Heating & Cooling

$$\rho c \frac{\partial T}{\partial t} = \dot{q} + k \frac{\partial^2 T}{\partial x^2}$$

$$Fo = \frac{k}{\rho c} \cdot \frac{t}{L^2} \qquad Br = \frac{\dot{q}}{k} \cdot \frac{L^2}{T}$$

$$k \sim 0.3 \frac{W}{m \cdot K}, \quad \rho c \sim 2 \times 10^6 \frac{J}{m^3 \cdot K}, \quad \alpha = \frac{k}{\rho c} \sim 10^{-7} \frac{m^2}{s}$$

Solution – Laplace Transform

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$s\bar{T} = \alpha \frac{d^2 \bar{T}}{dx^2}$$

$$T(x,t) = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right),$$

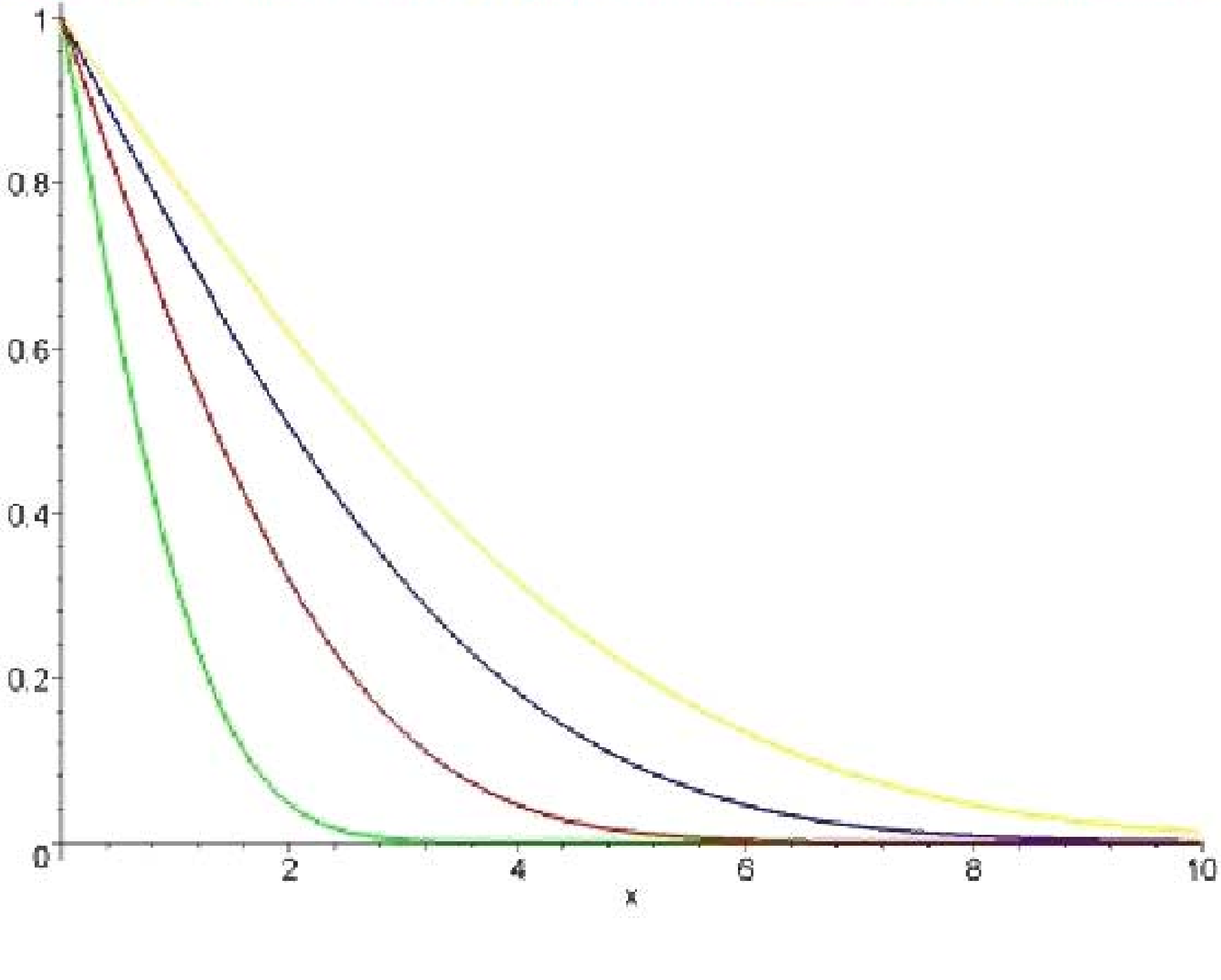
$$\operatorname{erf}(z) = \frac{2}{\pi} \int_0^z e^{-y^2} dy$$

