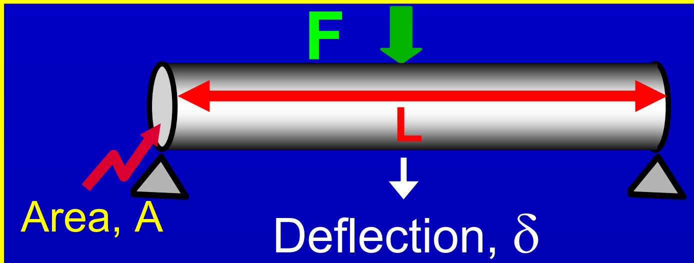
***P* from previous example of a light, strong tie:**

$$m \geq (F)(L)\left(\frac{\rho}{\sigma_y}\right)$$

The Material Index

Example: Materials for a stiff, light beam

- **Function:**
  - **Support a bending load**
- **Objective:**
  - **Minimize mass**
- **Constraints:**
  - **Length specified**
  - **Carry load F, without too much deflection**
- **Free variables:**
  - **Cross-section area**
  - **Material**



- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $S = \frac{F}{\delta} \geq \frac{CEI}{L^3}$

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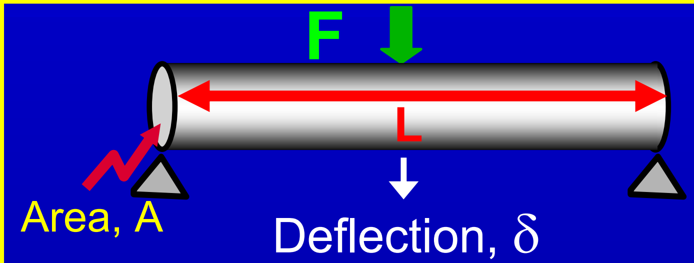
The Material Index

Example: Materials for a stiff, light beam

- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $S = \dfrac{F}{\delta} \geq \dfrac{CEI}{L^3}$
- **Rearrange to eliminate free variable**
  - $m = \left(\dfrac{4F\pi}{\delta}\right)^{1/2} \left(\dfrac{L^{5/2}}{C^{1/2}}\right) \left(\dfrac{\rho}{E^{1/2}}\right)$
- **Minimize weight by minimizing** $\left(\dfrac{\rho}{E^{1/2}}\right)$



Material Index

$$\left(\frac{E^{1/2}}{\rho}\right)$$

or maximize

Material Index Calculation Process Flow

Each combination of { Function, Constraint, Objective, Free variable } has a characterizing material index

FUNCTION

- Tie
- **Beam**
- Shaft
- Column

Mechanical, Thermal, Electrical...

CONSTRAINTS

- **Stiffness specified**
- Strength specified
- Fatigue limit
- Geometry specified

OBJECTIVE

- Minimum cost
- **Minimum weight**
- Maximum energy storage
- Minimum eco- impact

INDEX

$$M = \left[\frac{E^{1/2}}{\rho} \right]$$

Maximize this!

Material Index Examples

- An objective defines a performance metric: e.g. mass or resistance
- The equation for performance metric contains material properties
- Sometimes a single property
- Sometimes a combination

} Either is a Material Index

Objective:
Minimize Mass

Performance Metric:
Mass

Material Indices for a Beam

Loading	Stiffness Limited	Strength Limited
Tension	E/ρ	σ_f/ρ
Bending	$E^{1/2}/\rho$	$\sigma_f^{2/3}/\rho$
Torsion	$G^{1/2}/\rho$	$\sigma_f^{2/3}/\rho$

