MIT 3.071
Amorphous Materials

8: Mechanical Properties

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After-class reading list

- Fundamentals of Inorganic Glasses
  - Ch. 18
- Introduction to Glass Science and Technology
  - Ch. 9

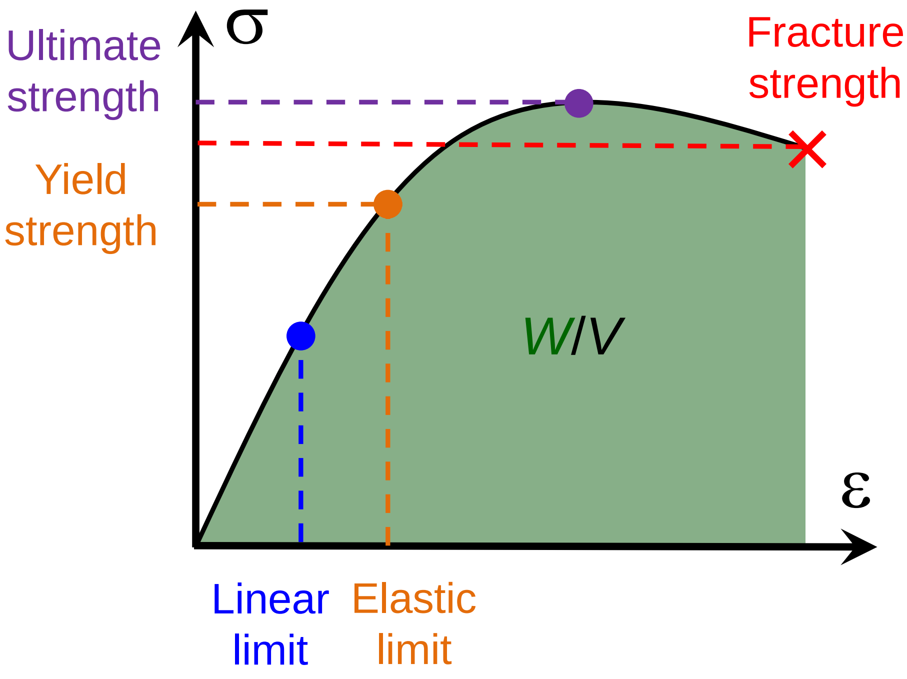
Glass = fragile?

Material	Iron	Structural steel	Glass fiber
Ultimate tensile strength	35 MPa	550 MPa	4890 MPa

~~Iron~~ man
Glass

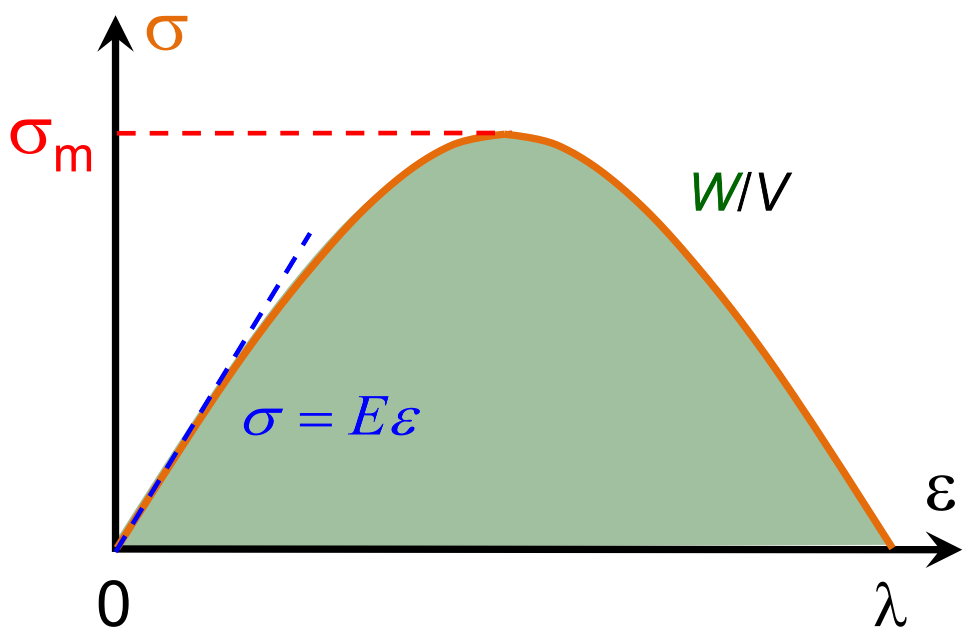
Strength and toughness

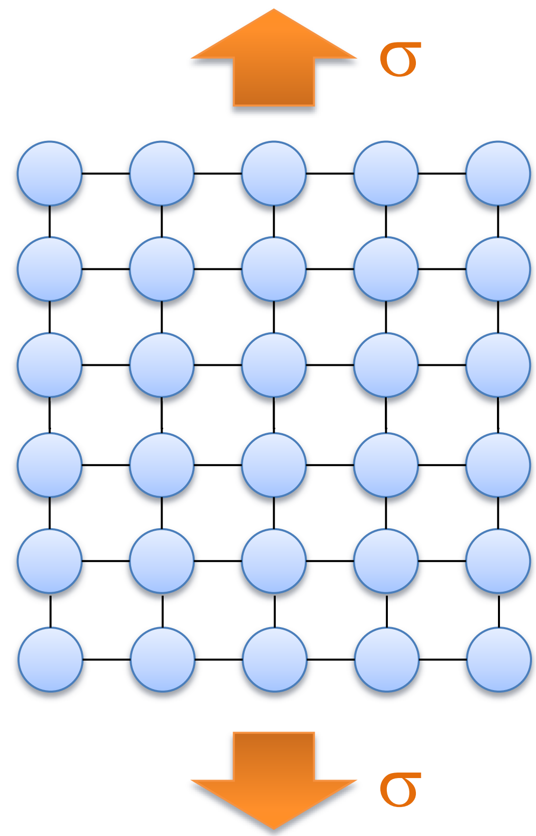
- Strength: applied stress a material can withstand
- Toughness: energy absorbed by (work performed to) a material per unit volume before fracture



Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms
- Work *W* performed to separate the solid equals to the energy of the fresh surfaces created during fracture





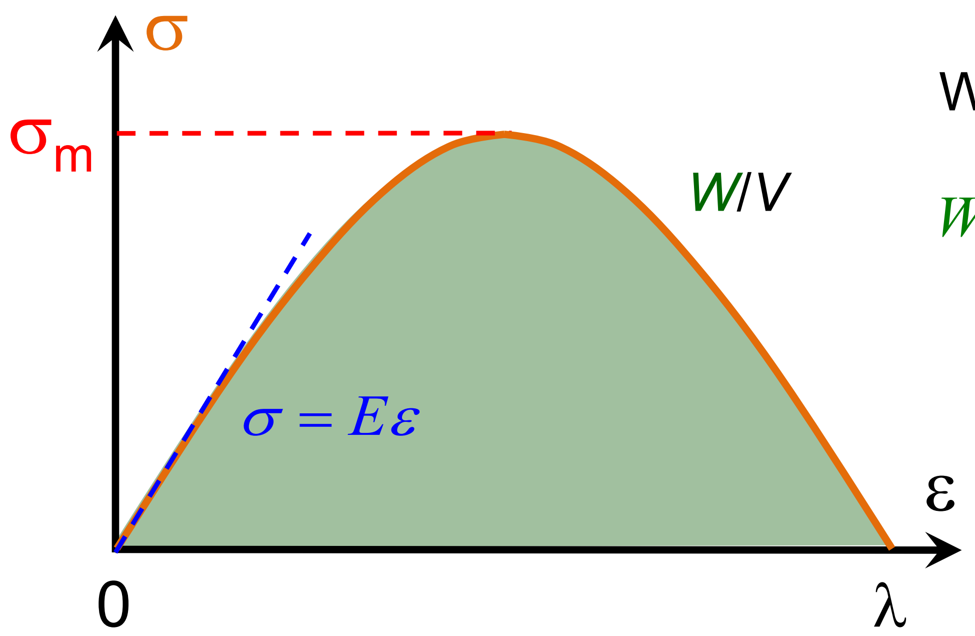
Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms
- Work *W* performed to separate the solid equals to the energy of the fresh surfaces created during fracture

$$\sigma = \sigma_m \sin\frac{\pi\varepsilon}{\lambda}$$

When $\sigma \ll \sigma_m$, $\sigma = E\varepsilon$

$$\sigma_m = \frac{\lambda E}{\pi}$$



Work *W* performed:

$$W = V \cdot \int_0^{\lambda} \sigma \, d\varepsilon = \frac{2\lambda^2 EV}{\pi^2} = 2\gamma S$$

$$V = a_0 S \;\Rightarrow\; \lambda = \pi\sqrt{\gamma / E a_0}$$

$$\Rightarrow \;\sigma_m = \sqrt{\frac{\gamma E}{a_0}}$$

Theoretical strength of a brittle material

- Consider silica glass
  - γ = 3.5 J/m^2, E = 70 GPa, a_0 = 0.2 nm

$$\sigma_m = \sqrt{\frac{\gamma E}{a_0}} = 35,000 \text{ MPa}$$

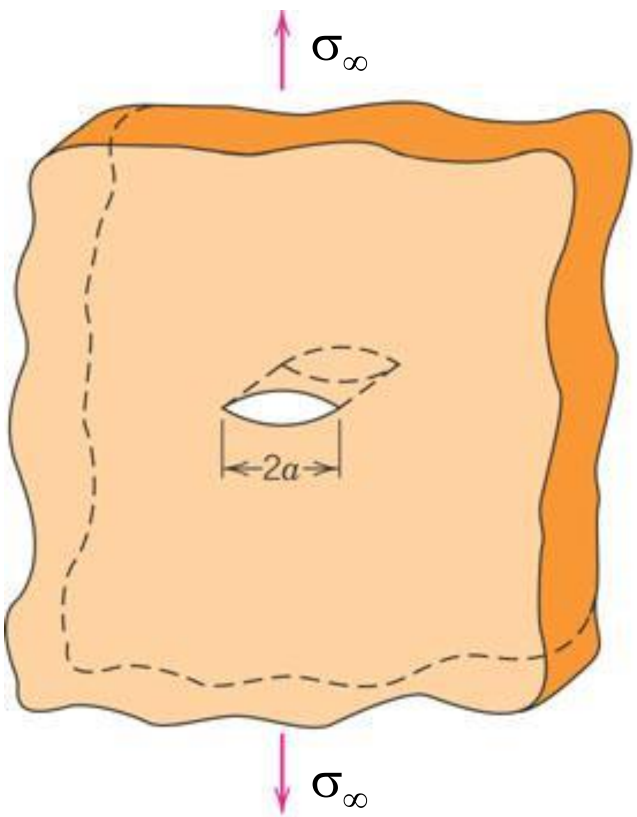
Material	Glass	Silica glass	Silica nanowire
Ultimate tensile strength	~ 30 MPa	110 MPa	26000 MPa†

