Lecture 15, Energy Absorption Notes, 3.054

Energy absorption in foams

- Impact protection must absorb the kinetic energy of the impact while keeping the peak stress below the threshold that causes injury or damage
- Direction of the impact may not be predictable
- Impact protection must itself be light e.g. helmet

- Capacity to undergo large deformation ($\epsilon \sim 0.8, 0.9$) at constant σ
- Absorb large energies with little increase in peak stress

- Foams — roughly isotropic — can absorb energy from any direction - light and cheap
- For a given peak stress, foam will always absorb more energy than solid it is made from
- Strain rates: Instron typically $\dot{\epsilon} \sim 10^{-8}$ to 10^{-2}/s

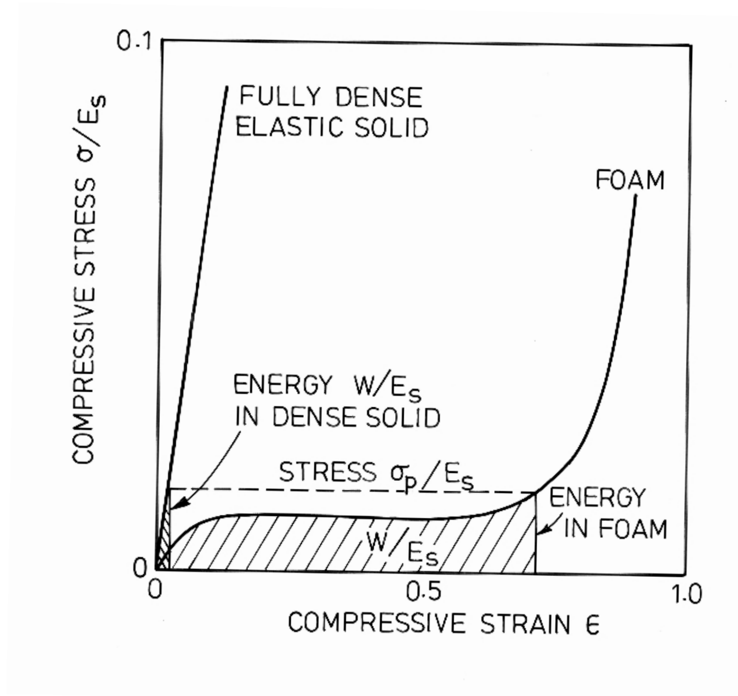
impact e.g. drop from height of 1 m, if thickness of foam=100mm

$$v_{\text{impact}} = \sqrt{2gh} = \sqrt{2(9.8)(1)} = 4.4\ \text{m/s}; \quad \dot{\epsilon}\frac{4.4\text{m/sec}}{0.1\ m} = 44/s$$

• servo controlled Instrons, drop hammer tests — up to $\dot{\epsilon} = 100/s$

blast: $\dot{\epsilon} = 10^3 - 10^4/s$ — inertial effects impt (we won't consider this)

Energy Absorption

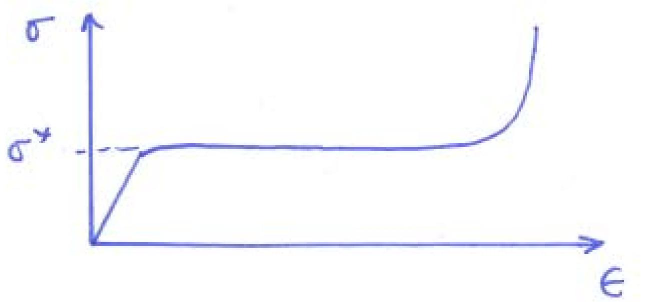


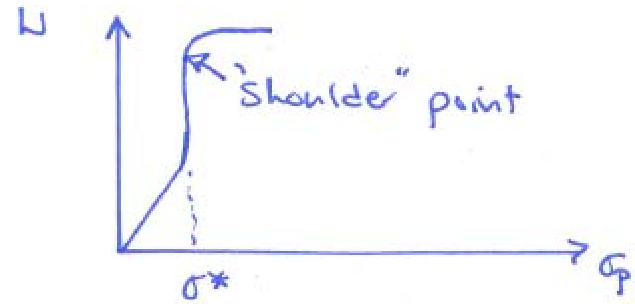
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Energy absorption mechanisms

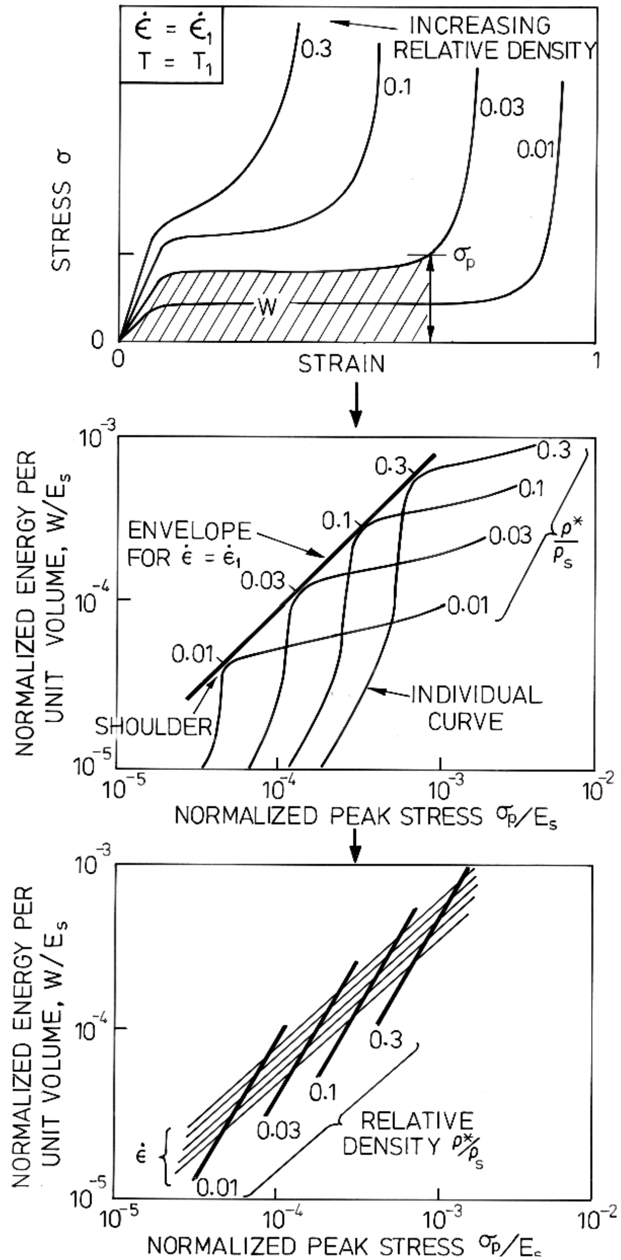
- Elastomeric foams — elastic buckling of cells
  — elastic deformation recovered → rebound
  — also have damping - energy dissipated as heat
- Plastic foams, brittle foams — energy dissipated as plastic work or work of fracture
  — no rebound
- Natural cellular materials — may have fiber composite cell walls
  — dissipate energy by fiber pullout and fracture
- Fluid within cells
  - open cell foams — fluid flow dissipation only important if fluid is viscous, cells are small or rates are high
  - closed cell foams — compression of cell fluid
    — energy recovered as unkading