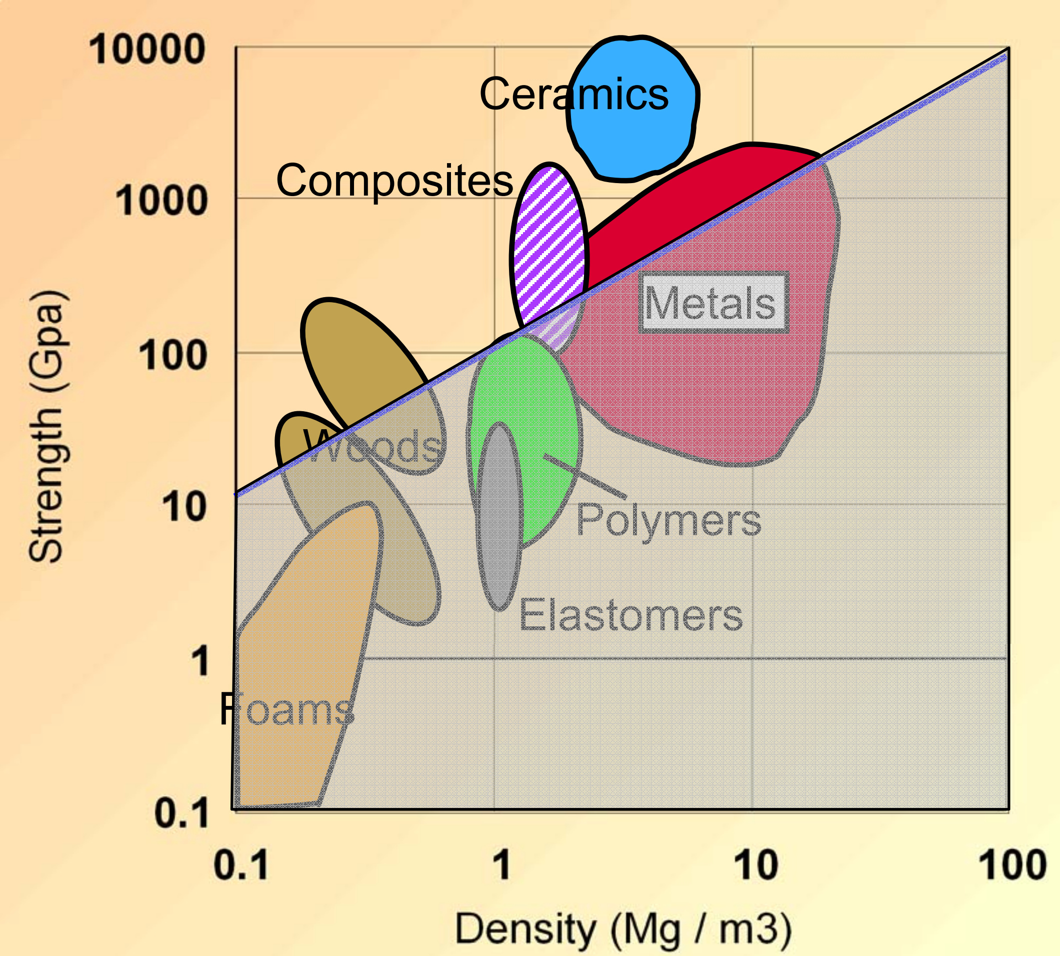
Maximize!

Optimized Selection Using Material Indices & Property Charts: Strength

Example:

Tension Load, strength limited

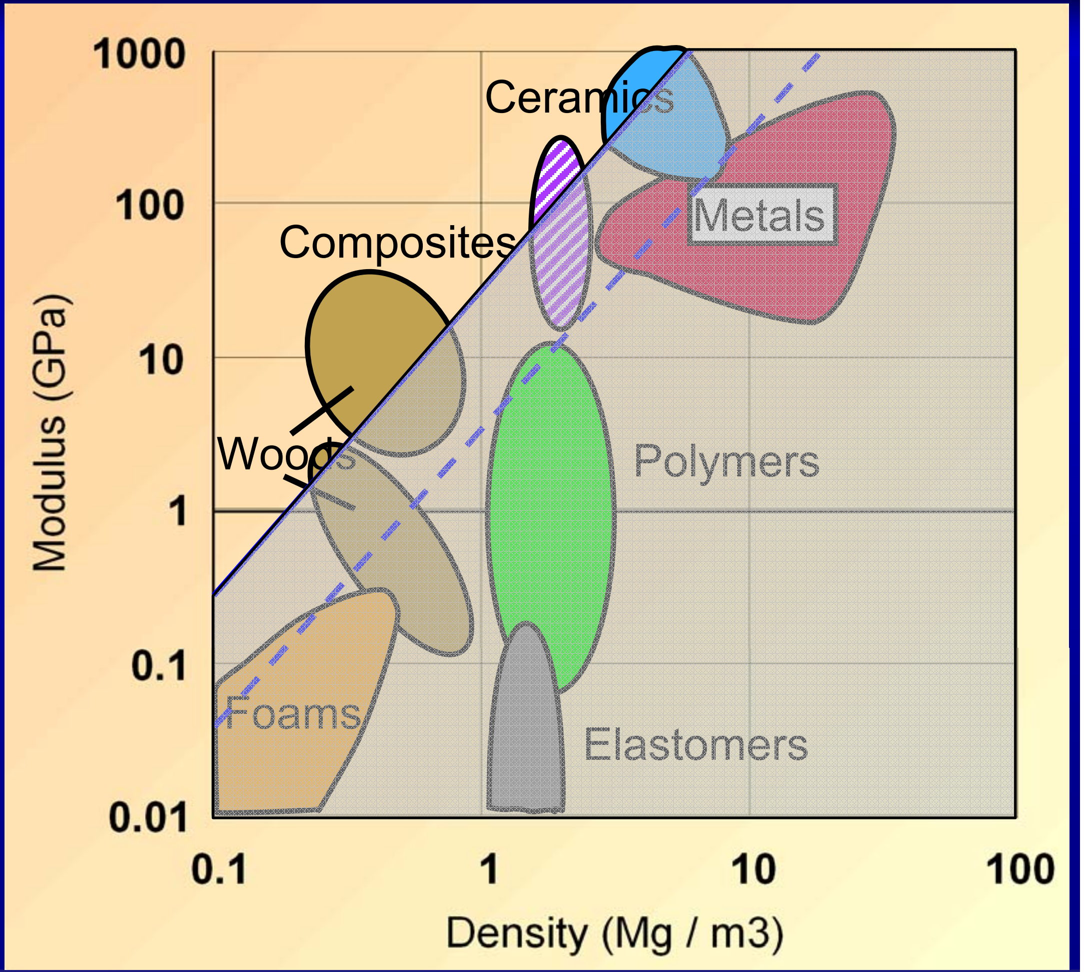
- **Maximize:** $M = \sigma/\rho$
- **In log space:** $\log \sigma = \log \rho + \log M$
- **This is a set of lines with slope=1**
- **Materials above line are candidates**