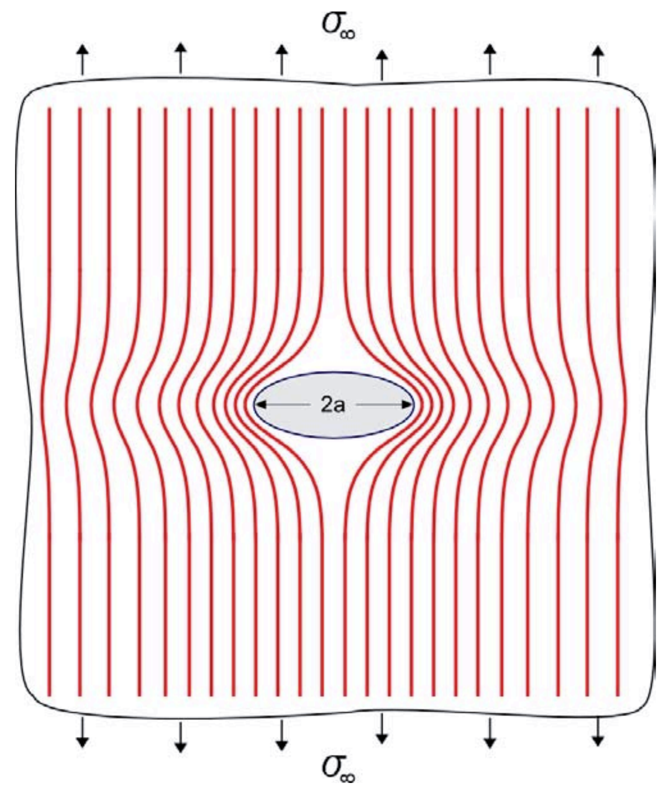
Practical strength of engineering materials is much less than their theoretical strength

† "The Ultimate Strength of Glass Silica Nanowires," *Nano Lett.* **9**, 831 (2009).

Griffith's theory

- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)





Griffith's theory

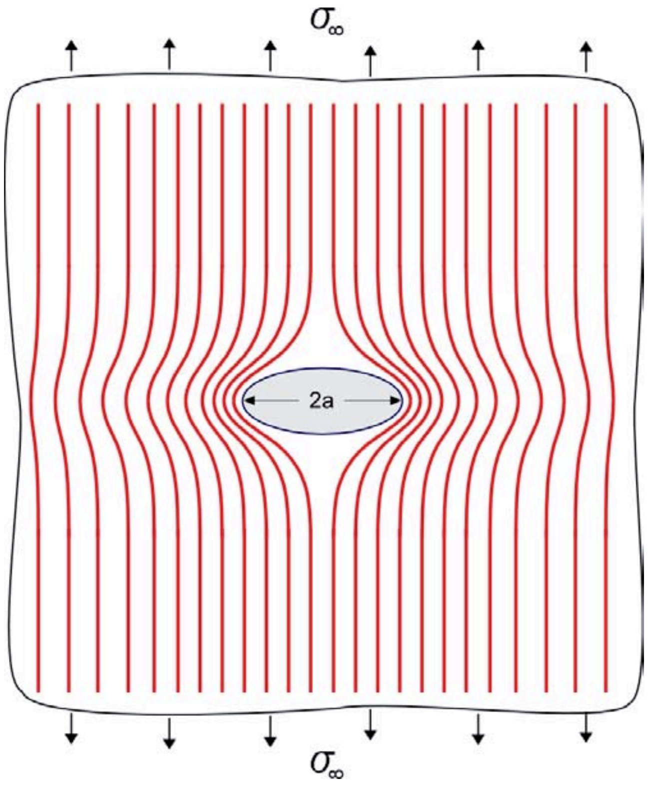
- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)

Stress concentration factor:

$$\frac{\sigma_{\max}}{\sigma_{\infty}} = 2\sqrt{\frac{a}{\rho}} = 2\sqrt{\frac{a}{a_0}}$$

Fracture strength of a flawed material:

$$\sigma_f = \frac{1}{2}\sqrt{\frac{\gamma E}{a}}$$



Griffith's theory

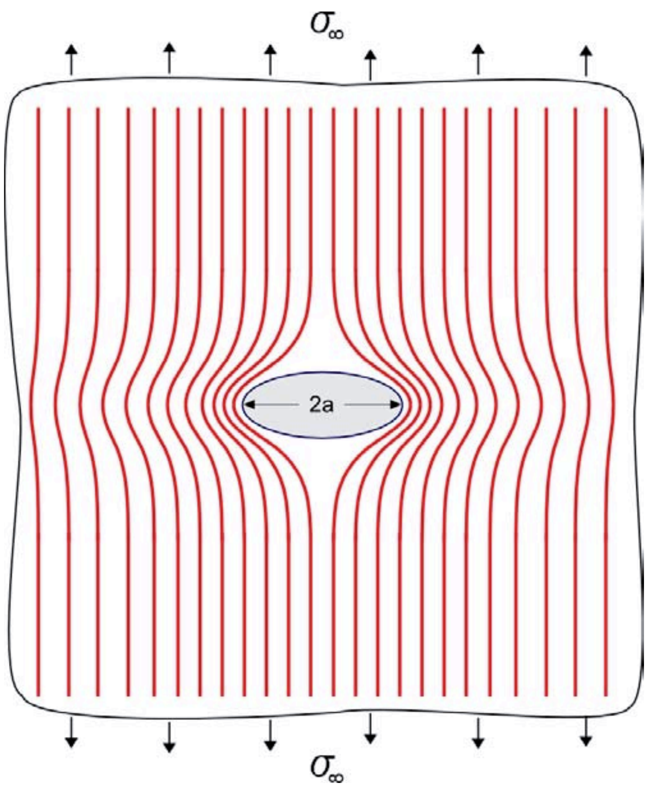
- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)