Energy absorption diagrams





- At stress plateau, energy W increases with little increase in peak stress, σ_p
- As foam densifies, $W \sim$ constant and σ_p increases sharply
- Ideally, want to be at "shoulder" point
- More generally — see Figure
- Test series of one type of foam of different ρ^*/ρ_s at constant $\dot{\epsilon}$ and temperature, T
- Plot W/E_s vs. σ_p/E_s for each curve (E_s at standard $\dot{\epsilon}$ and T)
- Heavy line joins the shoulder points for each curve
- Mark ρ^*/ρ_s for each foam on that line
- Repeat for varying $\dot{\epsilon}$ $\rightarrow$ join lines for constant ρ^*/ρ_s
- Build up family of optimum energy absorption curves
- Can treat different temperatures, T, in same way



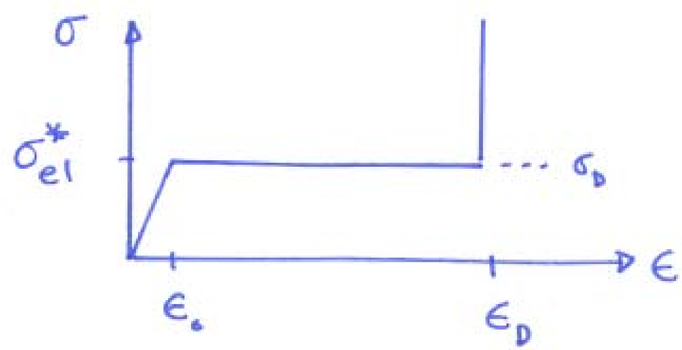
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Notes:

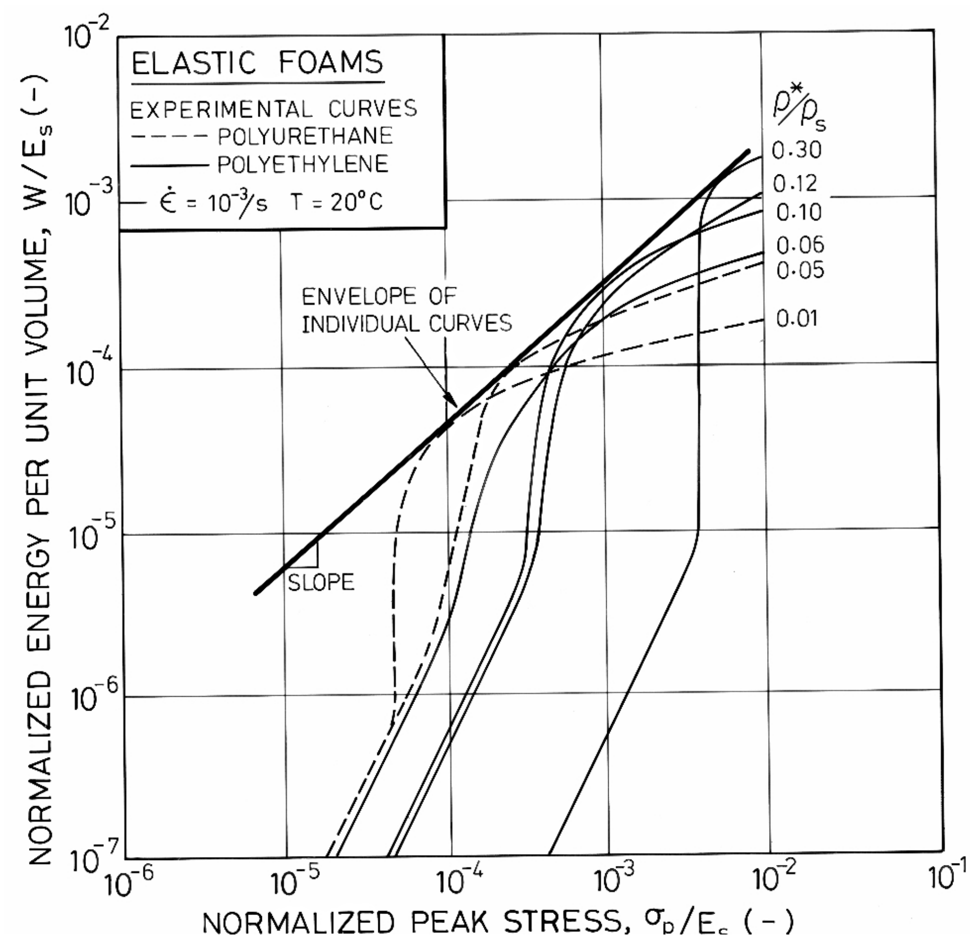
- Elastomeric foams can all be plotted on one curve since $E^* \propto E_s$ and $\sigma_{el}^* \propto E_s$ (normalize W/E_s and σ_p/E_s)
- Figure: polyurethane and polyethylene
- polymethacrylimid: σ_{pl}^* $\Rightarrow$ typical of foams with plastic collapse stress with $\sigma_{ys}/E_s = 1/30$
- Can generate energy absorption diagrams from data, or use models for foam properties