Material Indices & Property Charts: Stiffness

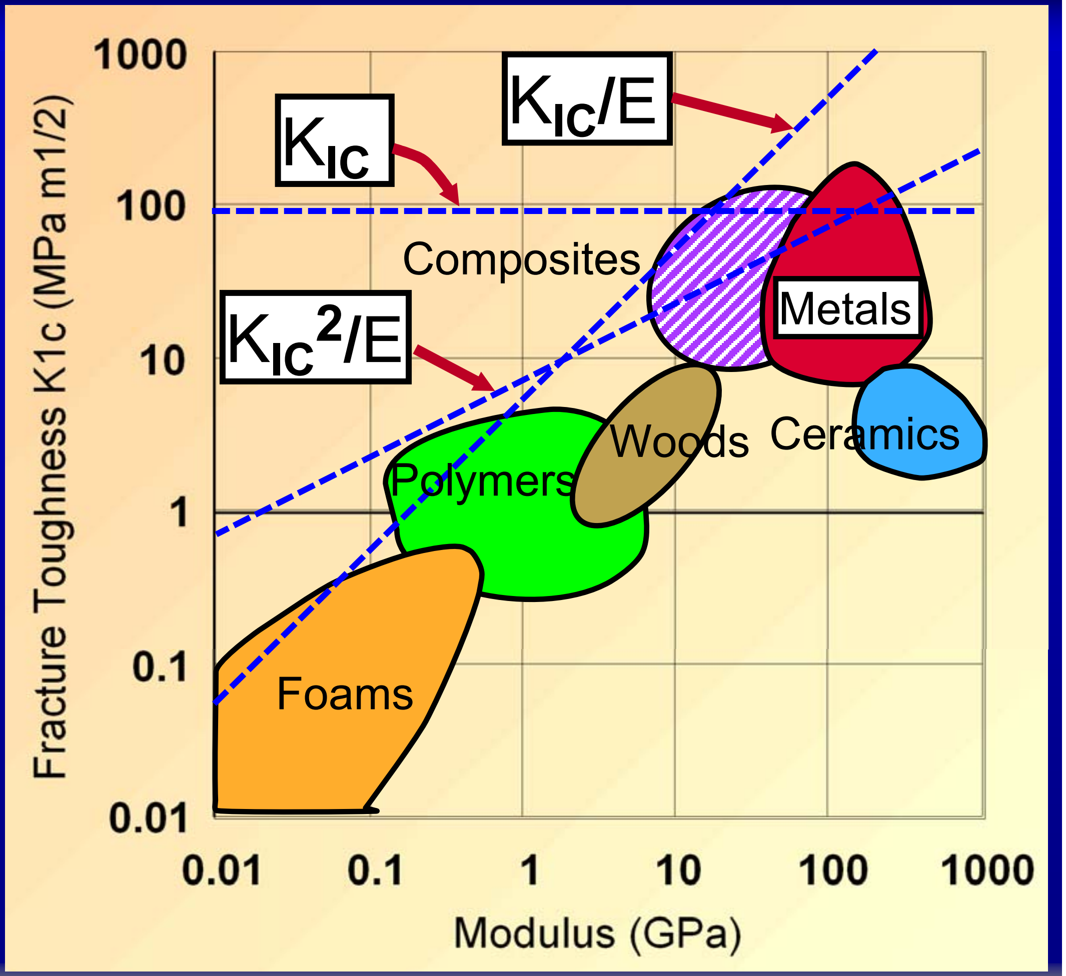
Example:
Stiff beam

- **Maximize:** $M = E^{1/2}/\rho$
- **In log space:**
  $\log E =$
  $2\,(\log \rho + \log M)$
- **This is a set of lines with slope=2**
- **Candidates change with objective**



Material Indices & Property Charts: Toughness

- **Load-limited**
  - $M = K_{IC}$
  - **Choose tough metals, e.g. Ti**
- **Energy-limited**
  - $M = K_{IC}^2 / E$
  - **Composites and metals compete**
- **Displacement-limited**
  - $M = K_{IC} / E$
  - **Polymers, foams**



