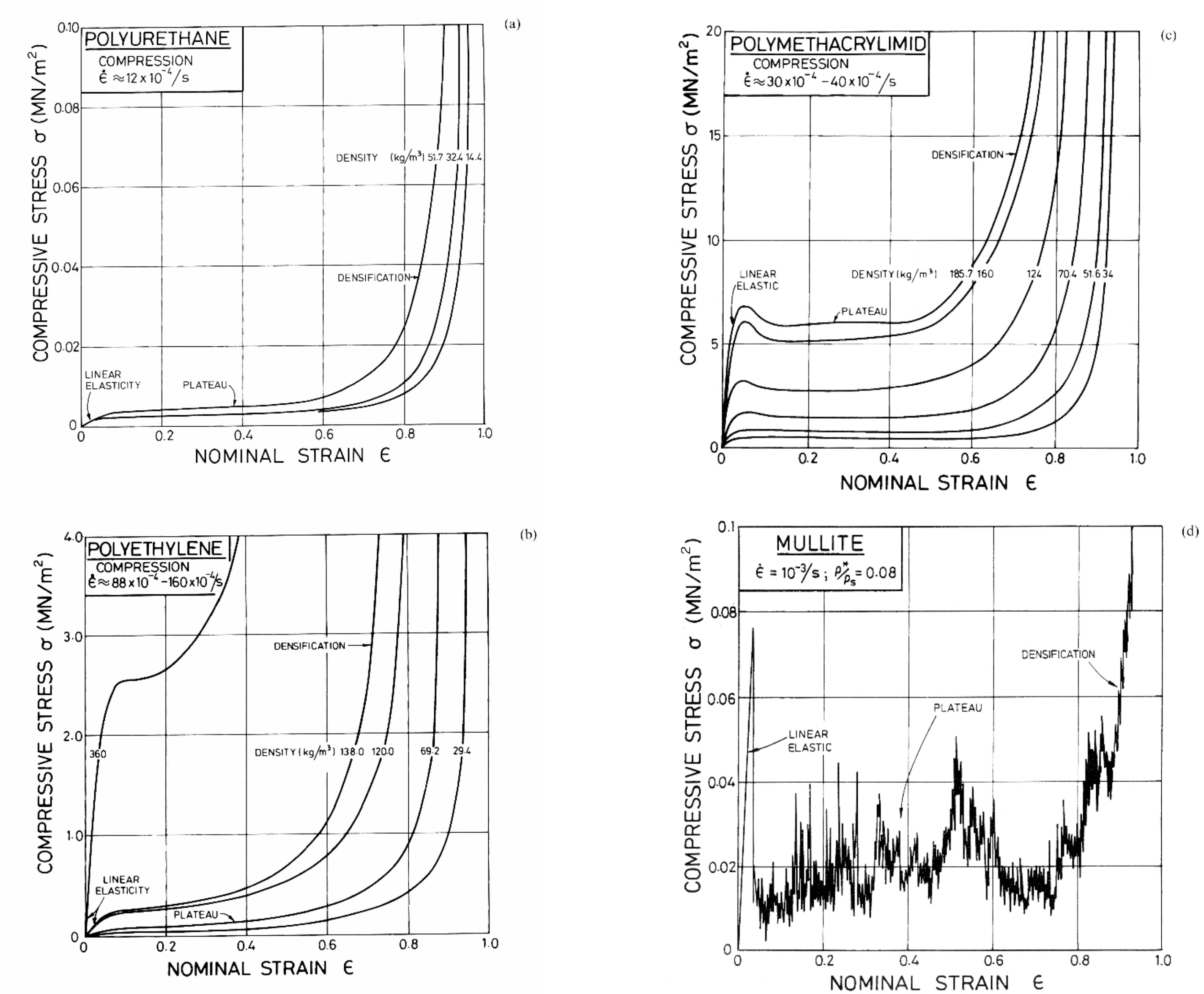
Lecture 7, Foams, 3.054

Open-cell foams

- Stress-Strain curve: deformation and failure mechanisms
- Compression - 3 regimes - linear elastic - bending
  - stress plateau - cell collapse by buckling
    yielding
    crushing
  - densification
- Tension - no buckling
  - yielding can occur
  - brittle fracture

Linear elastic behavior

- Initial linear elasticity - bending of cell edges (small t/l)
- As t/l goes up, axial deformation becomes more significant
- Consider dimensional argument, which models mechanism of deformation and failure, but not cell geometry
- Consider cubic cell, square cross-section members of area t^2, length l



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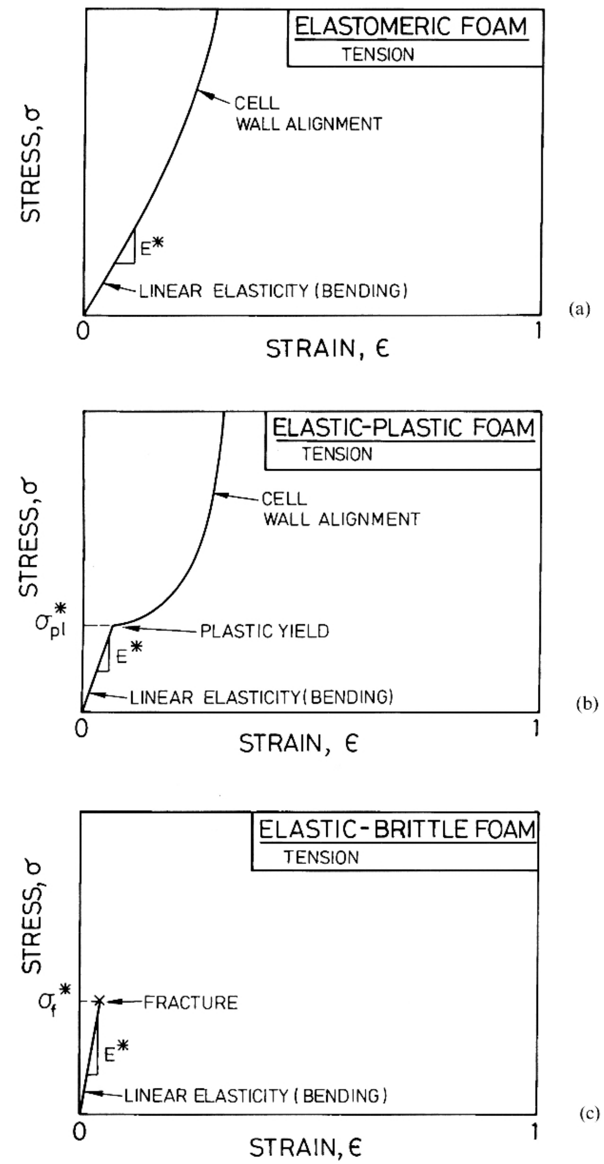