Modelling energy absorption diagrams

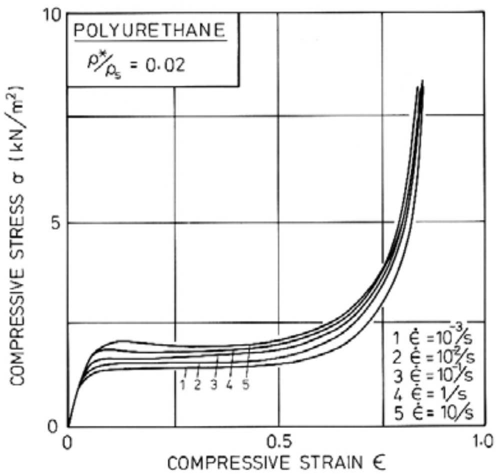
Open cell elastomeric foams

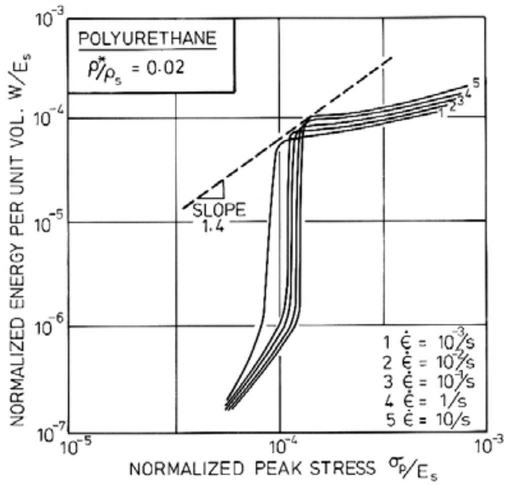


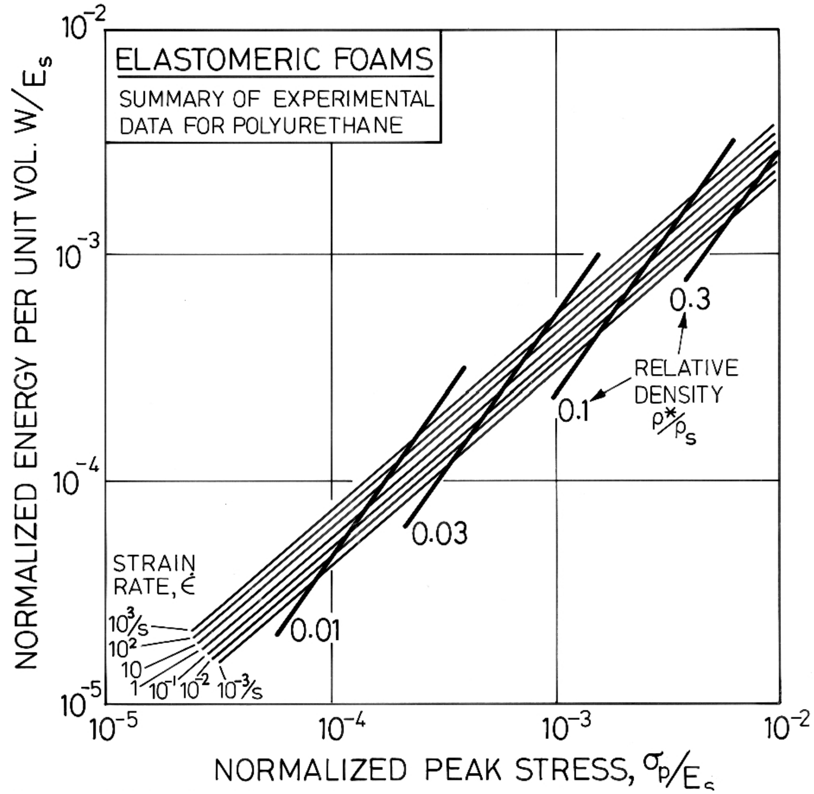
Elastomeric Foams



Flexible Polyurethane

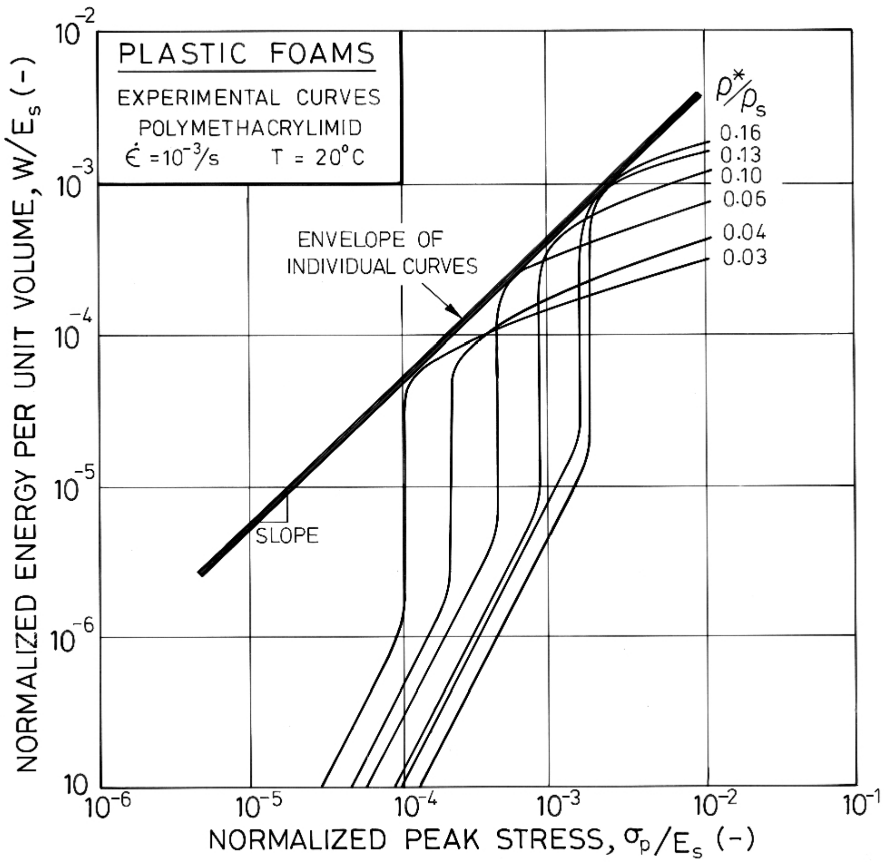


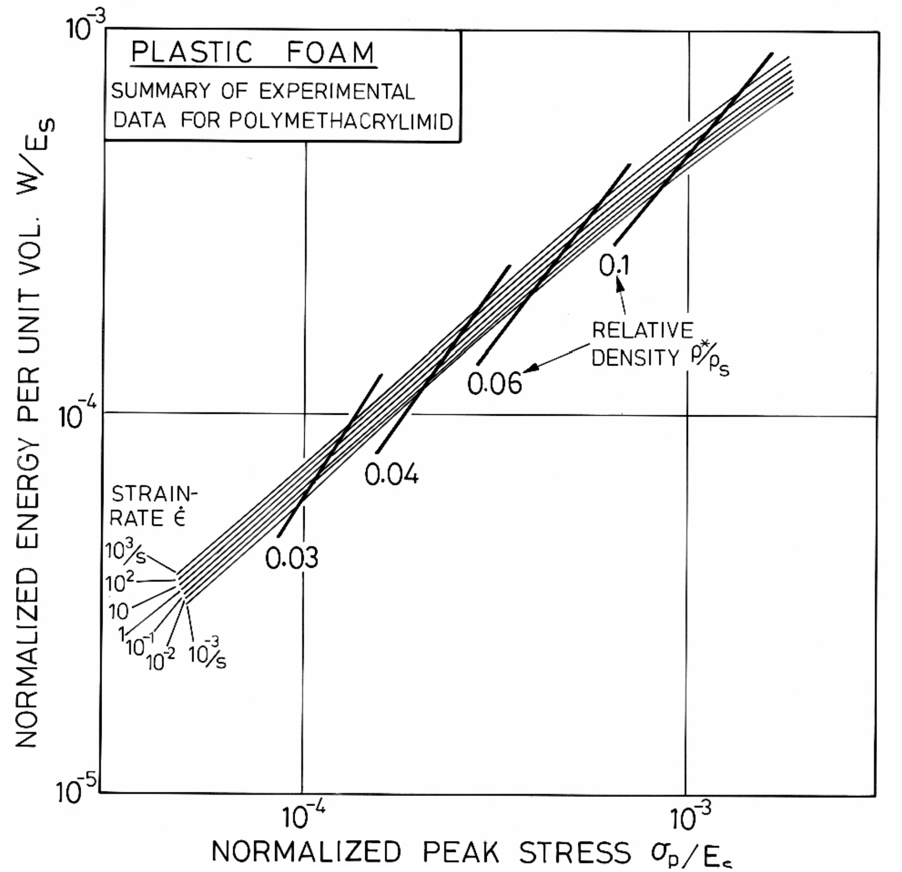




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Polymethacrylimid





(a) Linear elastic region $\epsilon < \epsilon_0$

$$W = \frac{1}{2}\frac{\sigma_p^2}{E^*} \qquad \frac{W}{E_s} = \frac{1}{2}\left(\frac{\sigma_p^2}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$$

(b) Stress plateau $\epsilon_0 < \epsilon < \epsilon_D$

$$dW = \sigma_{el}^* \, d\epsilon \qquad \frac{W}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(\epsilon - \epsilon_0)$$

- family of vertical lines on figure
- plateau ends at densification strain ϵ_D
- then W/E_s vs. σ_p/E_s becomes horizontal