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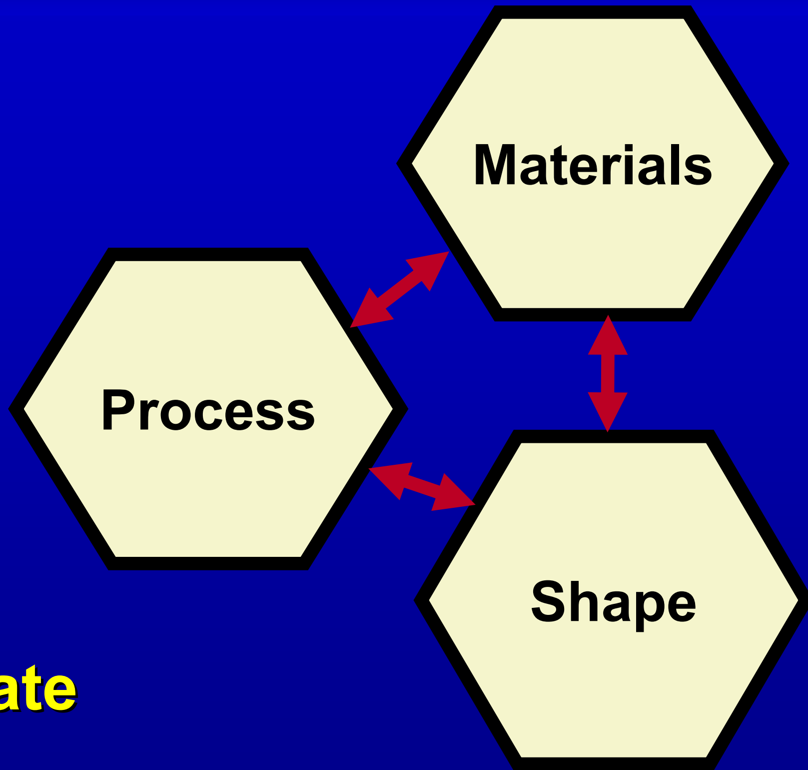
Considering Multiple Objectives/Constraints

- With multiple **constraints**:
  - Solve each individually
  - Select candidates based on each
  - Evaluate performance of each
  - Select performance based on most limiting
    - *May be different for each candidate*
- With multiple **objectives**:
  - Requires utility function to map multiple metrics to common performance measures

Method for Early Technology Screening

- Design performance is determined by the combination of:
  - Shape
  - Materials
  - Process
- Underlying principles of selection are unchanged
  - BUT, do not underestimate impact of shape or the limitation of process



Ashby Method for Early Material Selection:

Four basic steps

1. **Translation:** *express design requirements as constraints & objectives*
2. **Screening:** *eliminate materials that cannot do the job*
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Summary

- Material affects design based on
  - Geometric specifics
  - Loading requirements
  - Design constraints
  - Performance objective
- Effects can be assessed analytically
- Keep candidate set large as long as is feasible
- Materials charts give quick overview; software can be used to more accurately find options
- Remember, strategic considerations can alter best choice

Example Problem: Table Legs

Figure by MIT OCW.

- Want to redesign table with thin unbraced cylindrical legs
- Want to minimize cross-section and mass without buckling
- Toughness and cost are factors

Table Legs: Problem Definition

- **Function:**
  - Support compressive loads
- **Objective:**
  - Minimize mass
  - Maximize slenderness
- **Constraints:**
  - Length specified
  - Must not buckle
  - Must not fracture
- **Free variables:**
  - Cross-section area
  - Material

Performance Equation

$$m = \pi r^2 l \rho$$

$$P_{crit} = \frac{\pi^2 EI}{l^2} = \frac{\pi^3 E r^4}{4 l^2}$$

Table Legs: Material Indices

Use constraints to eliminate free variable, *r*

$$m \geq \left(\frac{4P}{\pi}\right)^{1/2} (l)^2 \left[\frac{\rho}{E^{1/2}}\right]$$

Functional Requirements — Geometric Parameters — Material Properties

Minimize mass by maximizing *M₁*

$$M_1 = \frac{E^{1/2}}{\rho}$$

For slenderness, calculate *r* at max load

$$r \geq \left(\frac{4P}{\pi^3}\right)^{1/4} (l)^{1/2} \left[\frac{1}{E}\right]^{1/4}$$