Transient temperature profiles

```
> T:=(x,t)-> 1-erf(x/sqrt(2*alpha*t));
```

$$T := (x, t) \rightarrow 1 - \operatorname{erf}\left(\frac{x}{\sqrt{2\,\alpha\,t}}\right)$$

```
> alpha:=1:plot({T(x,1),T(x,4),T(x,9),T(x,16)},x=0..10,thickness=3);
```



Maple solution of transient heat equation:

```
> pde:=diff(T(x,t),t)=diff(T(x,t),x,x);
```

$$pde := \frac{\partial}{\partial t} \mathrm{T}(x, t) = \frac{\partial^2}{\partial x^2} \mathrm{T}(x, t)$$

```
> with(DEtools):pdesolve(pde,T(x,t));
```

$$\mathrm{pdesolve}\left(\frac{\partial}{\partial t} \mathrm{T}(x, t) = \frac{\partial^2}{\partial x^2} \mathrm{T}(x, t), \mathrm{T}(x, t)\right)$$

Maple can't get a solution; try simplifying the problem - convert to ODE with Fourier transform::

```
> with(inttrans):ode1:=fourier(pde,x,w);
```

$$ode1 := \frac{\partial}{\partial t} \mathrm{fourier}(\mathrm{T}(x, t), x, w) = -w^2 \, \mathrm{fourier}(\mathrm{T}(x, t), x, w)$$

```
> ode:=subs( fourier(T(x,t),x,w)=U(t),ode1);
```

$$ode := \frac{\partial}{\partial t} \mathrm{U}(t) = -w^2 \, \mathrm{U}(t)$$

```
> bc:=U(0)=fourier(T_0*Dirac(x),x,w);
```

$$bc := \mathrm{U}(0) = T\_0$$

```
> dsolve({ode,bc},U(t));
```

$$\mathrm{U}(t) = T\_0 \, \mathbf{e}^{(-w^2 t)}$$

Invert back to x-plane:

```
> assume(t>0); T(x,t)=simplify(invfourier(rhs(%),w,x));
```

$$\mathrm{T}(x, t\sim) = \frac{1}{2} \frac{T\_0 \sqrt{\frac{\pi}{t\sim}} \, \mathbf{e}^{\left(-1/4 \frac{x^2}{t\sim}\right)}}{\pi}$$

```
> simplify(%);
>
```

$$\mathrm{T}(x, t\sim) = \frac{1}{2} \frac{T\_0 \, \mathbf{e}^{\left(-1/4 \frac{x^2}{t\sim}\right)}}{\sqrt{\pi} \sqrt{t\sim}}$$