(c) At end of stress plateau $\epsilon \sim \epsilon_D$

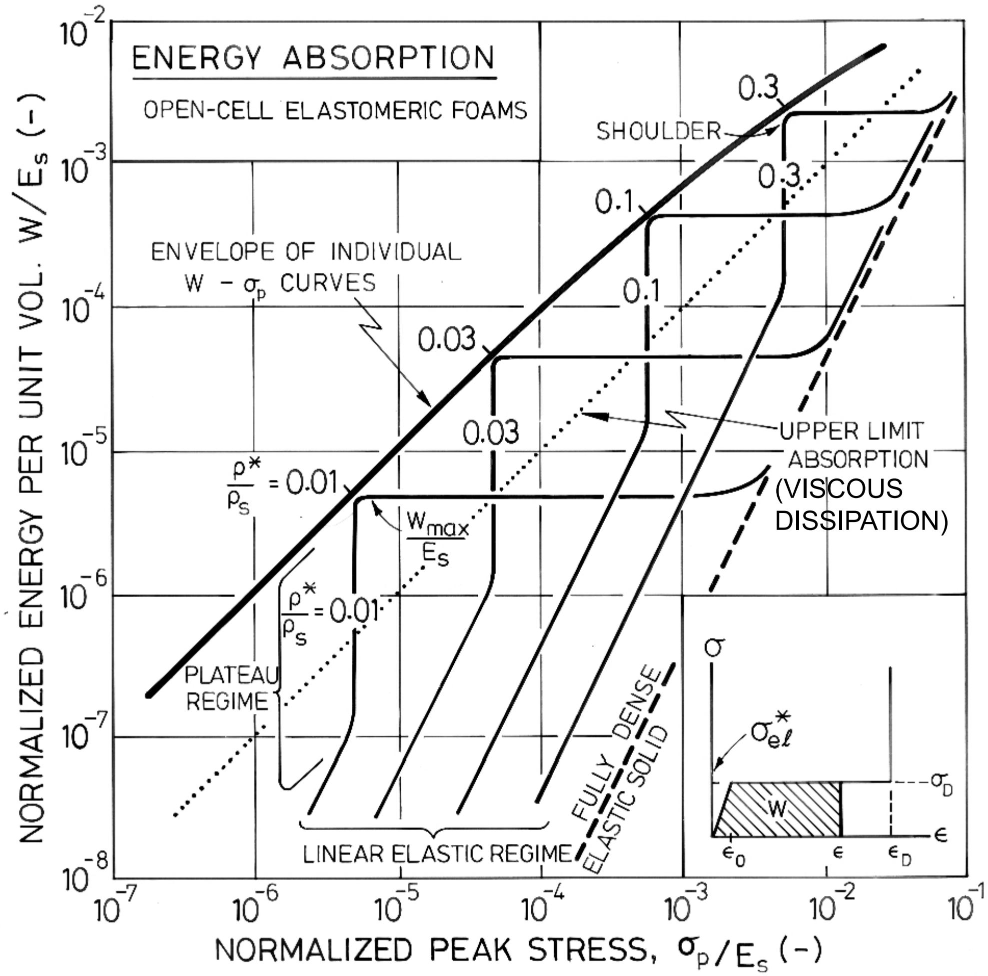
- maximum energy absorbed just before reach ϵ_D (shoulder point)

$$\frac{W_{\max}}{E_s} - = 0.05\,(\rho^*/\rho_s)^2(1 - 1.4\,\rho^*/\rho_s) \text{ (assuming } \epsilon_0 << \epsilon_D \text{ and neglecting } \epsilon_0\text{)}$$

- optimum choice of foam is one with shoulder point that lies at $\sigma_p = \sigma_D$
- envelope of shoulder points, for optimum foams, at:

$$\sigma_p = \sigma_D = 0.05\,E_s(\rho^*/\rho_s)^2 \qquad \rho^*/\rho_s = \left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}$$

Open-cell Elastomeric Foams: Modelling



Substituting into equation for $W_{\max}/E_s$:

$$\frac{W_{\max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 1.4\left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}\right] \qquad \frac{W_{\max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 6.26\left(\frac{\sigma_p}{E_s}\right)^{1/2}\right]$$

- Line of slope 1 at low stresses, falling to 7/8 at higher σ

(d) Densification

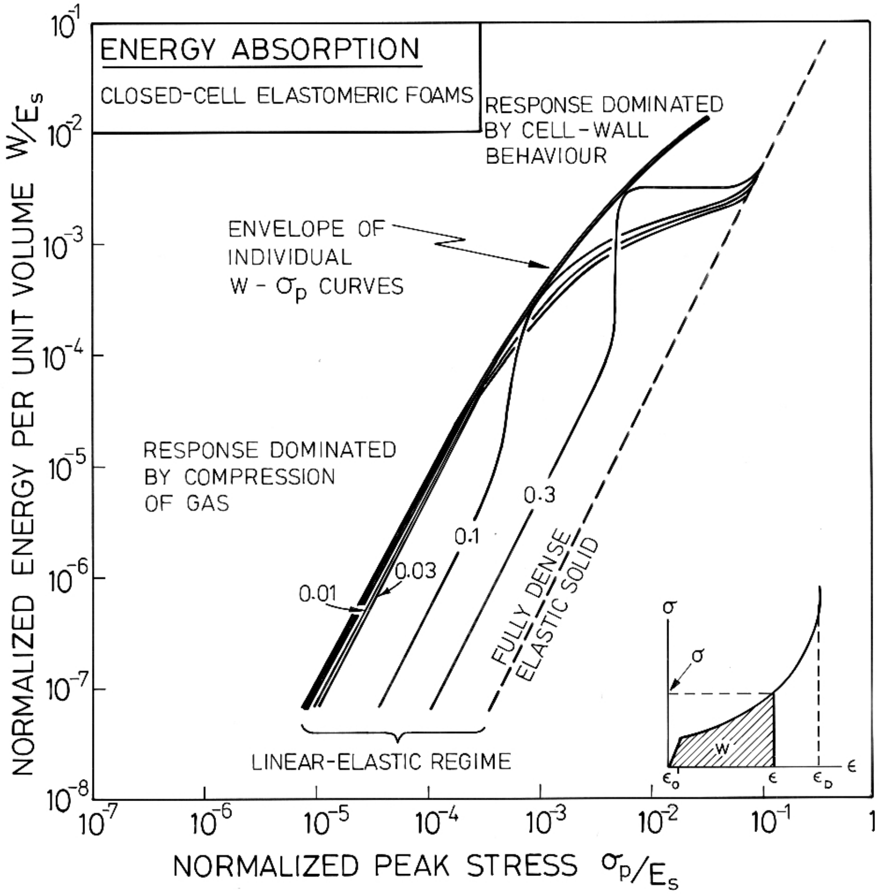
- when foam fully densified and compressed to a solid, then energy absorption curve joins that for the fully dense elastomer

$$\frac{W}{E_s} = \frac{1}{2}\frac{\sigma_p^2}{E_s}$$

Note:

- Model curves have same shape as expts.
- Model shows W/E_s depends on σ_p/E_s and ρ^*/ρ_s only — one diagram for all elastomer foams
- For a given W/E_s, σ_p/E_s for the foam less than that of the fully dense solid, by a factor of 10^{-3} to 10^{-1}

Closed-cell Elastomeric Foams: Modelling



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Modeling: open-cell foams that yield