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Foams: Bending, Buckling

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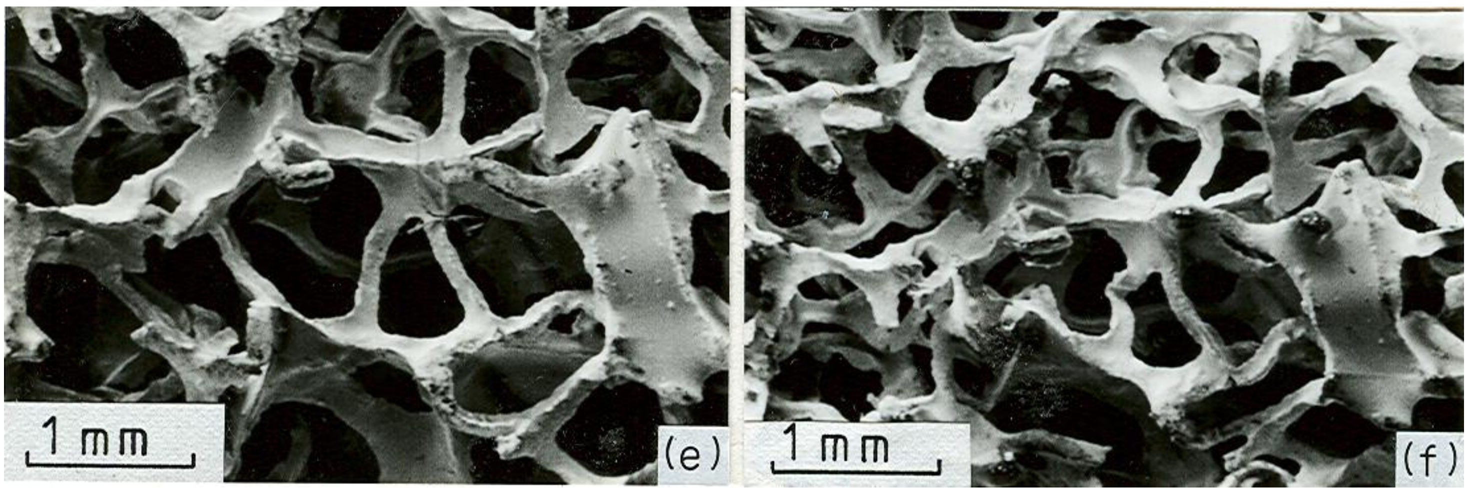
Foams: Plastic Hinges



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Foams: Cell Wall Fracture

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- Regardless of specific geometry chosen:

$\rho^*/\rho_s \propto (t/l)^2 \qquad I \propto t^4$

$\sigma \propto F/l^2 \qquad \epsilon \propto \delta/l \qquad \delta \propto \dfrac{Fl^3}{E_S I}$

$$E^* \propto \frac{\sigma}{\epsilon} \propto \frac{F}{l^2}\,\frac{l}{\delta} \propto \frac{F}{l}\,\frac{E_s t^4}{Fl^3} \propto E_s\left(\frac{t}{l}\right)^4 \propto E_s\left(\frac{\rho^*}{\rho_s}\right)^2$$

$\boxed{E^* = C_1 E_s (\rho^*/\rho_s)^2}$

C_1 includes all geometrical constants
Data: $C_1 \approx 1$

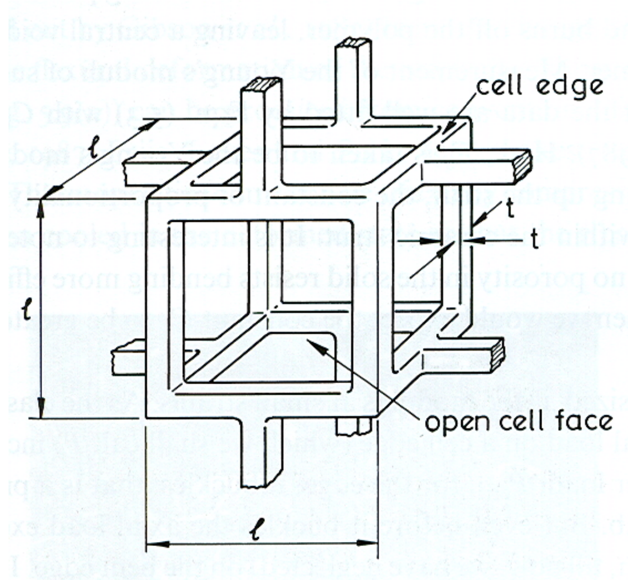
- Data suggests $C_1 = 1$
- Analysis of open cell tetrakaidecahedral cells with Plateau borders gives $C_1 = 0.98$
- Shear modulus $\boxed{G^* = C_2 E_s (\rho^*/\rho_s)^2}$ $\quad C_2 \sim 3/8$ if foam is isotropic

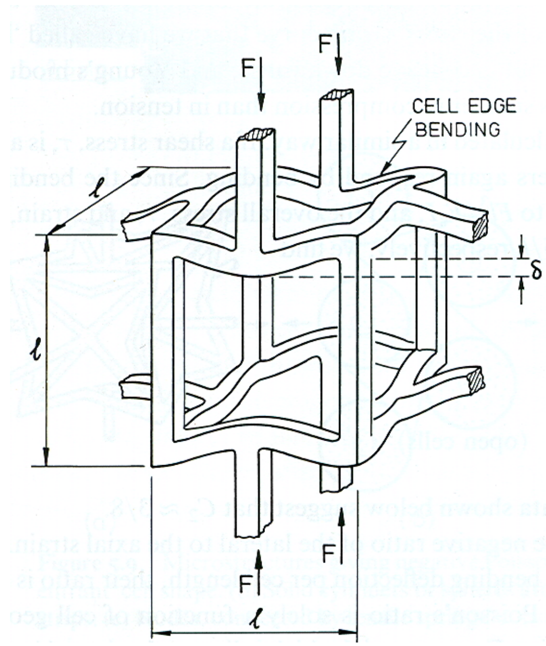
  Isotropy: $G = \dfrac{E}{2(1+\nu)}$

- Poisson's ratio: $\nu^* = \frac{E}{2G} - 1 = \frac{C_1}{2C_2} - 1 =$ constant, independent of E_s, t/l

$\boxed{\nu^* = C_3}$ $\qquad$ (analogous to honeycombs in-plane)

Foam: Edge Bending





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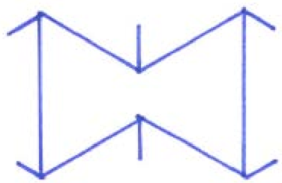
Poisson's ratio

- Can make negative Poisson's ratio foams
- Invert cell angles (analogous to honeycomb)
- Eg. thermoplastic foams - load hydrostatically

  and heat to $T > T_g$, then cool and release load

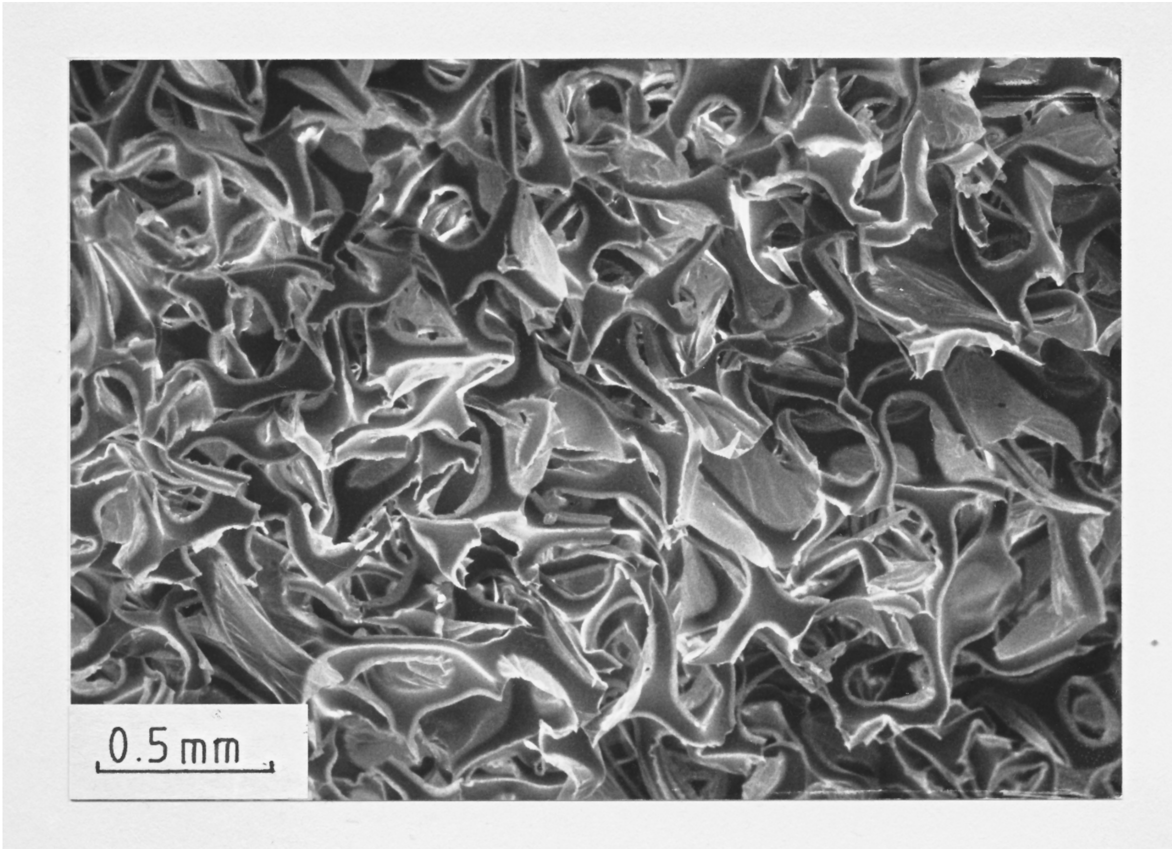
  so that edges of cell permanently point inward



