LECTURE 19: ELASTICITY OF SINGLE POLYMER CHAINS : THEORETICAL FORMULATIONS

Outline :

Objectives: To understand the theoretical formulations of single macromolecule elasticity

Readings: Course Reader Document 31, CR Documents 32-39 are the original theoretical papers for reference, English translations of CR 33 and 36 are handouts.

Multimedia : Podcast : Elasticity of fibronectin; Abu-Lail, et al. *Matrix Biology* **2006** 25 175

REVIEW : LECTURE 18 NANOMECHANICS AND BIOCOMPATIBILITY : PROTEIN-BIOMATERIAL INTERACTIONS 2

-**Two examples of biomaterials** : vascular graft and endotracheal tube (materials, design issues, relation to nanomechanics)

-**Kinetics of protein adsorption**; contributions to diffusion; ideal and activated (Szleifer model-CR 29,30); initial and secondary stage of protein adsorption

-**Modes of protein adsorption** : (I.) adsorption of proteins to the top boundary of the polymer brush (II.) local compression of the polymer brush by a strongly adsorbed protein (III.) protein interpenetration into the brush followed by the non-covalent complexation of the protein and polymer chain (IV.) adsorption of proteins to the underlying biomaterial surface via interpenetration with little disturbance of the polymer brush

-Use of **steric repulsion** (conformational entropy) to inhibit protein adsorption (Halperin model for **polymer brushes- Polymer brush** is a layer of polymers attached with one end to a surface whereby the distance between neighboring chains, $s<R_g$ where R_g is the radius of gyration of an isolated chain; this condition causes extension of the chains away from the surface)

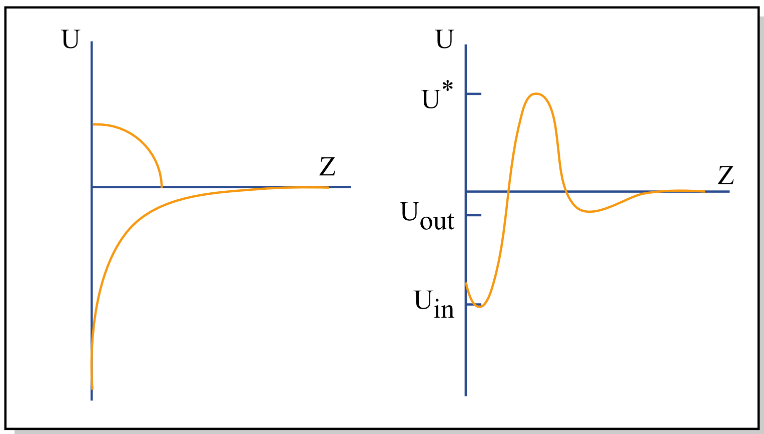


Figure by MIT OCW.
After Halperin, *Langmuir* 1999.

(Halperin, *Langmuir* **1999**) For a protein interacting with a planar surface : $U_{eff}(z) = U_{bare}(z) + U_{brush}(z)$

U^*=activation barrier determining rate of primary adsorption

k_{ads}= adsorption rate constant

Kramers rate theory: $k_{ads} \approx \exp\left(\frac{-U^*}{kT}\right)\frac{D}{\alpha L_0}$

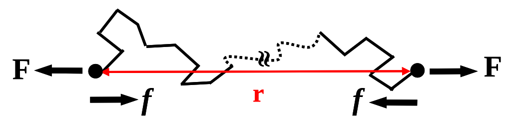
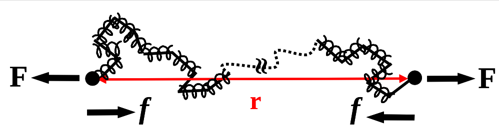
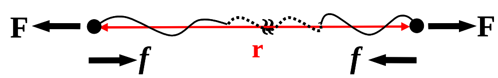
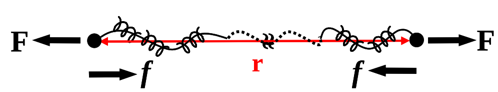
D= diffusion constant

α= width of barrier at U^*-kT

L_o= uncompressed height of polymer brush

-**Polyethylene oxide (PEO, PEG) - hydrophilic** and **water-soluble** at RT, forms an extensive **H-bonding** network; intramolecular H- bond bridges between -O- groups and HOH→ large excluded volume, locally (7/2) **helical supramolecular structure** (tgt axial repeat = 0.278 nm), high **flexibility**, molecular **mobility**, **low van der Waals attraction**, **neutral.** However: poor mechanical stability, protein adhesion reported under certain conditions (long implant times), maintains some hydrophobic character.