- Analysis similar to elastomeric foams, with σ_{pl} replacing σ_{el}
- Note that some closed cell foams that yield, face contribution to E^* σ_{pl} negligible
- Neglect fluid contribution

$$\sigma_{pl}^* = 0.3\, \sigma_{ys}(\rho^*/\rho_s)^{3/2}$$

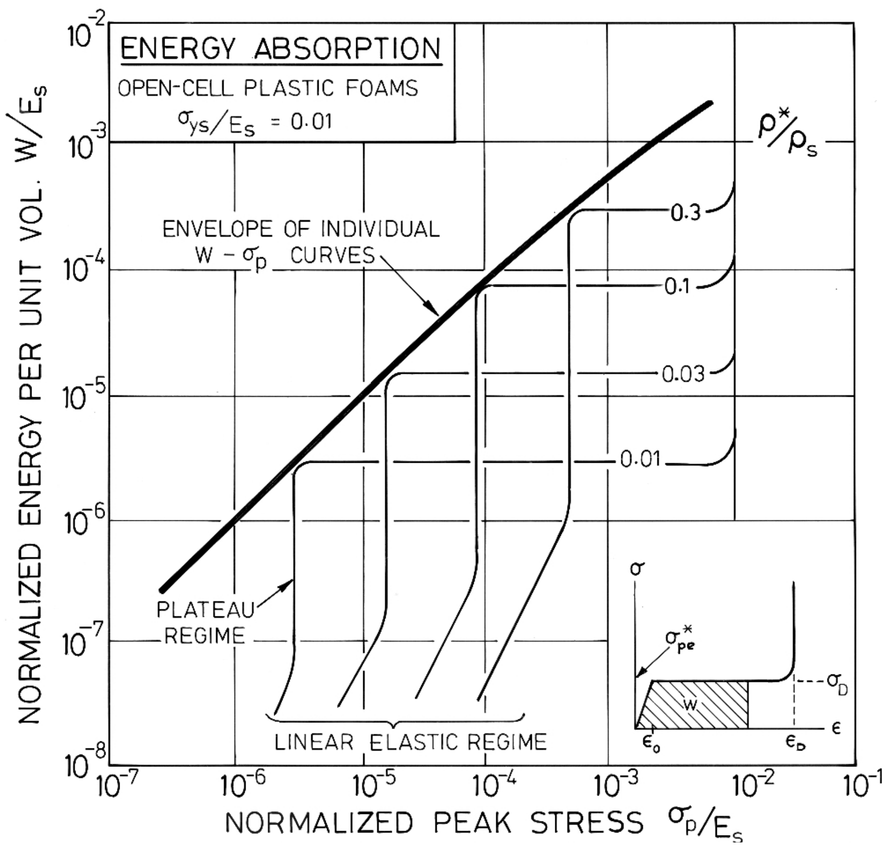
(a) Linear elastic regime: same as elastomeric foam: $\dfrac{W}{E_s} = \dfrac{1}{2}\left(\dfrac{\sigma_p}{E_s}\right)^2 \dfrac{1}{(\rho^*/\rho_s)^2}$

(b) Stress plateau: $\dfrac{W}{E_s} = 0.3\ \dfrac{\sigma_{ys}}{E_s}\left(\dfrac{\rho^*}{\rho_s}\right)^{3/2}(\epsilon - \epsilon_0)$

(c) End of stress plateau: $\dfrac{W_{\max}}{E_s} \approx 0.3\ \dfrac{\sigma_{ys}}{E_s}\left(\dfrac{\rho^*}{\rho_s}\right)^{3/2}(1 - 1.4\,\rho^*/\rho_s)$

- optimum choice of foam — absorbs maximum energy without σ_p rising sharply at ϵ_D

Plastic Foams: Modelling



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

- Curve of optimum energy absorption (heavy line on figure) is envelope that touches $W - \sigma_p$ curve at shoulder points
- For σ_p, $\dfrac{\rho^*}{\rho_s} = \left(\dfrac{3.3\,\sigma_p}{\sigma_{ys}}\right)^{2/3}$
- Substituting in $W_{\max}/E_s$ equation: $\dfrac{W_{\max}}{E_s} = \dfrac{\sigma_D}{E_s}\left\{1 - 3.1\left(\dfrac{\sigma_D}{\sigma_{ys}}\right)^{2/3}\right\}$
- Model curves explain general features of experimental curves
- Modeling — curves less general than for elastomers
  — this cure for a particular value of $\sigma_{ys}/E_s = 1/100$
  (typical value for polymers)

Design and selection of foams for impact protection

- Typically know object to be protected and some details about it

mass, m
contact area, A
max drop height, h
(or energy to be absorbed, u)

max allowable acceleration, a
(e.g. head injury - 100g)
peak stress allowable, σ_p

- Variables: foam material, density, thickness

Example 1

Given: mass, m=0.5 kg
contact area, A=0.01 m^2
drop height, h=1 m
max deceleration, a=10g
foam: flexible polyurethane $E_s = 50$ MPa

Find: optimum foam density
optimum foam thickness

Example 1:
Find Foam Density and Thickness

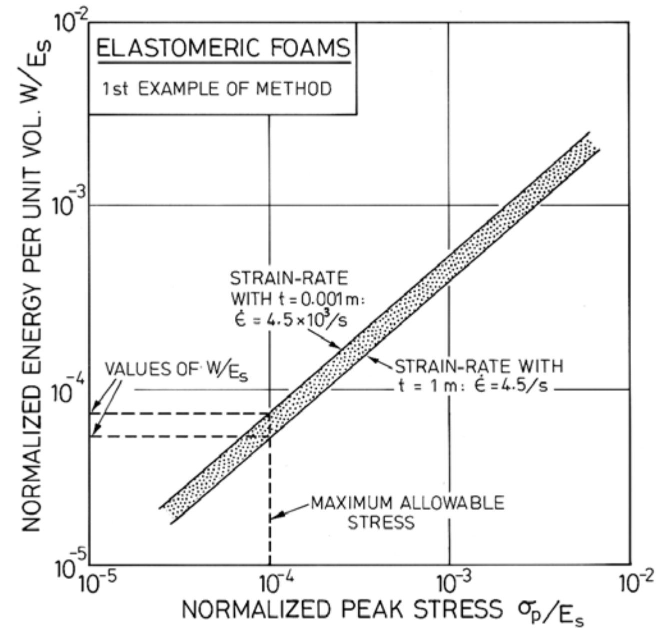
Table 8.2 Example 1: selection of foams

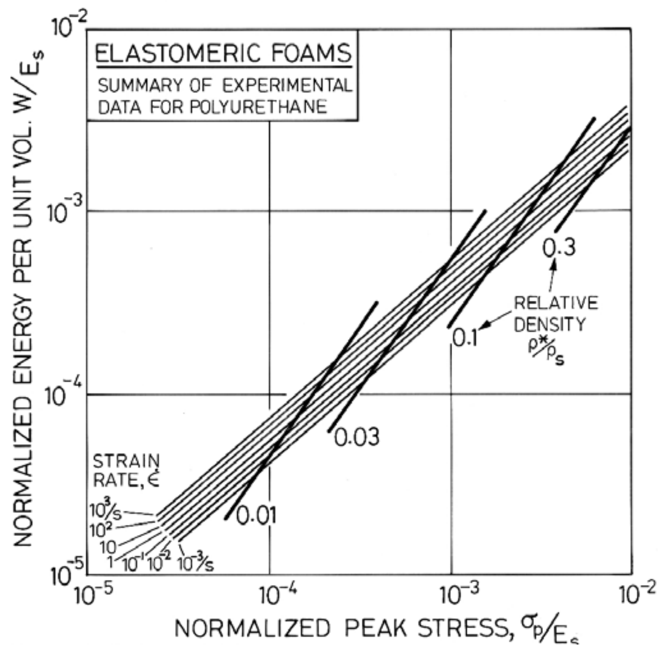
Specification of the problem

Mass of the package object, $m = 0.5\,\text{kg}$
Area of contact between foam and object, $A = 0.01\,\text{m}^2$
Velocity of package on impact (drop height $h = 1\,\text{m}$), $v = 4.5\,\text{m/s}$
Energy to be absorbed, $U = mv^2/2 = 5\,\text{J}$
Maximum allowable package force (based on deceleration of $10g$), $F = ma = 50\,\text{N}$
Maximum allowable peak stress, $\sigma_p = F/A = 5\,\text{kN/m}^2$
Solid modulus in foam (flexible polyurethane), $E_s = 50\,\text{MN/m}^2$
Maximum allowable normalized peak stress, $\sigma_p/E_s = 10^{-4}$