Closed-cell foams

- Edge bending as for open cell foams
- Also: face stretching and gas compression
- Polymer foams: surface tension draws material to edges during processing
  - define t_e, t_f in figure
- Apply F to the cubic structure

Negative Poisson's Ratio



- External work done $\propto F\delta$.
- Internal work bending edges $\propto \frac{F_e}{\delta_e}\delta_e^2 \propto \frac{E_s I}{l^3}\delta^2$
- Internal work stretching faces $\propto \sigma_f\,\epsilon_f\,v_f \propto E_s\,\epsilon_f^2\,v_f \propto E_s\left(\delta/l\right)^2 t_f\,l^2$

$$\therefore \quad F\delta = \alpha\,\frac{E_s t_e^4}{l^3}\delta^2 + \beta\,E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* \propto \frac{F}{l^2}\,\frac{l}{\delta} \rightarrow \quad F \propto E^*\,\delta l$$

$$\therefore \quad E^*\delta^2\,l = \alpha\,\frac{E_s t_e^4}{l^3}\delta^2 + \beta\,E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* = \alpha E_s\left(\frac{t_e}{l}\right)^4 + \beta E_s\left(\frac{t_f}{l}\right)$$

Note: Open cells, uniform t:
$\rho^*/\rho_s \propto (t/l)^2$

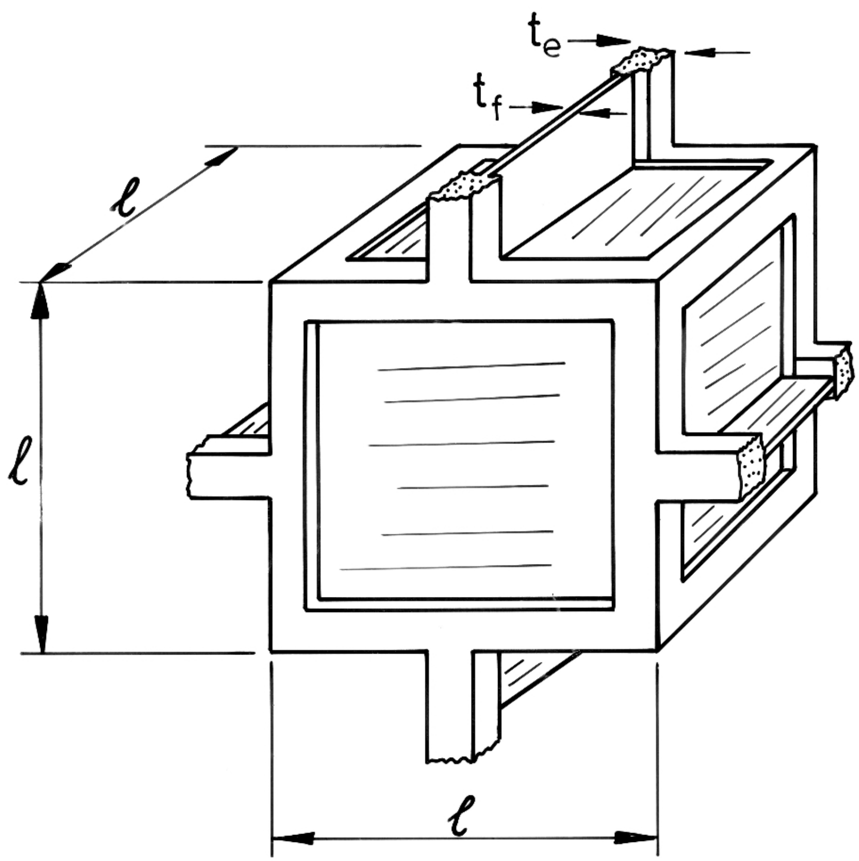
Closed cells, uniform t:
$\rho^*/\rho_s \propto (t/l)$

If ϕ is volume fraction of solid in cell edges:

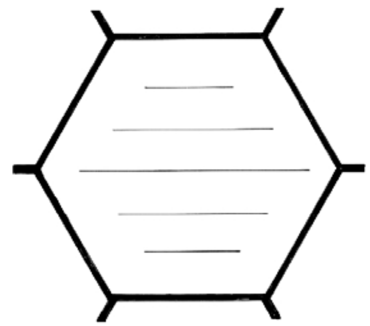
$$t_e/l = C\phi^{1/2}(\rho^*/\rho_s)^{1/2}$$
$$t_f/l = C'(1-\phi)\,(\rho^*/\rho_s)$$

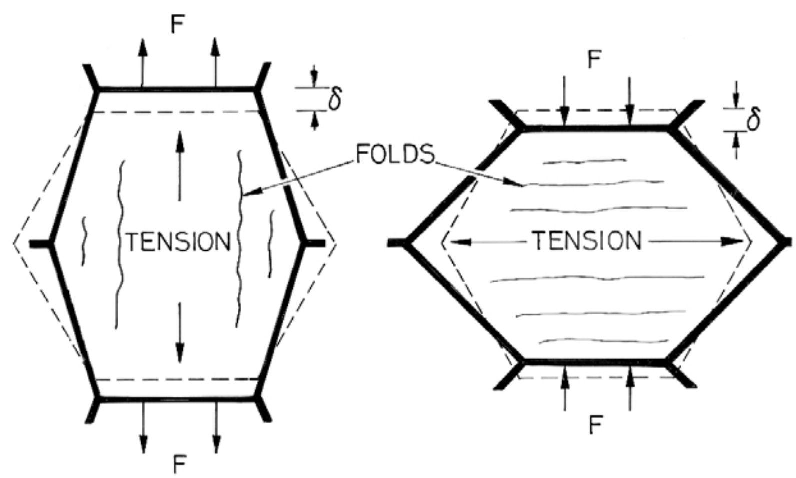
$$\boxed{\frac{E^*}{E_s} = C_1\phi^2\left(\rho^*/\rho_s\right)^2 + C_1'(1-\phi)\rho^*/\rho_s}$$

Closed-Cell Foam



Cell Membrane Stretching





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Closed cell foams - gas within cell may also contribute to E^*

- Cubic element of foam of volume V_0
- Under uniaxial stress, axial strain in direction of stress is ϵ
- Deformed volume V is:

$$\frac{V}{V_0} = 1 - \epsilon(1 - 2\nu^*)$$

taking compressive strain as positive,

neglecting ϵ^2, ϵ^3 terms

$$\frac{V_g}{V_g^0} = \frac{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}{1 - \rho^*/\rho_s}$$

V_g = volume gas

V_g^0 = volume gas initially

- Boyle's law: $pV_g = p_0 V_g^0$

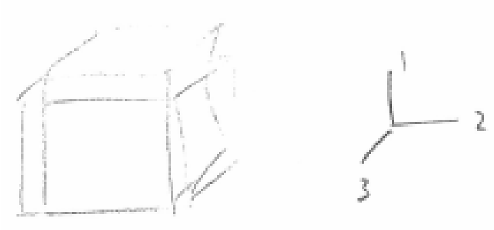
p = pressure after strain ϵ
p_0 = pressure initially