In flawed silica glass:

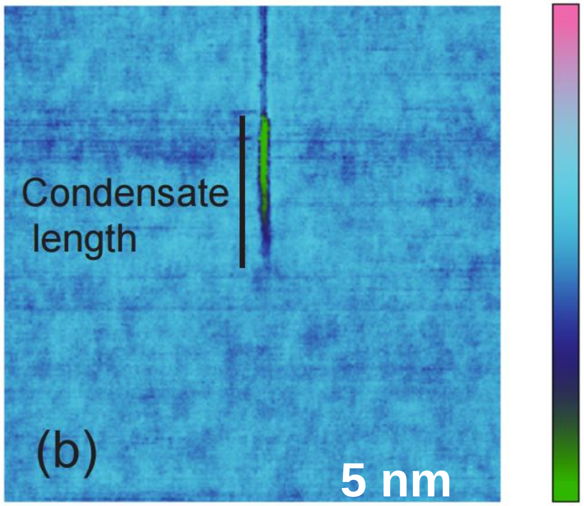
$$\sigma_f = \frac{1}{2}\sqrt{\frac{\gamma E}{a}} = 110 \text{ MPa}$$

$$\Rightarrow \quad a = 5 \ \mu\text{m}$$

A. Griffith, "The Phenomena of Rupture and Flow in Solids," *Philos. Trans. Roy. Soc. London, A* **221**, 163 (1921).



Visualizing Griffith cracks in glass



AFM phase image

Displacement field near a crack tip

Water condensation at crack tip

Europhys. Lett. **89**, 66003 (2010);
J. Am. Ceram. Soc. **94**, 2613 (2011).

Stress intensity factor and fracture toughness

- Stress intensity factor (tensile):

$K_I = \sigma_\infty \sqrt{\pi a}$ $\quad$ $K_{Ic} = \sigma_f \sqrt{\pi a}$ critical stress intensity factor

- Strain energy release rate:

$G_I = K_I^{\,2} / E$ $\quad$ $G_{Ic} = K_{Ic}^{\,2} / E$ work of fracture

- Fracture condition:

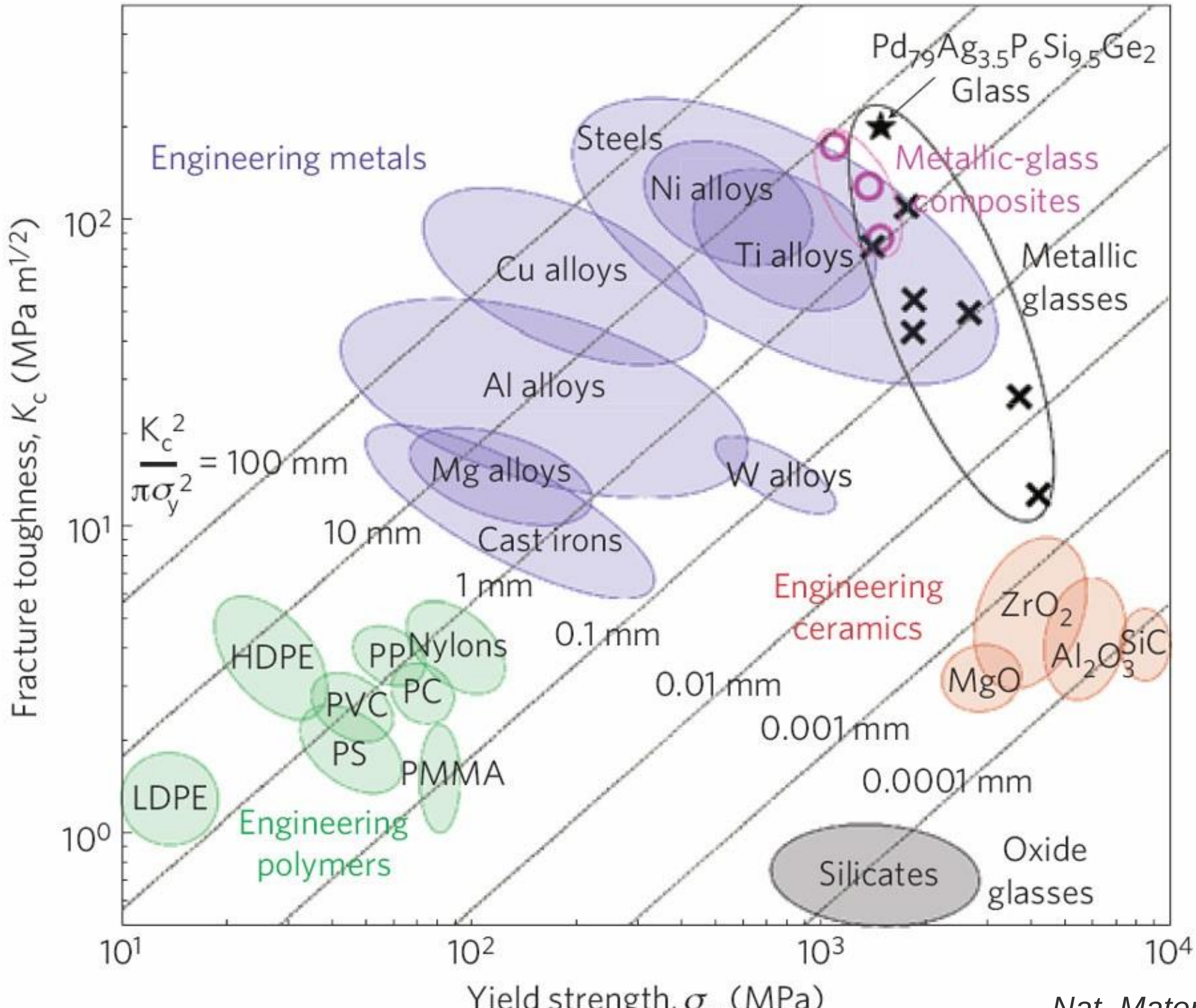
$K_I > K_{Ic}$ $\quad$ $G_I > G_{Ic}$

K_{Ic} is a material constant and is independent of crack length

Intrinsic plasticity in amorphous metals

- Lack of global plasticity
- Intrinsic plasticity
- When *G*/*K* < 0.42: plastic; *G*/*K* > 0.42: brittle (*K*: bulk modulus)