Iterative procedure

1st Iteration	$t_1 \gg t$	$t_1 \ll t$
Initial choice of t_1	1 m	0.001 m
Resulting strain-rate, $\dot{\epsilon} = v/t_1$	$4.5\,\text{s}^{-1}$	$4.5 \times 10^3\,\text{s}^{-1}$
Resulting (W/E_s) at $\sigma_p/E_s = 10^{-4}$	5.25×10^{-5}	7.4×10^{-5}
Energy absorbed per unit volume, W	$2620\,\text{J/m}^3$	$3700\,\text{J/m}^3$
2nd Iteration		
Revised t_2 (from $U = WAt$)	0.19 m	0.14 m
Revised $\dot{\epsilon} = v/t_2$	$24\,\text{s}^{-1}$	$32\,\text{s}^{-1}$
Revised (W/E_s)	6.6×10^{-5}	6.7×10^{-5}
Revised W	$3300\,\text{J/m}^3$	$3350\,\text{J/m}^3$
3rd Iteration		
Revised t_3 (from $U = WAt$)	0.15 m	0.15 m
Optimum density, ρ^*/ρ_s (Fig. 8.8)	A little below 0.01	





- Energy to be absorbed, u=mgh=(0.5 kg)(10 m/s ^{2})(1m)=5 J
- Maximum allowable force on package = F=ma=(0.5 kg)(10g)=50 N
- Peak stress, σ_p=F/A=50 N / 0.01 m^2=5 kN/m^2
- Normalized peak stress, σ_p/E_s=5 kPa / 50 MPa = 10^{-4}
- Draw vertical line on energy absorption diagram at $\sigma_p/E_s = 10^{-4}$
- Need to know $\dot\epsilon \approx v/t$ velocity $v = \sqrt{2gh} = 4.5$ m/s
- Iterative approach — choose arbitrary thickness, t

- e.g. $t_1 = 1$ m
  - $\dot\epsilon = 4.5$/s
  - $W/E_s = 5.25 \times 10^{-5}$
  - $W = 2620$ J/m^3
  - (u=WAt$_1$) $t_2 = WA/u = 0.19$ m
  - $\dot\epsilon_2 = 24$/s
  - $W/E_s = 6.6 \times 10^{-5}$
  - $W = 3300$ J/m^3

$t_1 = 0.001$ m
$\dot\epsilon = 4.5 \times 10^3$/s
$W/E_s = 7.4 \times 10^{-5}$
$W = 3700$ J/m^3
$t_2 = 0.14$ m
$\dot\epsilon_2 = 32$/s
$W/E_s = 6.7 \times 10^{-5}$
$W = 3350$ J/m^3

Third iteration: t_3=0.15 m
(both W)
Optimum density (Fig)
$\rho^*/\rho_s \sim 0.01$
Note: t converges quickly even from very different initial guesses for t

Example 2

Given m = 2.5 kg   Find foam material

A = 0.025 m^2 foam density

t = 20 mm

h = 1 m

a = 100 g

Calculate W, σ_p, $\dot{\epsilon}$

$$W = \frac{mgh}{At} = \frac{(2.5\text{ kg})(10\text{ m/s}^2)(1\text{ m})}{0.025\text{ m}^2(0.02\text{ m})} = 5 \times 10^{-4}\text{ J/m}^3$$

$$\sigma_p = \frac{F_{\max}}{A} = \frac{ma}{A} = \frac{(2.5\text{ kg})(100)(10\text{ m/s}^2)}{0.025\text{ m}^2} = 10^5\text{ N/m}^2$$

$$\dot{\epsilon} = \frac{v}{t} = \frac{\sqrt{2gh}}{t} = \frac{\sqrt{2(10\text{ m/s}^2)(1\text{ m})}}{0.02\text{ m}} = \frac{4.5\text{ m/s}}{0.02\text{ m}} = 225\text{ /s}$$