- Pressure that must be overcome is $p' = p - p_0$:

$$p' = \frac{p_0\, \epsilon\, (1 - 2\nu^*)}{1 - \epsilon\, (1 - 2\nu^*) - \rho^*/\rho_s}$$

- Contribution of gas compression to the modulus E^*:

$$E_g^* = \frac{dp'}{d\epsilon} = \frac{p_0(1 - 2\nu^*)}{1 - \rho^*/\rho_s}$$



$$V_0 = l_0^3 \qquad \epsilon_1 = \frac{l_1 - l_0}{l_0} \quad \rightarrow \quad l_1 = l_0 + \epsilon_1 l_0 = l_0(1+\epsilon_1)$$

$$V = l_1 l_2 l_3 \qquad \epsilon_2 = \frac{l_2 - l_0}{l_0} \quad \rightarrow \quad \begin{aligned} l_2 &= l_0 + \epsilon_2 l_0 \qquad \nu = -\frac{\epsilon_2}{\epsilon_1} \\ &= l_0 - \nu\epsilon_1 l_0 \qquad \epsilon_2 = -\nu\epsilon_1 \\ &= l_0(1 - \nu\epsilon_1) \end{aligned}$$

$$\epsilon_3 = l_0(1 - \nu\epsilon_1)$$

$$\begin{aligned} V &= l_1 l_2 l_3 = l_0(1+\epsilon_1) l_0(1-\nu\epsilon_1) l_0(1-\nu\epsilon_1) = l_0^3(1+\epsilon_1)(1-\nu\epsilon_1)^2 \\ \frac{V}{V_0} &= \frac{l_0^3(1+\epsilon)(1-\nu\epsilon)^2}{l_0^3} = (1+\epsilon)(1-2\nu\epsilon+\nu^2\epsilon^2) \\ &= (1-2\nu\epsilon+\nu^2\epsilon^2) + \epsilon - 2\nu\epsilon^2 + \nu^2\epsilon^3 \\ &= 1 - \epsilon + 2\nu\epsilon \\ &= 1 - \epsilon(1-2\nu) \end{aligned}$$

$$
\begin{aligned}
p' &= p - p_0 \\
p &= \frac{p_0 V_g'}{V_g} \\
p' &= p - p_0 = \frac{p_0 V_g'}{V_g} - p_0 = p_0\Big(\frac{V_g^0}{V_g} - 1\Big) \\
&= p_0\Big[\frac{1 - \rho^*/\rho_s}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s} - 1\Big] \\
&= p_0\Big[\frac{1 - \rho^*/\rho_s(1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s)}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}\Big] \\
&= p_0\Big[\frac{\epsilon(1 - 2\nu^*)}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}\Big]
\end{aligned}
$$

Closed cell foam

$$\frac{E^*}{E_s} = \underbrace{\phi^2\left(\frac{\rho^*}{\rho_s}\right)^2}_{\text{edge bending}} + \underbrace{(1-\phi)\left(\frac{\rho^*}{\rho_s}\right)}_{\text{face stretching}} + \underbrace{\frac{p_0(1-2\nu^*)}{E_s(1-\rho^*/\rho_s)}}_{\text{gas compression}}$$

- Note: if $p_0 = p_{\text{atm}} = 0.1$ MPa, gas compression term is negligible, except for closed-cell elastomeric foams
- Gas compression can be significant if $p_0 >> p_{\text{atm}}$; also modifies shape of stress plateau in elastomeric closed-cell foams

Shear modulus: edge bending, face stretching; shear $\Delta V = 0$ gas contribution is 0

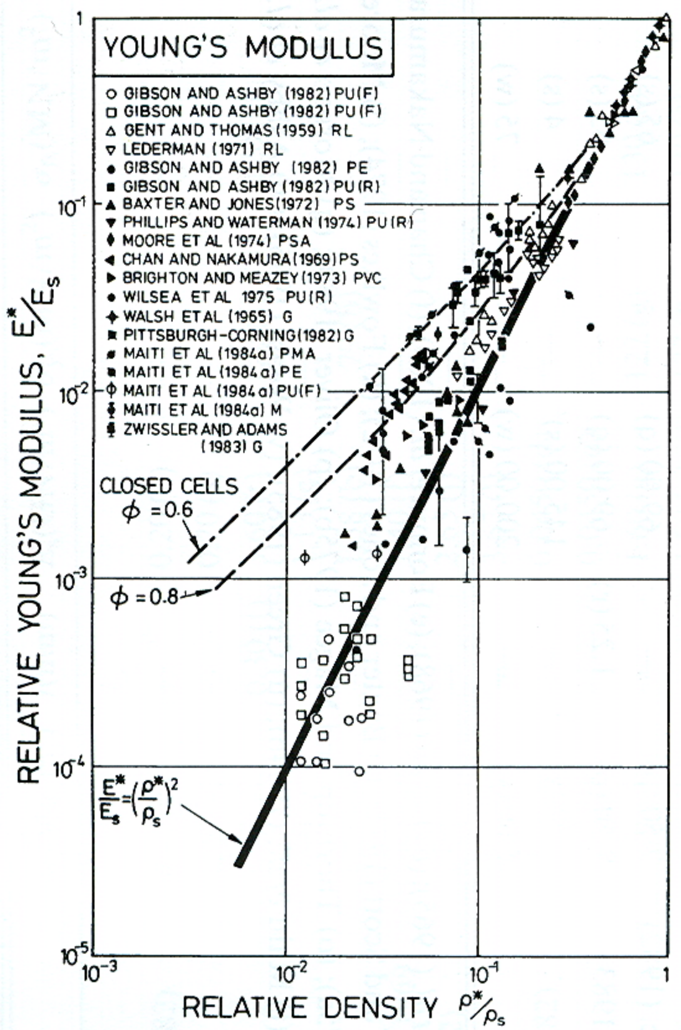
$$\frac{\epsilon^*}{E_s} = \frac{3}{8}\left[\phi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + (1-\phi)\left(\frac{\rho^*}{\rho_s}\right)\right] \qquad \text{(isotropic foam)}$$

Poison's ratio $= f$ (cell geometry only) $\qquad \nu^* \approx 1/3$

Comparison with data

- Data for polymers, glasses, elastomers
- E_s, ρ_s - Table 5.1 in the book
- Open cells — open symbols
- Closed cells — filled symbols