Finite Difference Solution

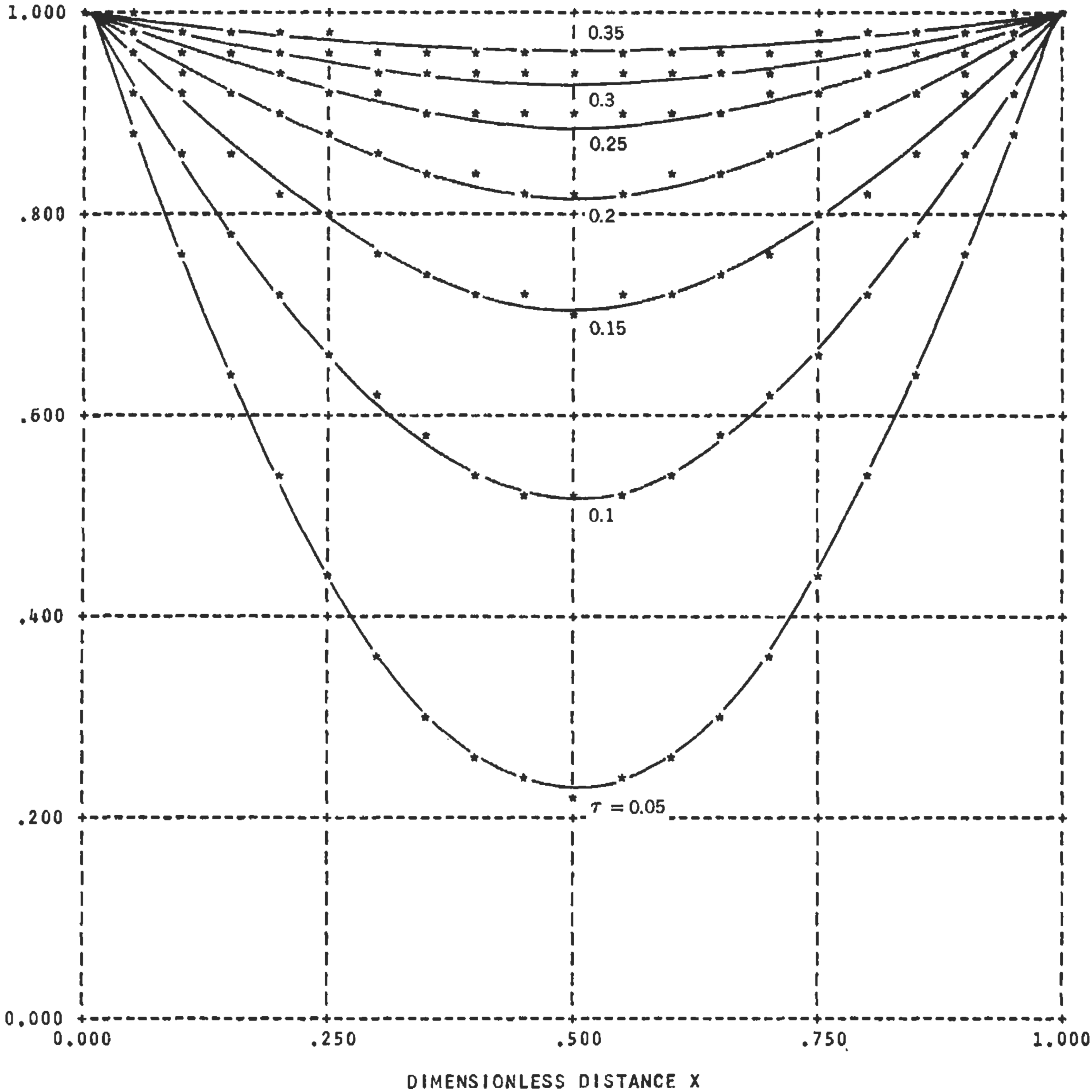
$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \frac{T_x^{t+1} - T_x^{t}}{\Delta t} + O(\Delta t)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{x-1}^{t} - 2T_x^{t} + T_{x+1}^{t}}{(\Delta x)^2} + O\left[(\Delta x)^2\right]$$

$$T_x^{t+1} = \lambda T_{x-1}^{t} + (1-2\lambda)T_x^{t} + \lambda T_{x+1}^{t}$$

$$\lambda = \frac{\Delta t}{(\Delta x)^2}$$


1.000
.800
.600
.400
.200
0.000
0.000
.250
.500
.750
1.000
0.35
0.3
0.25
0.2
0.15
0.1
τ = 0.05
DIMENSIONLESS DISTANCE X