Select arbitrary value of E_s=100 MPa

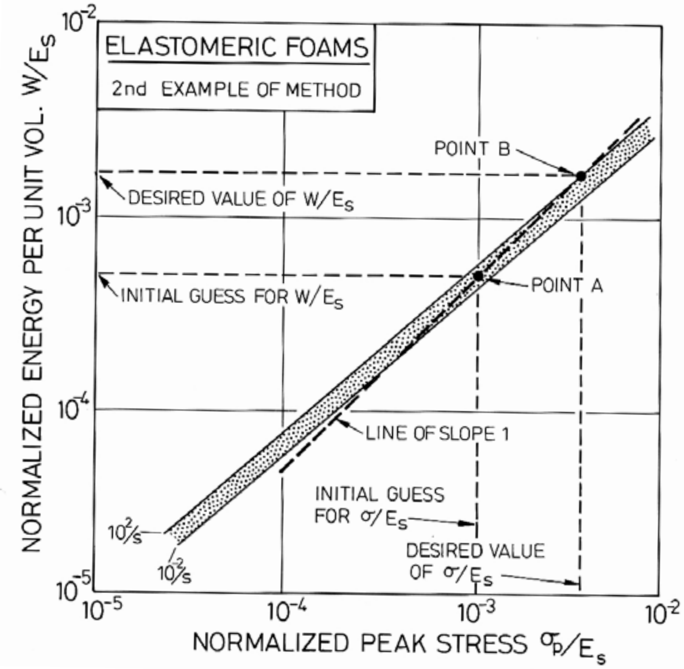
Plot $W/E_s = 5 \times 10^{-4}$ point A

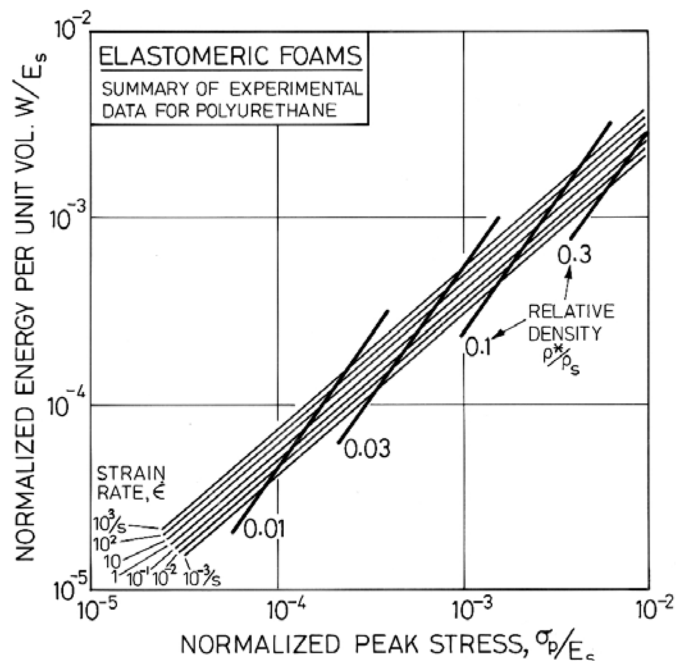
$\sigma_p/E_s = 10^{-3}$

Example 2: Find Foam Material and Density

Table 8.3 Example 2: selection of foams

Specification of the problem
Mass of the package object, $m = 2.5\,\mathrm{kg}$
Area of contact between foam and object, $A = 0.025\,\mathrm{m}^2$
Thickness of foam, $t = 20\,\mathrm{mm}$
Drop height, $h = 1\,\mathrm{m}$
Velocity of impact $v = (2gh)^{1/2} = 4.5\,\mathrm{m/s}$
Strain-rate $\dot{\epsilon} = v/t = 225/\mathrm{s}$
Energy to be absorbed $U = mgh = 25\,\mathrm{J}$
Energy to be absorbed per unit volume of foam $W = U/At = 5 \times 10^4\,\mathrm{J/m^3}$
Maximum allowable force (based on decleration of $100g$) $= 2500\,\mathrm{N}$
Maximum allowable peak stress $\sigma_p = F/A = 10^5\,\mathrm{N/m^2}$
Trial design point A, using $E_s = 100\,\mathrm{MN/m^2}$
Normalized energy $W/E_s = 5 \times 10^{-4}$
Normalized peak stress $\sigma_p/E_s = 10^{-3}$
Final design point B, read from diagram
Normalized energy $W/E_s = 1.8 \times 10^{-3}$
Normalized stress $\sigma_p/E_s = 3.7 \times 10^{-3}$
Resulting derived value of $E_s = 28\,\mathrm{MN/m^2}$
Desired foam density ≈ 0.1





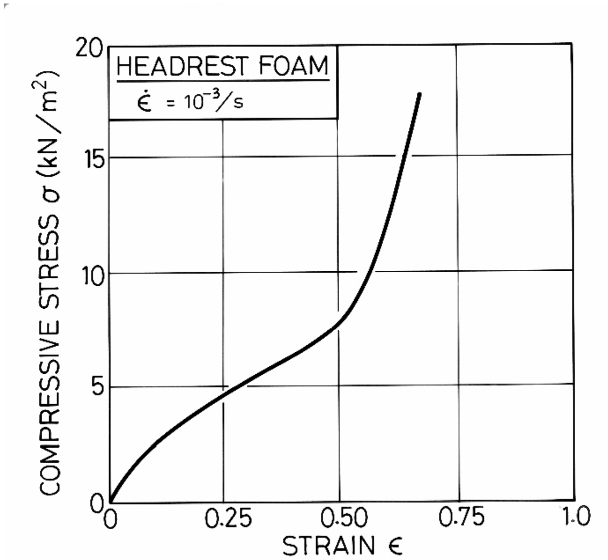
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

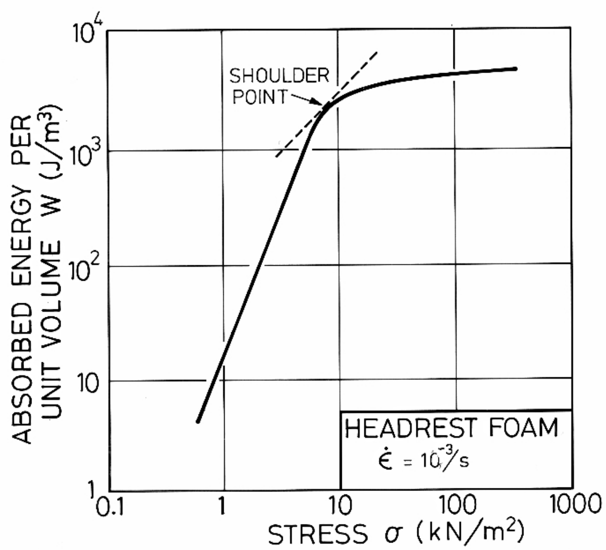
- Construct a line of slope 1 through this point (broken line)
- Moving along this line simply changes E_s
- Select the point where the broken line intersects the appropriate $\dot{\epsilon} \sim 10^2$/s (point B)
- Read off values of $W/E_s = 1.8 \times 10^{-3}$
  $\sigma_p/E_s = 37 \times 10^{-3}$
- Resulting value of $E_s = 28$ MPa $\Rightarrow$ low modulus, flexible polyurethane
- Replotting on more detailed figure: $\rho^*/\rho_s = 0.1$
- If point A above all energy contours and lone of slope 1 does not intersect them, specification cannot be achieved, A or t has to increase
- If point A below all contours, then A and t larger than need to be — can be reduced

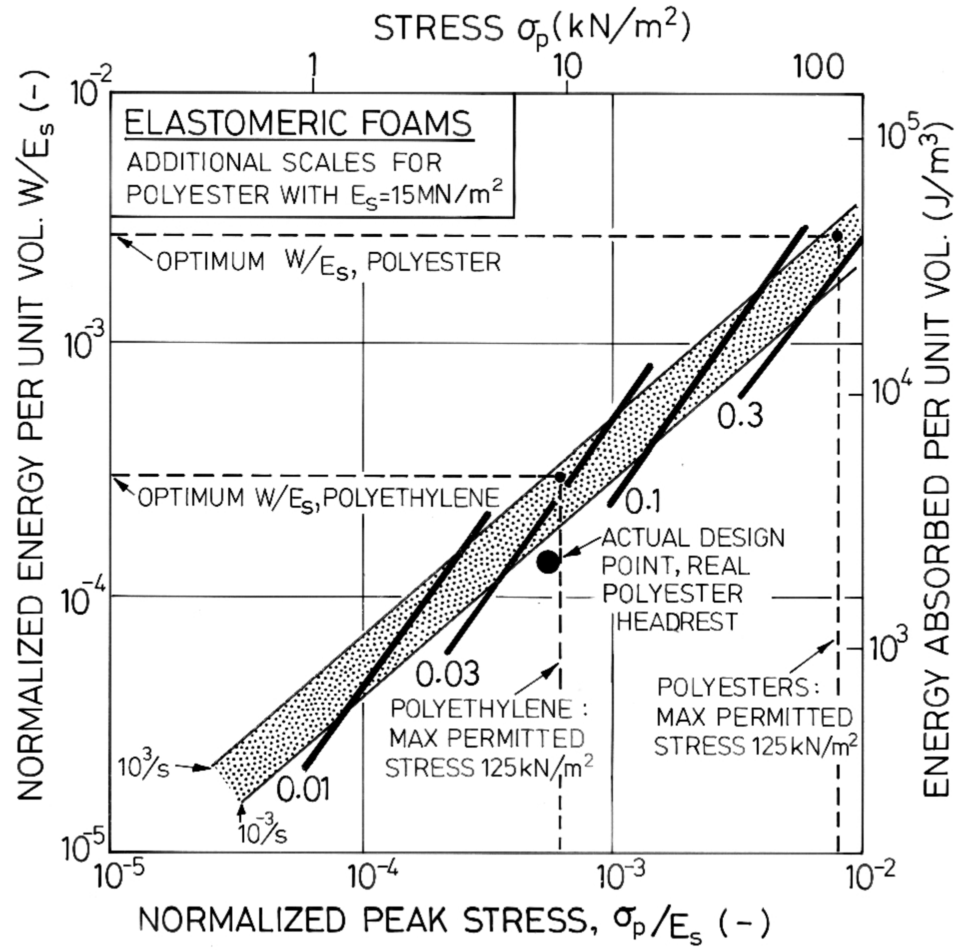
Case study: design of car head rest

- Head rest should absorb kinetic energy of head while keeping force less than that which would cause injury
- Example in book:
  mass of head = 25 kg
  max. deceleration = a = 50 g = 500 m/s^2
  area of contact, A = 0.01 m^2
  thickness of padding t = 0.17 m
  max. allowable force F = ma = 1250 N
  max. allowable stress σ_p = F/A = 125 kN/m^2
  energy to be absorbed/vol, W = $\frac{1/2\,mv^2}{At}$ = 735 v^2 J/m^3
  peak strain rate $\dot{\epsilon}$=v/t [s^{-1}]
  current material — flexible polyester foam $\rho^*/\rho_s = 0.06$
  from plot: for σ_p = 125 kN/m^2 W = 5×10^3 J/m^3
  maximum collision velocity = v = $\sqrt{\frac{W}{735}} = \sqrt{\frac{5\times10^3}{735}}$ = 2.6 m/s = 5.8 mph