Young's Modulus



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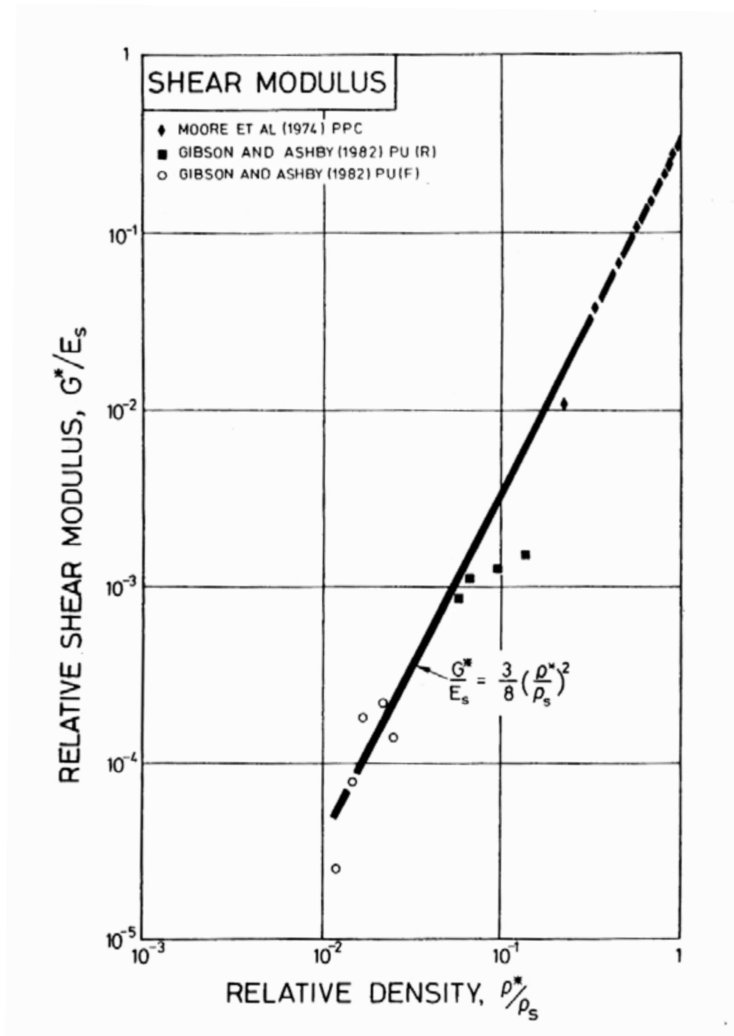
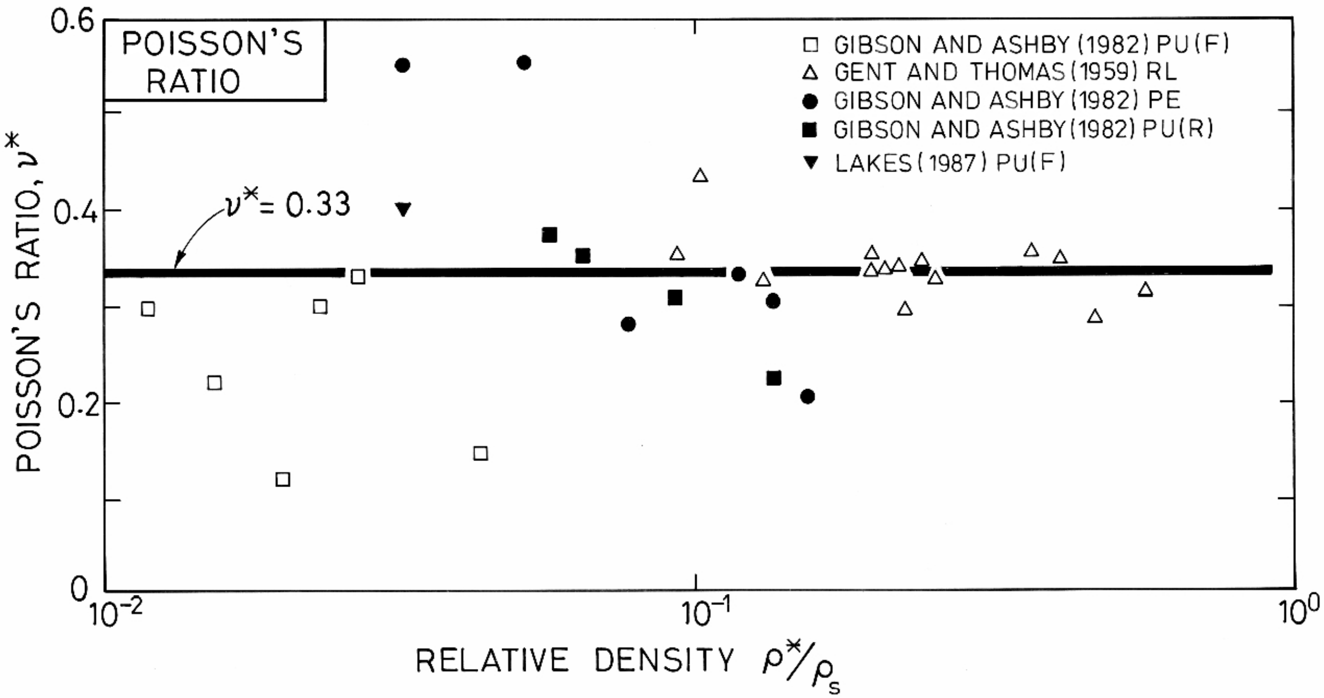
Shear Modulus



Poisson's Ratio



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Non-linear elasticity

Open cells:

$$P_{\mathrm{cr}} = \frac{n^2\pi^2 E_s I}{l^2}$$

$$\sigma^*_{\mathrm{ei}} \propto \frac{P_{\mathrm{cr}}}{l^2} \propto E_s \left(\frac{t}{l}\right)^4$$

$$\boxed{\sigma^*_{\mathrm{el}} = C_4\, E_s \left(\frac{\rho^*}{\rho_s}\right)^2}$$

Data: $C_4 \approx 0.05$, corresponds to strain when buckling initiates, since $E^* = E_S \left(\frac{\rho^*}{\rho_s}\right)^2$

Closed cells:

- t_f often small compared to t_e (surface tension in processing) - contribution small
- If $p_0 >> p_{\mathrm{atm}}$, cell walls pre-tensioned, bucking stress has to overcome this

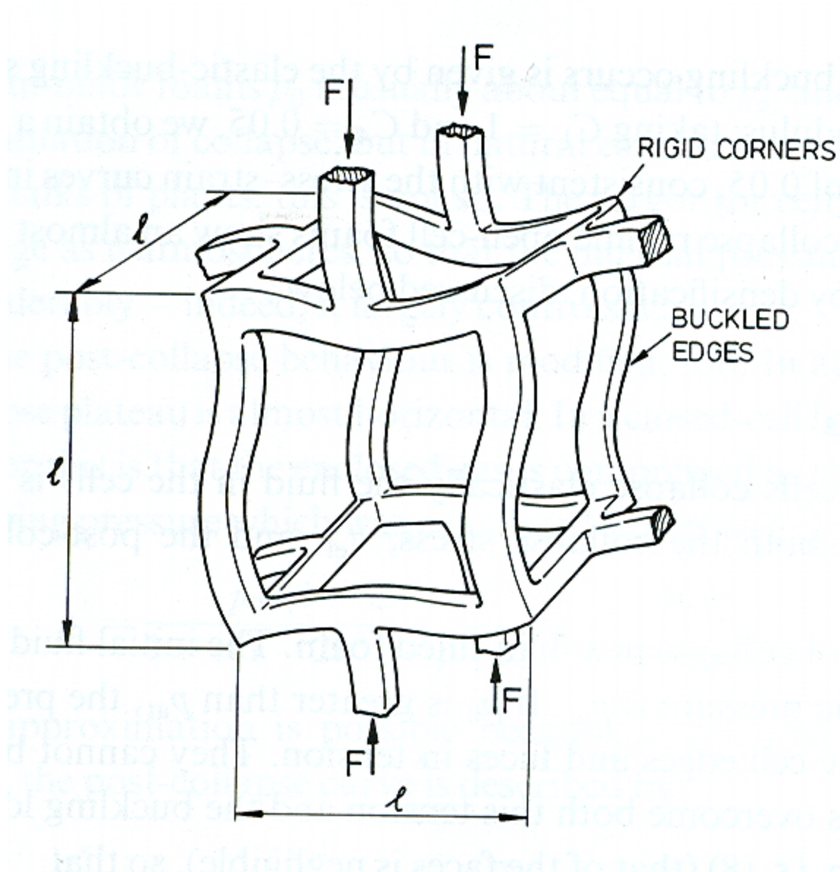
$$\sigma^*_{\mathrm{el}} = 0.05 E_s \left(\frac{\rho^*}{\rho_s}\right)^2 + p_0 - p_{\mathrm{atm}}$$

- Post-collapse behavior - stress plateau rises due to gas compression (if faces don't rupture) $\nu^* = 0$ in post-collapse regime

$$p' = \frac{p_0\, \epsilon(1-2\nu^*)}{1-\epsilon(1-2\nu^*) - \rho^*/\rho_s} = \frac{p_0 \epsilon}{1-\epsilon-\rho^*/\rho_s}$$

$$\boxed{\sigma^*_{\text{post-collapse}} = 0.05\, E_s \left(\frac{\rho^*}{\rho_s}\right)^2 + \frac{p_0\epsilon}{1-\epsilon-\rho^*/\rho_s}}$$