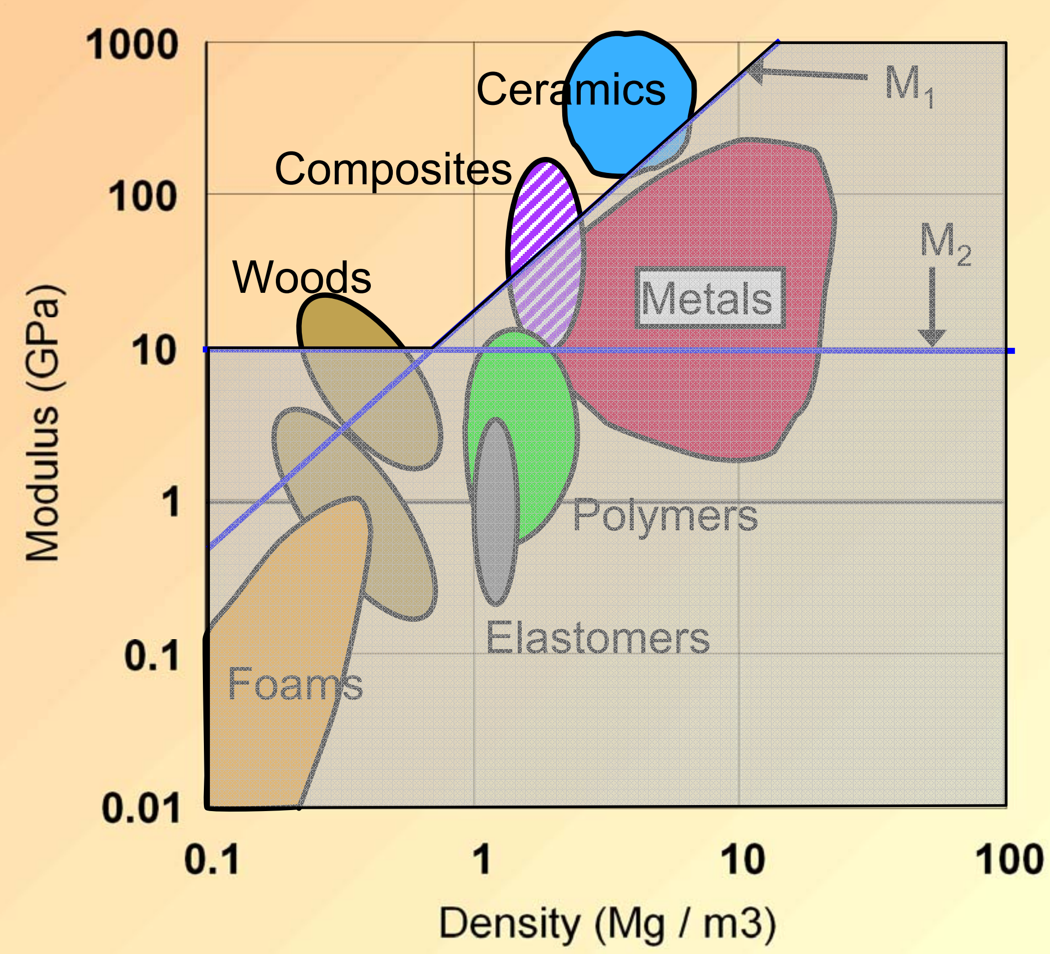
Functional Requirements — Geometric Parameters — Material Properties

Maximize slenderness by maximizing *M₂*

$$M_2 = E$$

Table Legs: Material Selection

- Eliminated
  - Metals (too heavy)
  - Polymers (not stiff enough)
- Possibilities: Ceramics, wood, composites
- Final choice: **wood**
  - Ceramics too brittle
  - Composites too expensive
- Note: higher constraint on modulus eliminates wood