Phil. Mag. Lett. **85**, 77 (2006)



Nat. Mater. **10**, 123 (2011)

Brittle fracture of glass

- When a crack exceeds the critical length, the crack becomes unstable and propagates catastrophically through the material

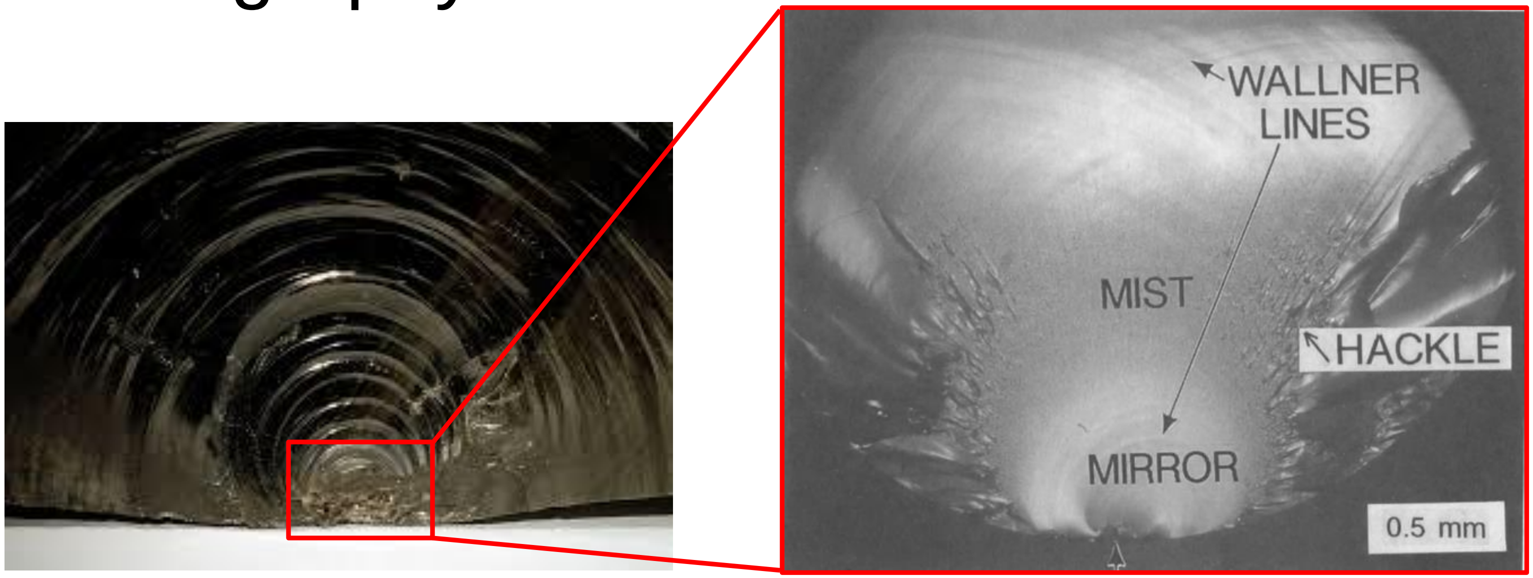
Glass cracking at 231,000 fps

Crack propagation velocity:

1540 m/s

J. Am. Cer. Soc. **22**, 302-307 (1939).

Fractography



Conchoidal fracture

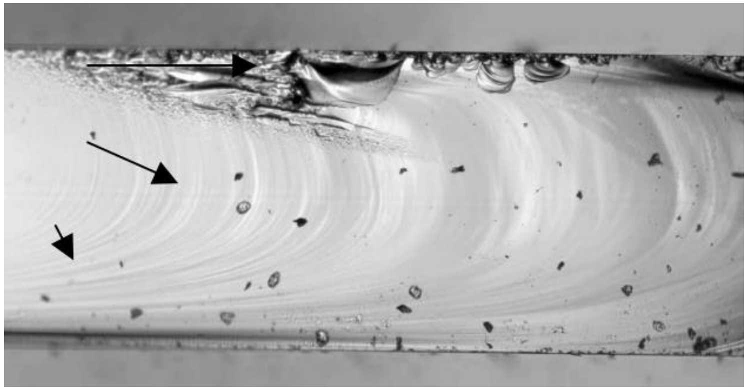
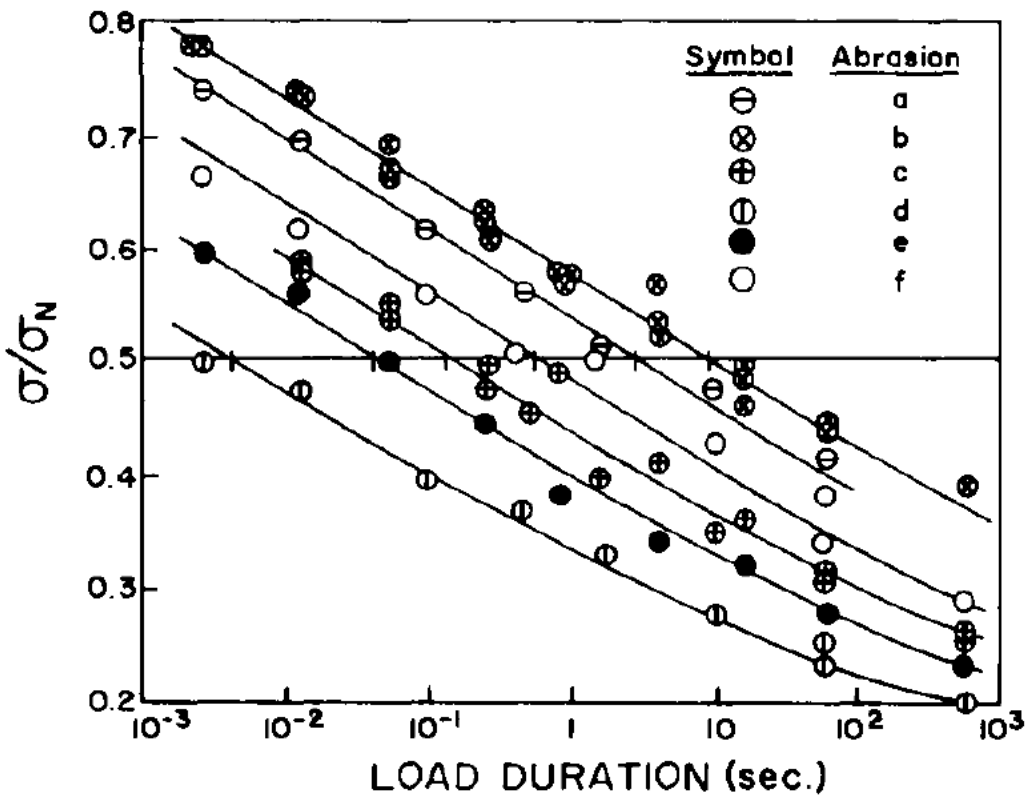


Image from "*Fracture analysis, a basic tool to solve breakage issues*"

Static fatigue in glass

- Under constant load, the time-to-failure varies inversely with the load applied



Sub-critical crack growth: crack length increases over time even when $\sigma_\infty < \sigma_f$

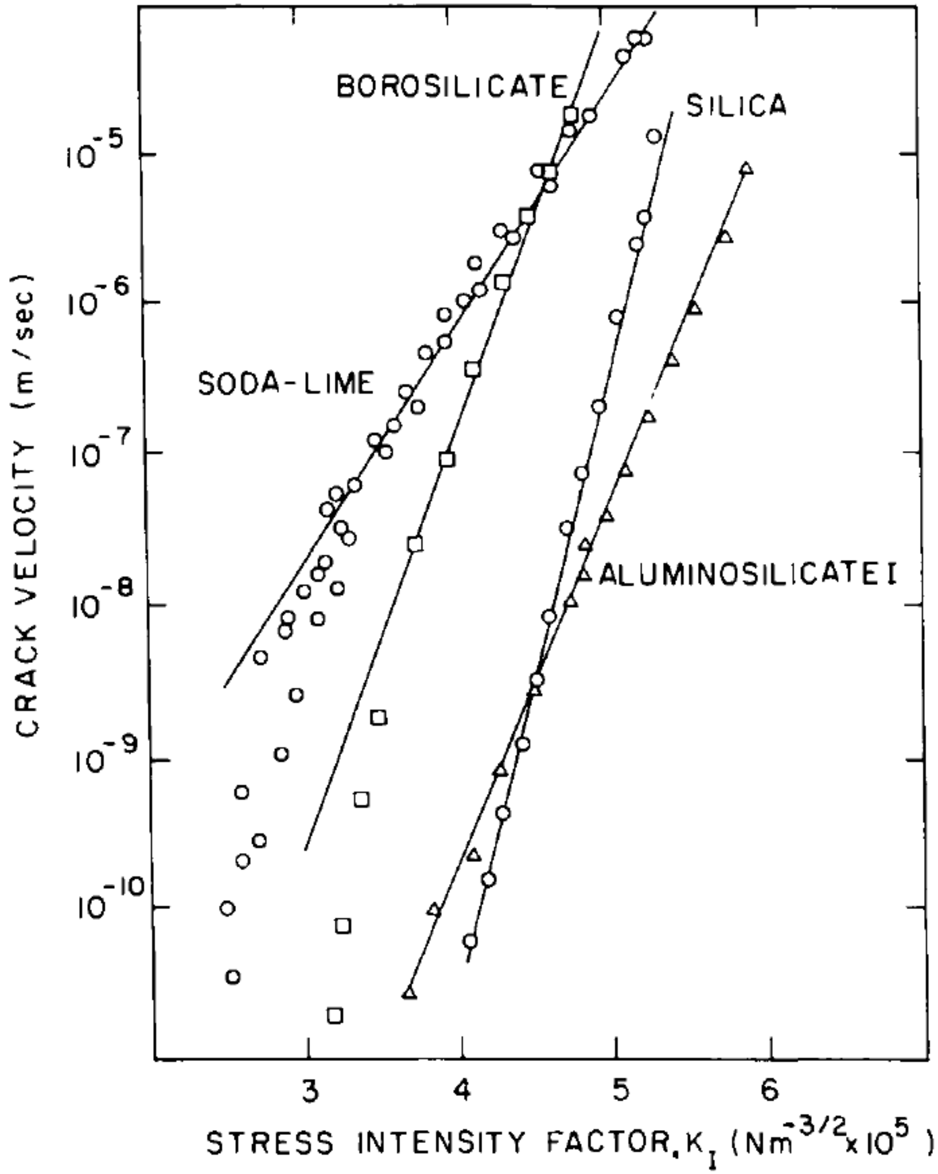
Stress corrosion

- Reaction at crack tip:

  $$\text{-Si-O-Si-} + H_2O \rightarrow$$
  $$\text{-Si-OH} + \text{HO-Si-}$$

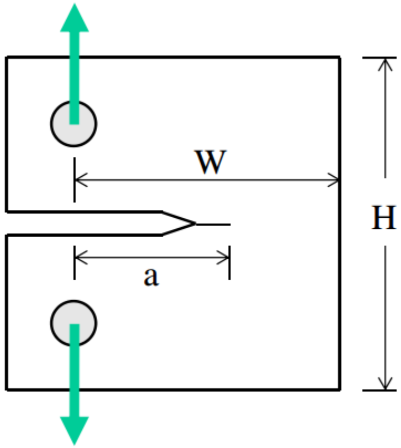
- Higher alkaline content generally reduces fatigue resistance
- Higher susceptibility to stress corrosion in basic solutions
- Thermally activated process

J. Non-Cryst. Solids **316**, 1 (2003)
J. Am. Ceram. Soc. **53**, 544 (1970)

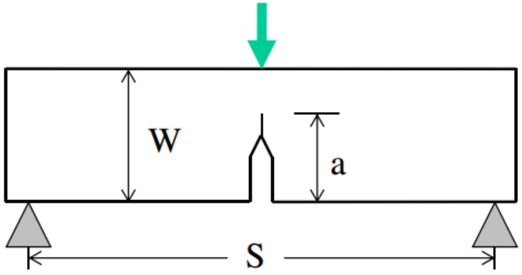


Fracture toughness measurement

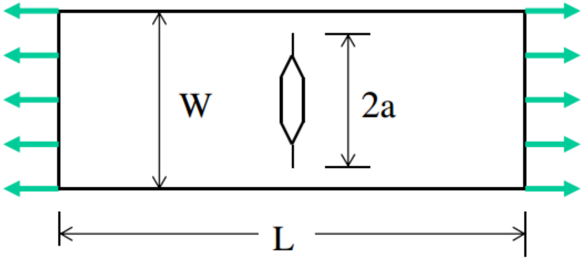
- ASTM Standard E1820-15: Standard Test Method for Measurement of Fracture Toughness
- Standard specimen geometries to obtain load-displacement plot



Compact tension specimen



Single edge-notched bend specimen (for three-point bending)

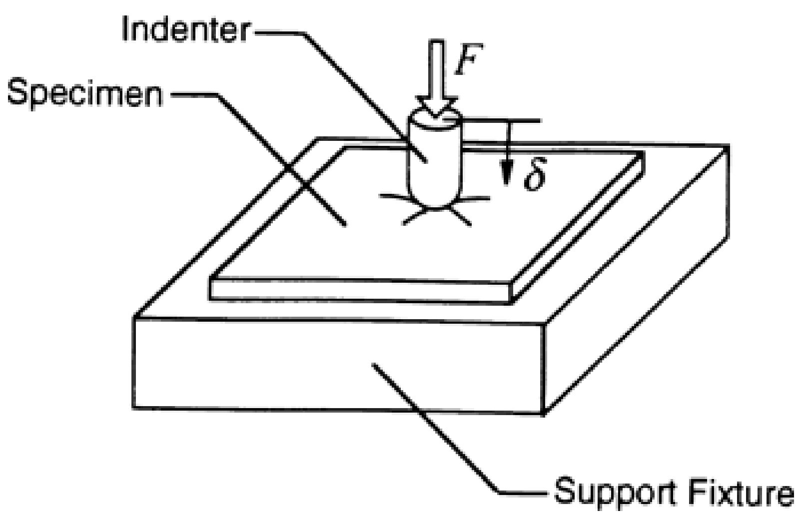


Middle-cracked tension specimen

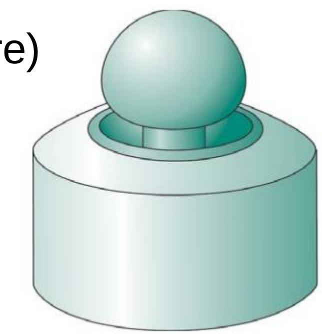
Eng. Fract. Mech. **85**, 1 (2012)

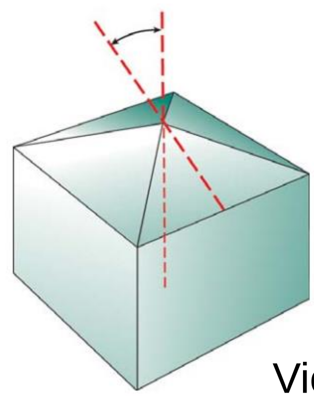
Indentation of glass samples

- Mechanical properties evaluated through indented crack size or crack-opening displacement based on empirical equations
- Poor correlation with conventional test results can be a concern