Elastic Collapse Stress



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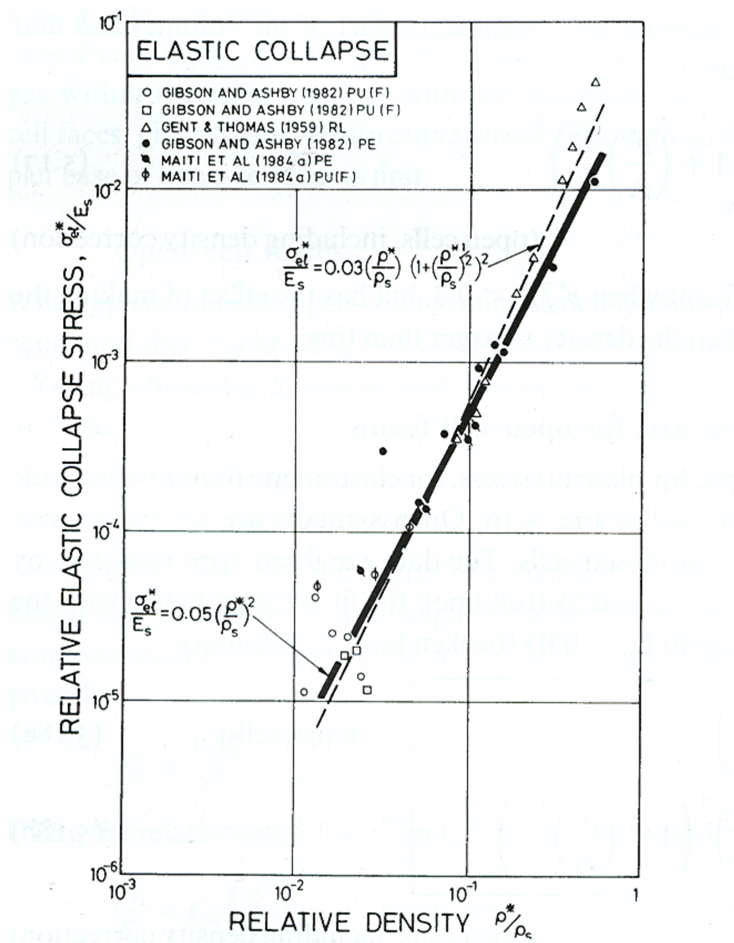
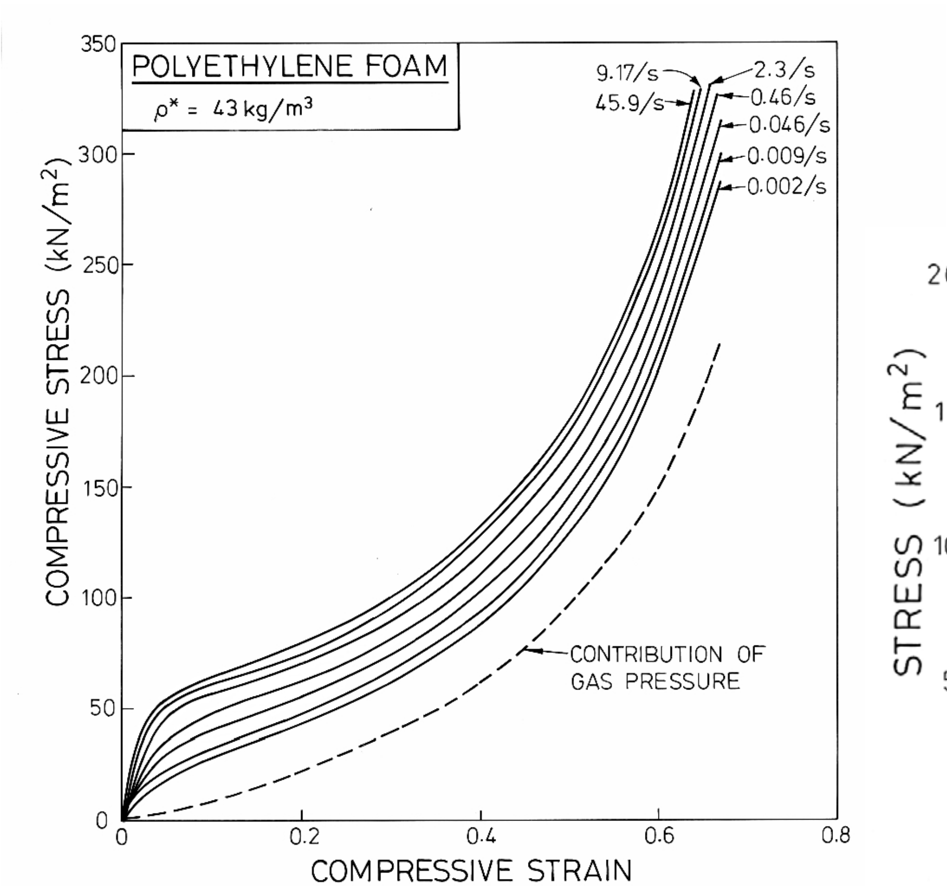
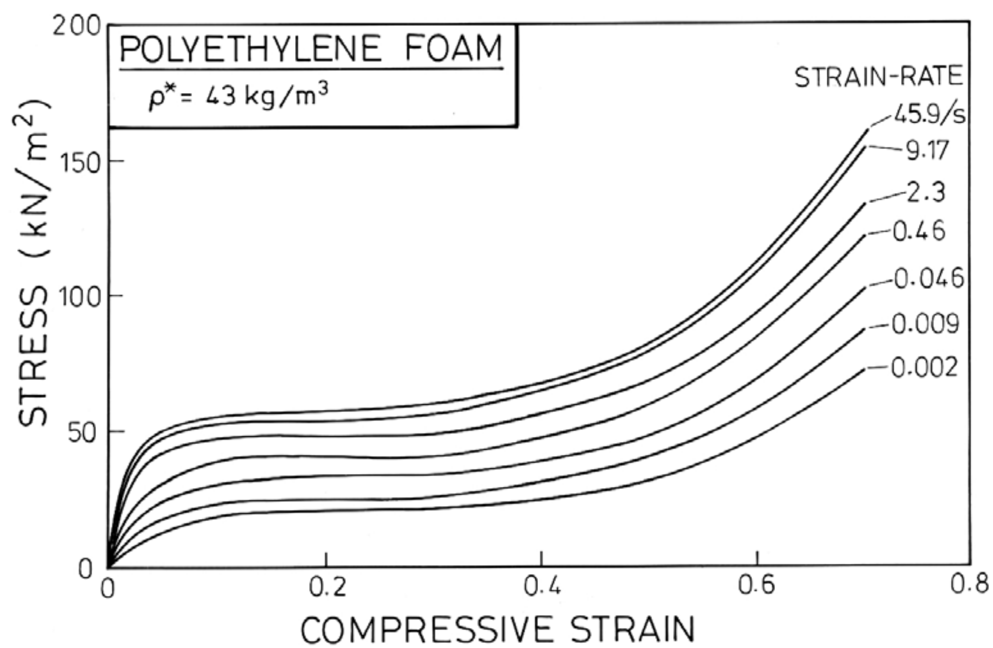
Elastic Collapse Stress



Post-collapse stress strain curve





Plastic collapse

Open cells:

- Failure when $M = M_p$
- $M_p \propto \sigma_{ys} t^3 \qquad M \propto \sigma^*_{pl} l^3$