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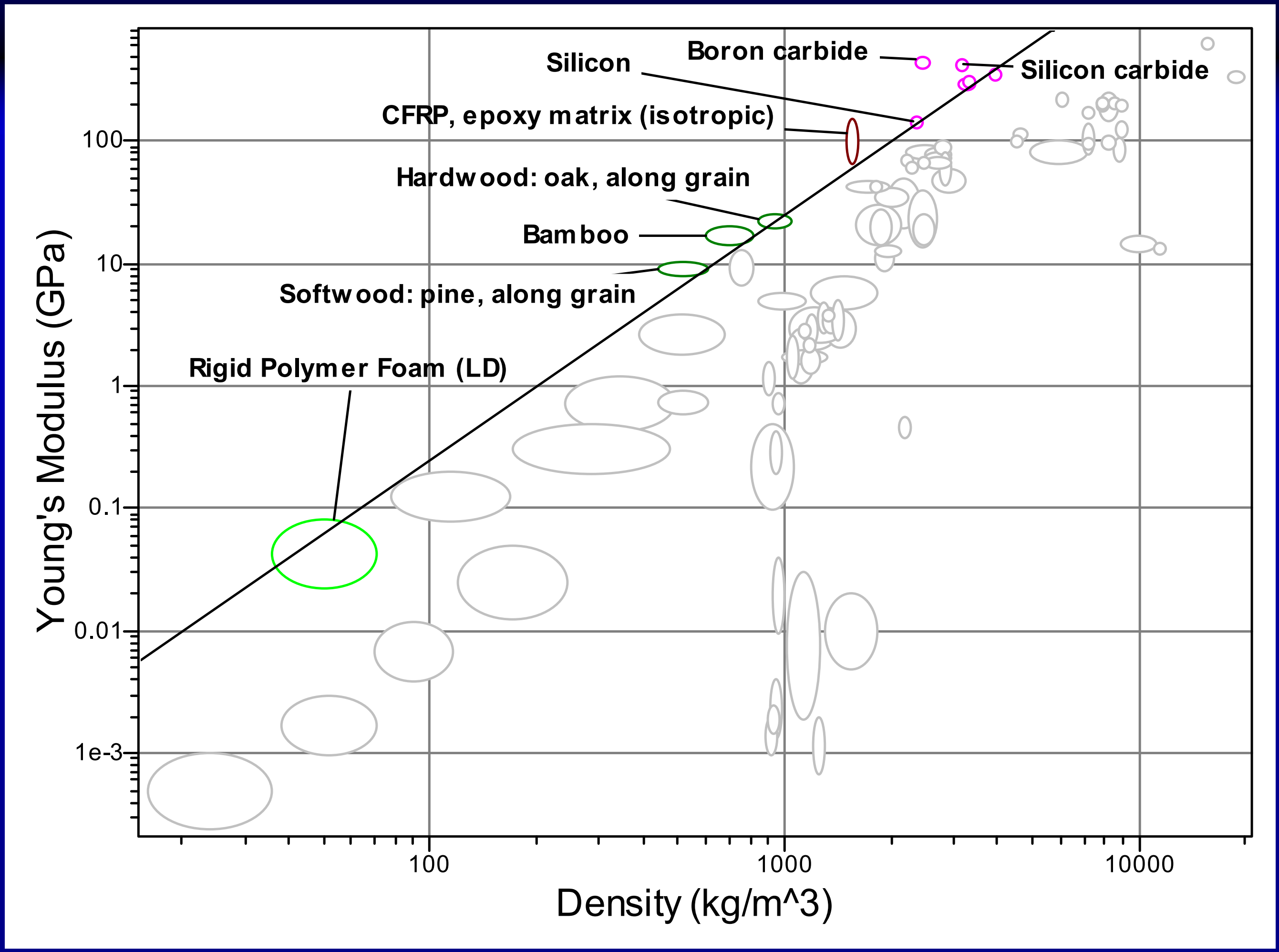
Material Index 1



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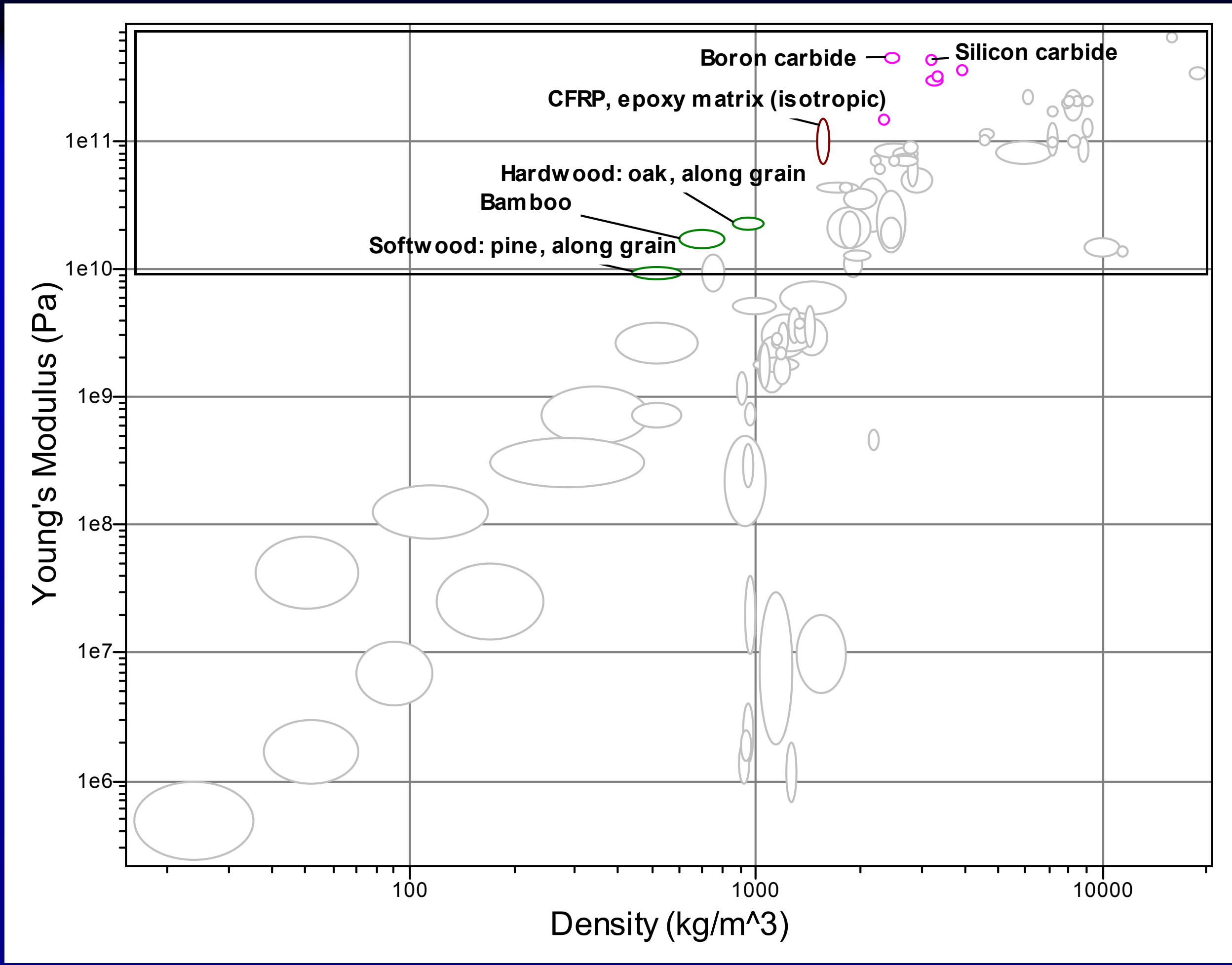
Material Index 2