Hertzian (sphere) indenter tip





Vickers indenter tip

J. Mech. Behav. Biomed. Mater. **2**, 384 (2009)

Indentation of glass samples

- Vickers indentation of soda-lime glass

J. Am. Ceram. Soc. **73**, 787 (1990)

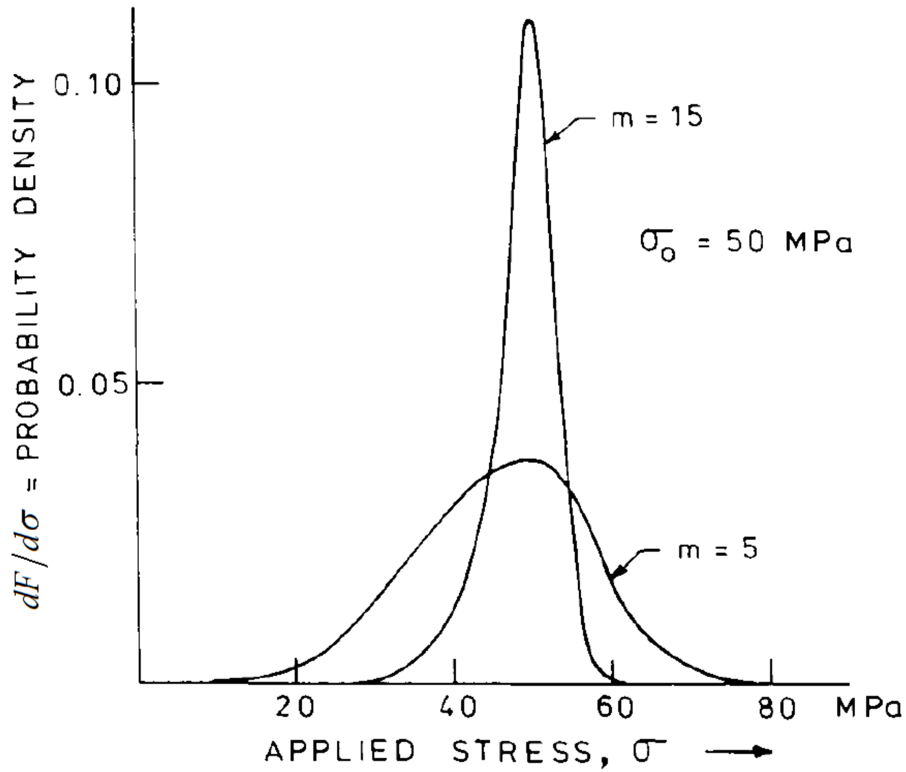
Fracture statistics

- Experimental results of fracture strength can often be described by the Weibull distribution
- The fraction F of samples which fracture at stresses below σ is given by:

$$F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

m : Weibull modulus

- Probability density $dF/d\sigma$
  - Probability of samples fracture at stress σ



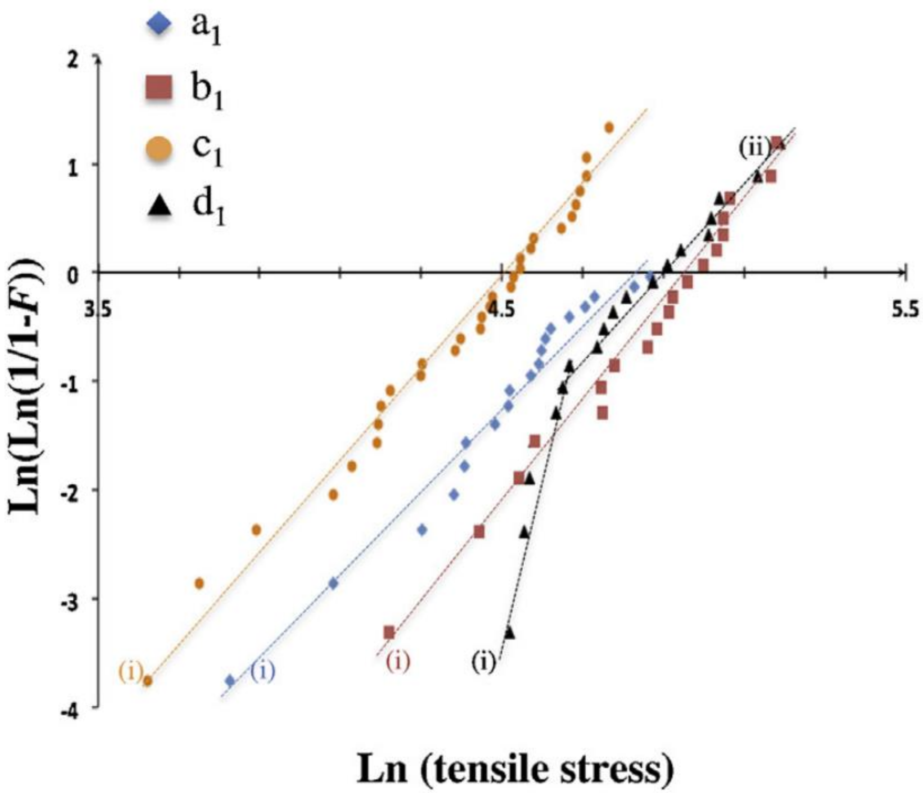
Weibull plot

$$F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0} \right)^m \right] \Rightarrow \ln\left[-\ln(1-F) \right] = m(\ln\sigma - \ln\sigma_0)$$

m : slope of the Weibull plot

σ_0 : intercept with horizontal axis

Mater. Res. Bull. **49**, 250 (2014)



Summary

- Theoretical and practical strengths of materials
  - Practical strength of brittle materials is usually much lower than the theoretical strength due to the presence of defects
  - Oxide glasses are extremely sensitive to surface defects
  - Intrinsic ductility in select BMGs contributes to high toughness
- Basics of fracture mechanics
  - Griffith crack theory
  - Fracture toughness $K_{Ic} = \sigma_f \sqrt{\pi a} = \frac{1}{2}\sqrt{\pi \gamma E}$
- Fatigue and stress corrosion
- Fracture toughness measurement
- Fracture statistics: Weibull plot