  $\boxed{\sigma^*_{pl} = C_5 \sigma_{ys} (\rho^*/\rho_s)^{3/2}} \qquad C_5 \sim 0.3$ from data.
- Elastic collapse precedes plastic collapse if $\sigma^*_{el} < \sigma^*_{pl}$
- $$\begin{aligned} 0.05\, E_s (\rho^*/\rho_s)^2 &\leq 0.3\, \sigma_{ys} (\rho^*/\rho_s)^{3/2} \\ (\rho/\rho_s)_{\text{critical}} &\leq 36\, (\sigma_{ys}/E_s)^2 \end{aligned}$$

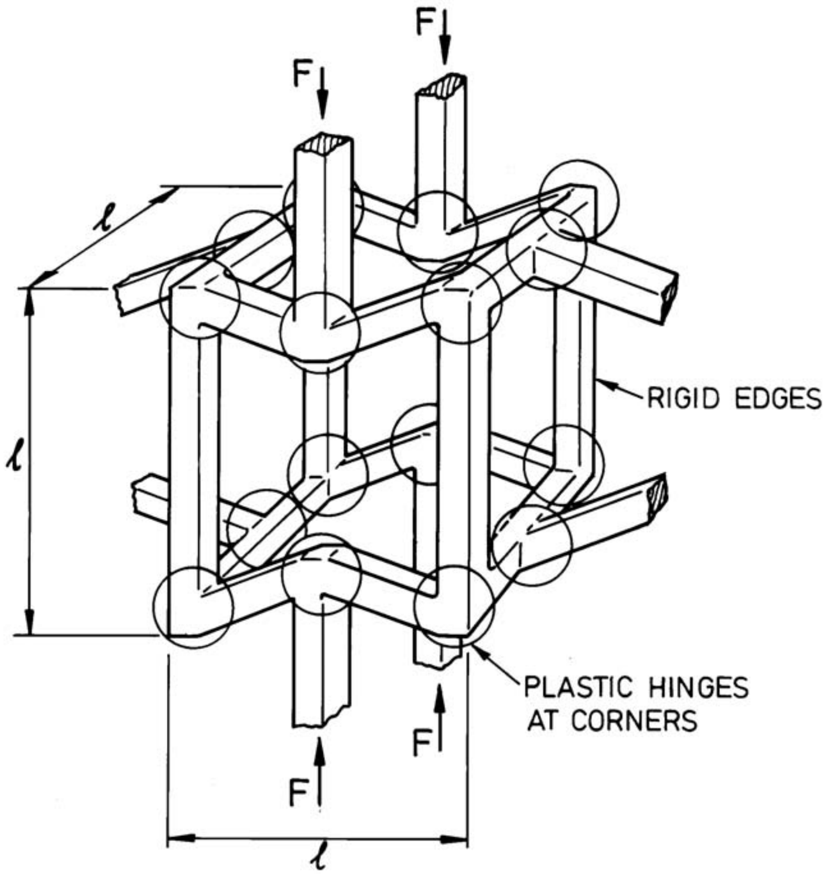
  rigid polymers $(\rho^*/\rho_s)_{cr} < 0.04 \left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{30}\right)$

  metals $(\rho^*/\rho_s)_{cr} < 10^{-5} \left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{1000}\right)$

Closed cells:

- Including all terms: $$\sigma^*_{pl} = \underbrace{C_5\, \sigma_{ys} \left(\phi \frac{\rho^*}{\rho_s}\right)^{3/2}}_{\uparrow \text{ edge bending}} + \underbrace{C'_5\, \sigma_{ys} (1-\phi) \left(\frac{\rho^*}{\rho_s}\right)}_{\uparrow \text{ face stretching}} + \underbrace{p_0 - p_{\text{atm}}}_{\uparrow \text{ gas}}$$
- But in practice, faces often rupture around σ^*_{pl} - often $\sigma^*_{pl} = 0.3\, (\rho^*/\rho_s)^{3/2}\, \sigma_{ys}$

Plastic Collapse Stress



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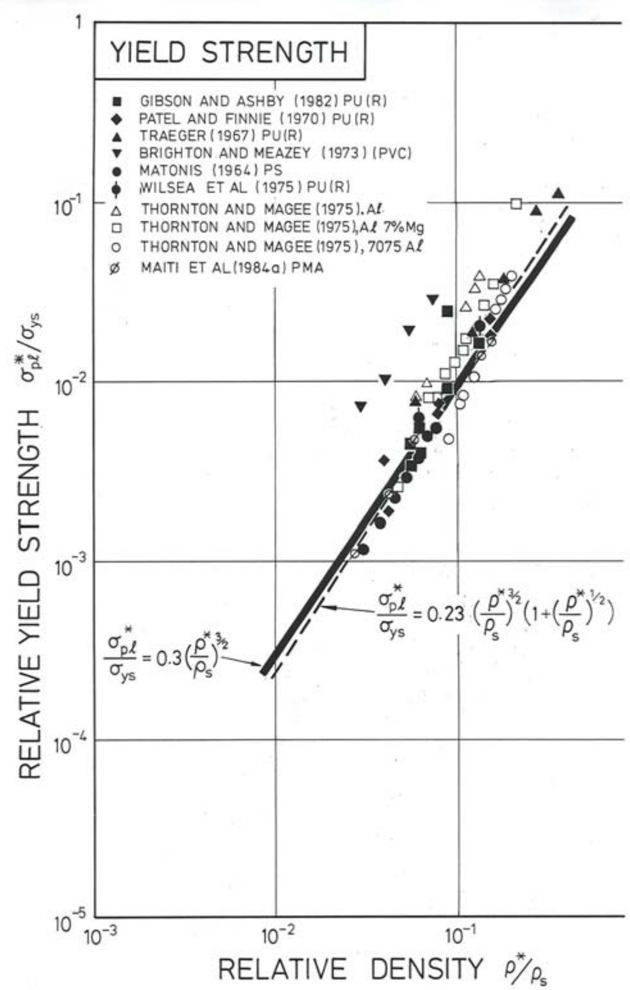
Plastic Collapse Stress



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Brittle crushing strength

Open cells:

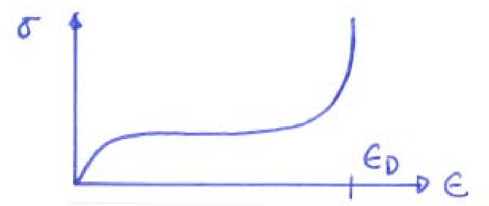
- Failure when $M = M_f$ $\quad M \propto \sigma^*_{cr}\, l^2 \quad M_f \propto \sigma_{fs}\, t^3$

$$\boxed{\sigma_{cr} = C_6\, \sigma_{fs}\, (\rho^*/\rho_s)^{3/2}} \qquad C_6 \approx 0.2$$

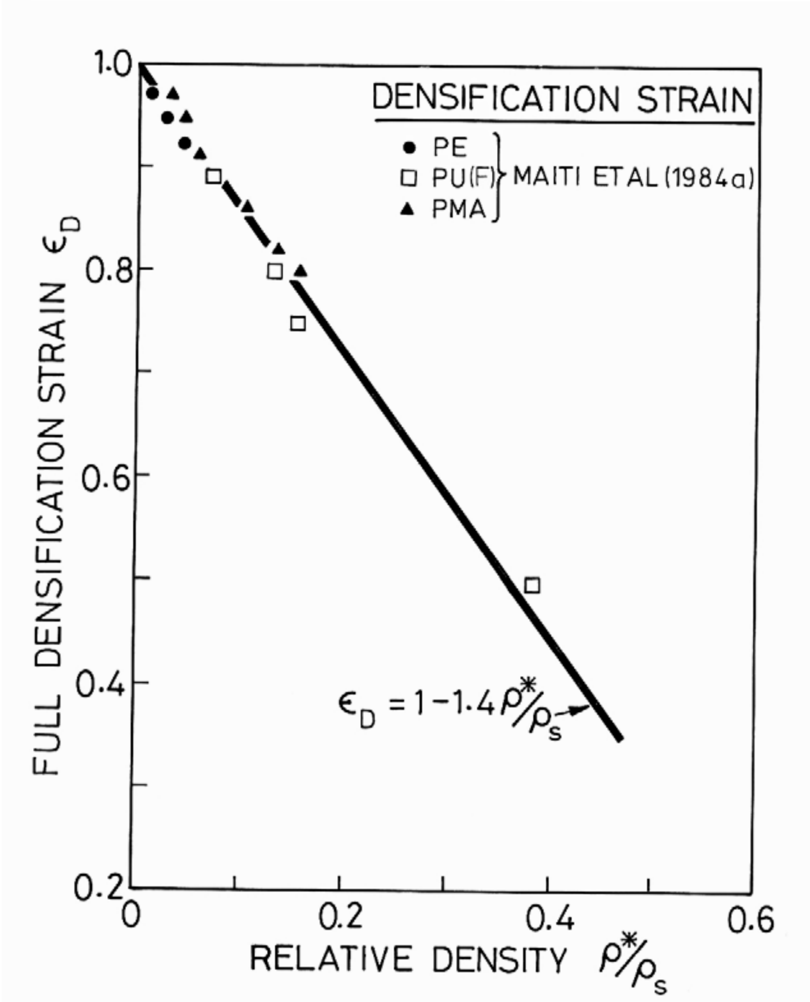
Densification strain, ϵ_D:

- At large comp. strain, cell walls begin to touch, $\sigma - \epsilon$ rises steeply
- $E^* \rightarrow E_s$; $\sigma - \epsilon$ curve looks vertical, at limiting strain
- Might expect $\epsilon_D = 1 - \rho^*/\rho_s$
- Walls jam together at slightly smaller strain than this:

$$\boxed{\epsilon_D = 1 - 1.4\, \rho^*/\rho_s}$$



Densification Strain



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Fracture toughness

Open cells: crack length $2a$, local stress σ_l, remote stress σ^∞

$$\sigma_l = \frac{C\sigma^\infty\sqrt{\pi a}}{\sqrt{2\pi r}} \qquad \text{a distance } r \text{ from crack tip}$$

- Next unbroken cell wall a distance $r \approx l/2$, a head of crack tip subject to a force (integrating stress over next cell)

  $$F \propto \sigma_l\, l^2 \propto \sigma^\infty \sqrt{\tfrac{a}{l}}\, l^2$$

- Edges fail when applied moment, M = fracture moment, M_f

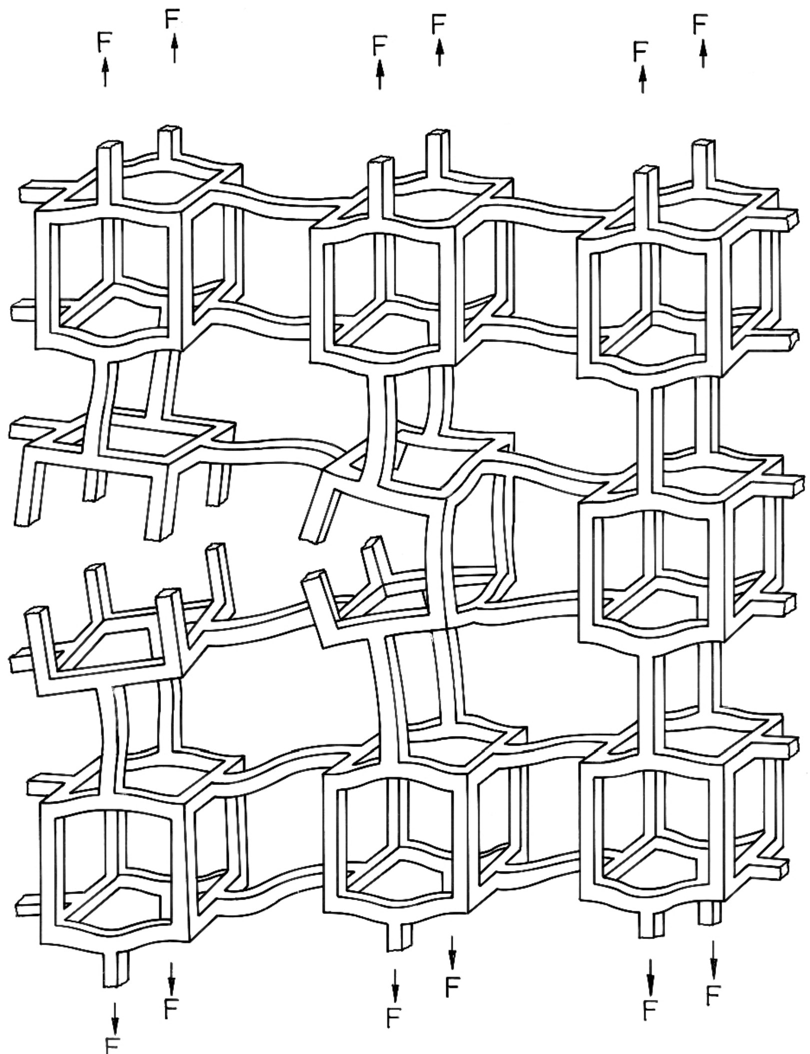
  $$M_f = \sigma_{\text{fs}}\, t^3$$

  $$M \propto F\, l \propto \sigma^\infty \left(\frac{a}{l}\right)^{1/2} l^3 \qquad M = M_f \quad \rightarrow \quad \sigma^\infty \left(\frac{a}{l}\right)^{1/2} l^3 \propto \sigma_{\text{fs}}\, t^3$$

  $$\sigma^\infty \propto \sigma_{\text{fs}} \left(\frac{l}{a}\right)^{1/2} \left(\frac{t}{l}\right)^3$$

$$\boxed{K^*_{IC} = \sigma^\infty \sqrt{\pi a} = C_8\, \sigma_{\text{fs}} \sqrt{\pi l}\left(\rho^*/\rho_s\right)^{3/2}} \qquad \text{Data: } C_8 \sim 0.65$$

Fracture Toughness



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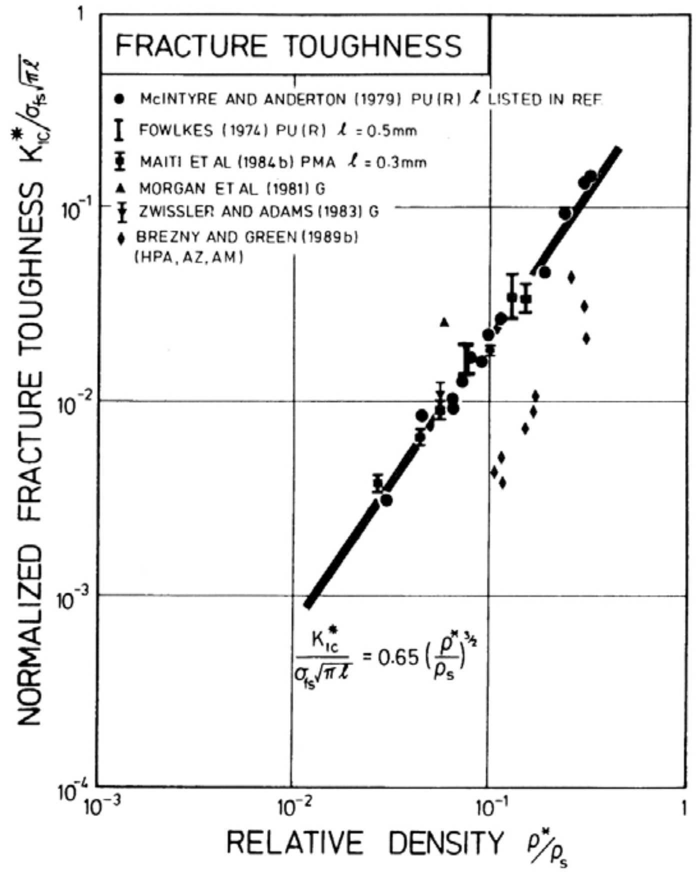
Fracture Toughness



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