Car Head Rest Design







Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Alternative design #1

- Consider energy absorption diagram for elastomeric foams
- Add sales for polyester (using E_s=15 MPa)
- For $\sigma_p = 125$ kN/m^2 could use polyester foam ρ^*/ρ_s=0.2

  then $W/E_s = 2.6 \times 10^{-3}$ and v = 7.3 m/s =16 mph

Alternative design #2

- Use different material e.g. low density open cell polyethylene $E_s = 200$ MPa
- $\sigma_p/E_s = \frac{0.125}{200} = 6.3 \times 10^{-4}$
- At $\dot{\epsilon} = v/t \approx 100$/s (estimated)

  $W/E_s = 3.2 \times 10^{-4}$ (from fig)

  $W = (3.2 \times 10^{-4})(200 MPa) = 6.4 \times 10^4$ J/m^4

  $v = \sqrt{\frac{W}{735}} = \sqrt{\frac{6.4\times10^4}{735}} = 9.3$ m/s $= 21$ mph

- Reading from figure 1: $\rho^*/\rho_s = 0.03$

Case study: foams for bicycle helmets

US: 600-700 bicycle deaths/year
$>$ 90% not wearing a helmet
$\sim$ 50,000 cyclists injured (2009)

(US Nat. Hwy Traffic Safety Admin Bicycle Helmet Safety Inst.)

- Helmets consist of solid outer shell and foam liner (e.g. expanded PS)
- Liner thickness typically 20 mm
- Wish to absorb as much energy as possible while keeping peak acceleration less than that to cause head injury
- Foam liner
  - Redistributes load over larger area, reducing stress on head
  - Peak stress on head limited by plateau stress of foam (as long as don't reach densification)
  - Max. tolerable acceleration = 300 g (if for a few milliseconds)
  - Mass of head $\approx$ 3 kg

    $$\mathrm{F_{max}} = \mathrm{ma} = (3\text{ kg})(300)(10\text{ m/s}^2) = 9\text{ kN}$$

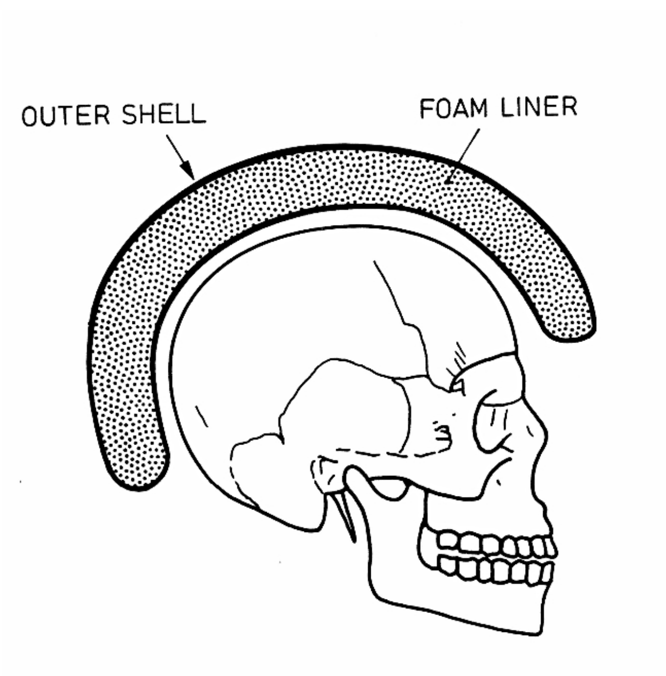
  - As foam crushes, it distributes load over area $\sim A \sim 0.01\mathrm{m}^2$ (may be high)

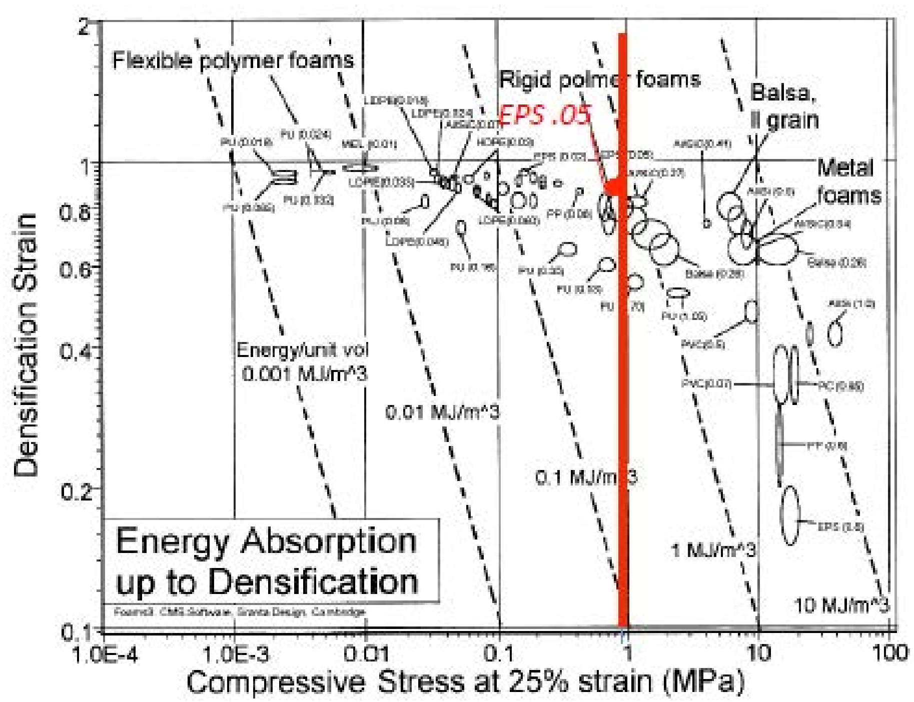
    $$\sigma_p = \frac{9\mathrm{kN}}{0.01\mathrm{m}^2} = 0.9\text{ MPa}$$

Figure $\Rightarrow$ EPS $\rho^* = 0.05$ Mg/m^3
absorbs W = 0.8 MJ / m^3

- Diagram allows easy identification of possible candidate materials
- More complete analysis can then be done
- Energy absorbed U=0.8 $\times$ 10^6 J/m^3 $\times$ 0.01 m^2 $\times$ 0.02 m = 160 J (u=WAt)
- $1/2\, mv^2 = U$; $v_{\max} = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(160)\text{kg}}{3\text{ kg}}\frac{\text{m}^2}{\text{s}^2}}$=10 m/s $\approx$ 22 mph

Case Study:
Foams for Bicycle Helmets





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2015