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Example: Heat-Storing Wall

- Outer surface heated by day
- Air blown over inner surface to extract heat at night
- Inner wall must heat up ~12h after outer wall

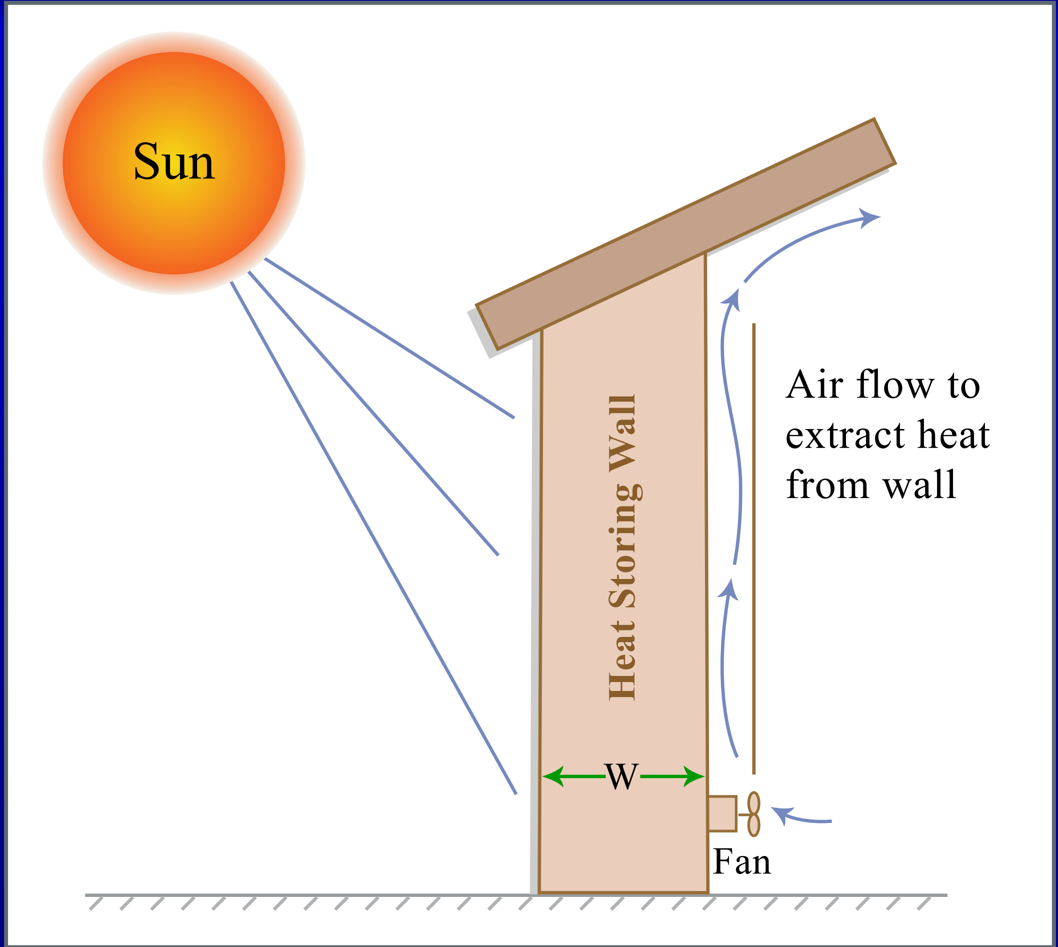

Figure by MIT OCW.