SUMMARY : THEORETICAL MODELS FOR SINGLE POLYMER CHAIN ELASTICITY

MODEL	SCHEMATIC	FORMULAS
Freely-Jointed Chain (FJC) (Kuhn and Grün, 1942 James and Guth, 1943)	 *(a, n)*	**Gaussian :** $f(r)=\left(\frac{3k_BT}{na^2}\right)r=\left(\frac{3k_BT}{aL_{contour}}\right)r$ (1) **Non - Gaussian :** Exact Formula : $r(f)=na\left(\coth(x)-\frac{1}{x}\right)$ where : $x=\left(\frac{fa}{k_BT}\right)$ (2) Langevin Expansion : $f(r)=\left(\frac{k_BT}{a}\right)\beta$ (3) $\beta=\mathcal{L}^{-1}\left(\frac{\mathbf{r}}{na}\right)$ = Inverse Langevin Function $=3\left(\frac{\mathbf{r}}{na}\right)+\left(\frac{9}{5}\left(\frac{\mathbf{r}}{na}\right)^3\right)+\left(\frac{297}{175}\left(\frac{\mathbf{r}}{na}\right)^5\right)+\left(\frac{1539}{875}\left(\frac{\mathbf{r}}{na}\right)^7+...\right)$ High Stretch Approximation : $f(r)=\left(\frac{k_BT}{a}\right)\left(1-\frac{r}{L_{contour}}\right)^{-1}$ (4)
Extensible Freely-Jointed Chain (Smith, et. al, 1996)	 *(a, n, $k_{segment}$)*	Non - Gaussian : $f(r)=\left(\frac{k_BT}{a}\right)\mathcal{L}^{-1}\left(\frac{\mathbf{r}}{L_{total}}\right)$; $L_{total}=L_{contour}+n\left(\frac{f}{k_{segment}}\right)$
Worm-Like Chain (WLC) (Kratky and Porod, 1943 Fixman and Kovac, 1973 Bustamante, et. al 1994)	 *(p, n)*	Exact : Numerical Solution Interpolation Formula : $f(r)=\left(\frac{k_BT}{p}\right)\left(\frac{r}{L_{contour}}+\frac{1}{4\left(1-\frac{r}{L_{contour}}\right)^2}-\frac{1}{4}\right)$
Extensible Worm-Like Chain (Odijk, 1995)	 *(p, n, $k_{segment}$)*	Interpolation Formula : $f(r)=\left(\frac{k_BT}{p}\right)\left(\frac{r}{L_{total}}+\frac{1}{4\left(1-\frac{r}{L_{total}}\right)^2}-\frac{1}{4}\right)$; $L_{total}=L_{contour}+n\left(\frac{f}{k_{segment}}\right)$

VARIOUS MATHEMATICAL FORMS FOR THE (INEXTENSIBLE) FREELY JOINTED CHAIN (FJC) MODEL

Gaussian: $f(r) = \left(\frac{3k_BT}{na^2}\right)r = \left(\frac{3k_BT}{aL_{contour}}\right)r \quad (1)$

Non-Gaussian:

Analytical Formula: $r(f) = na\left(\coth(x) - \frac{1}{x}\right)$ where: $x = \left(\frac{fa}{k_BT}\right) \quad (2)$

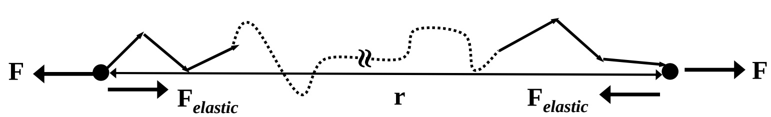
Langevin Expansion: $f(r) = \left(\frac{k_BT}{a}\right)\beta \quad (3)$

$$\beta = \mathcal{L}^{-1}\left(\frac{\mathbf{r}}{na}\right) = \text{Inverse Langevin Function}$$