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Heat-Storing Wall: Problem Definition

- **Function:**
  - Heat storing medium
- **Objective:**
  - Maximize thermal energy stored per unit cost
- **Constraints:**
  - Heat diffusion time ~12h
  - Wall thickness ≤ 0.5 m
  - Working temp T_{max}>100 C
- **Free variables:**
  - Wall thickness, *w*
  - Material

Heat content: $Q = w\rho C_p \Delta T$

Heat diffusion distance:

$$w = \sqrt{2at}$$

$C_p =$ **Specific Heat**

$a =$ **Thermal Diffusivity** $= \dfrac{\lambda}{\rho C_p}$

$\lambda =$ **Thermal Conductivity**

Heat-Storing Wall: Material Indices

Eliminate free variable:

$$Q = \sqrt{2t}\,\Delta T a^{1/2} \rho C_p$$

Insert λ to obtain Performance Eqn:

$$Q = \sqrt{2t}\,\Delta T \left(\frac{\lambda}{a^{1/2}} \right)$$

Maximize: $M_1 = \dfrac{\lambda}{a^{1/2}}$

Thickness restriction:

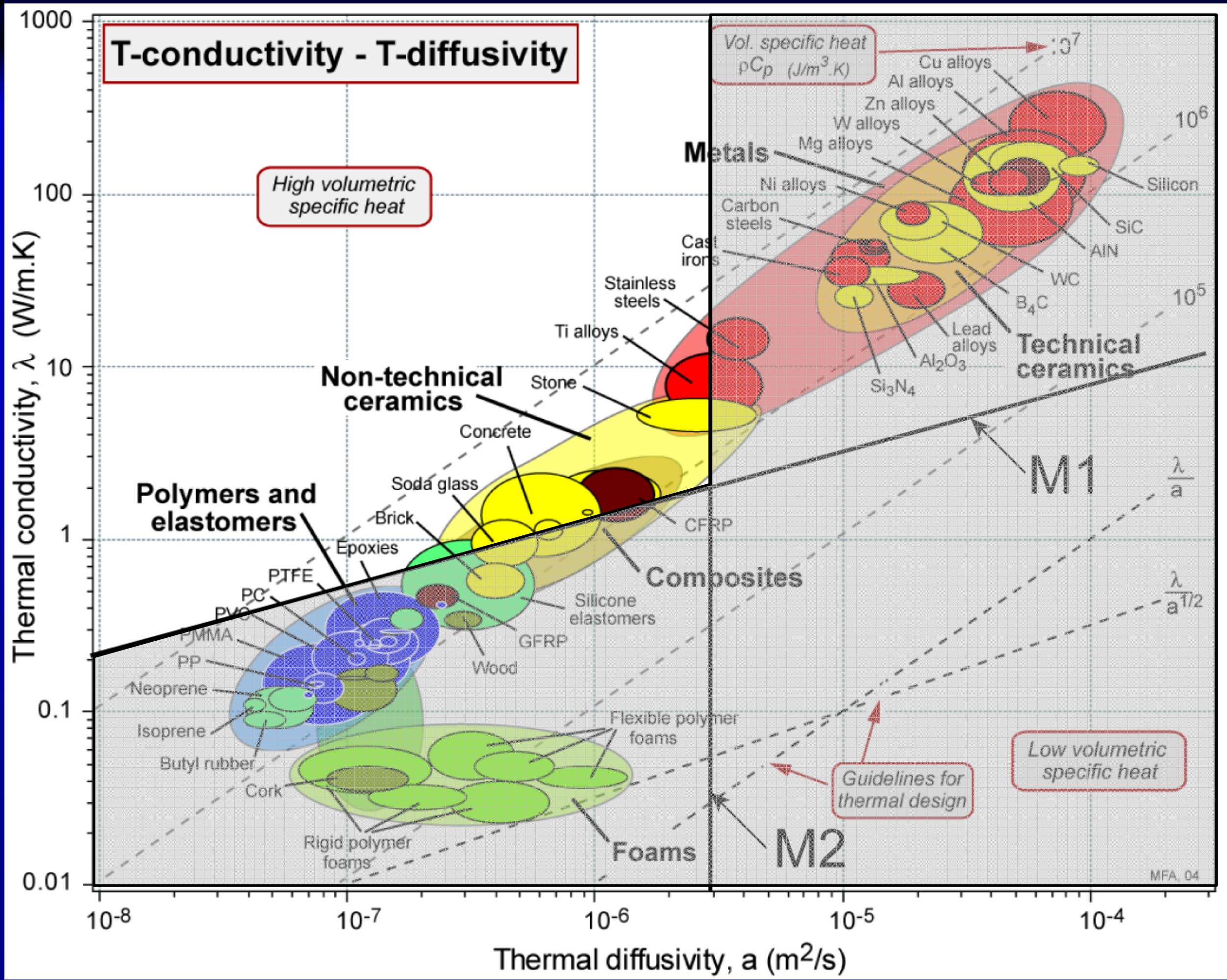
$$a \le \frac{w^2}{2t}$$

For $w \le 0.5$ m and $t = 12$ h:

$M_2 = a \le 3 \times 10^{-6}$ **m²/s**

Heat-Storing Wall: Material Selection

- **Eliminated**
  - **Foams: Too porous**
  - **Metals: Diffusivity too high**
- **Possibilities: Concrete, stone, brick, glass, titanium(!)**
- **Final Choices**
  - **Concrete is cheapest**
  - **Stone is best performer at reasonable price**



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