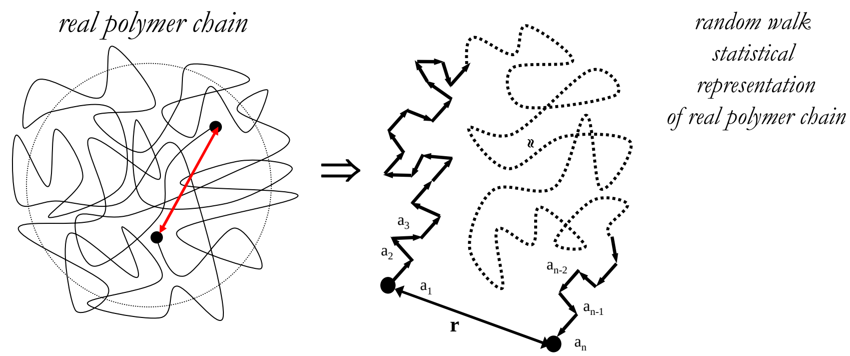
$$= 3\left(\frac{\mathbf{r}}{na}\right) + \left(\frac{9}{5}\left(\frac{\mathbf{r}}{na}\right)^3\right) + \left(\frac{297}{175}\left(\frac{\mathbf{r}}{na}\right)^5\right) + \left(\frac{1539}{875}\left(\frac{\mathbf{r}}{na}\right)^7 + \ldots\right)$$

High Stretch Approximation:

$$f(r) = \left(\frac{k_BT}{a}\right)\left(1 - \frac{r}{L_{contour}}\right)^{-1} \quad (4)$$

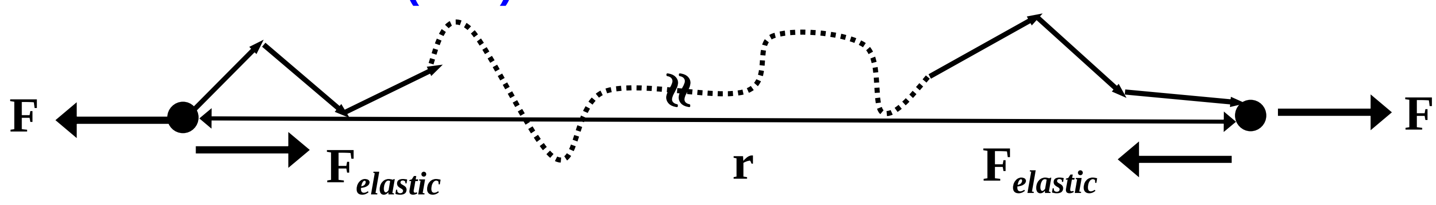


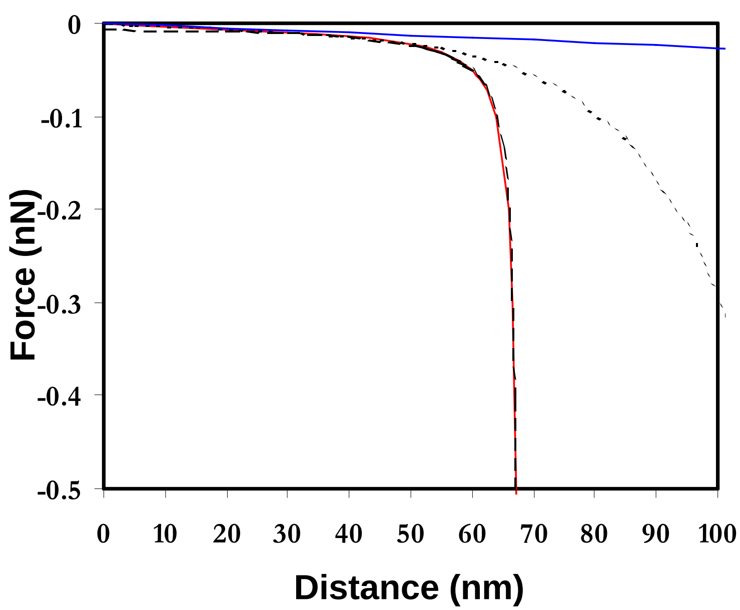
-entropic elasticity; chain wants to maximum # of conformations (random coil), when stretched;
links rotate to uncoil, align, extend, polymer chain along stretching axis
available conformations ↓
disorder and entropy ↓
restoring force driving back to random coil↑
assume no enthalpy change (no stretching of backbone bonds)

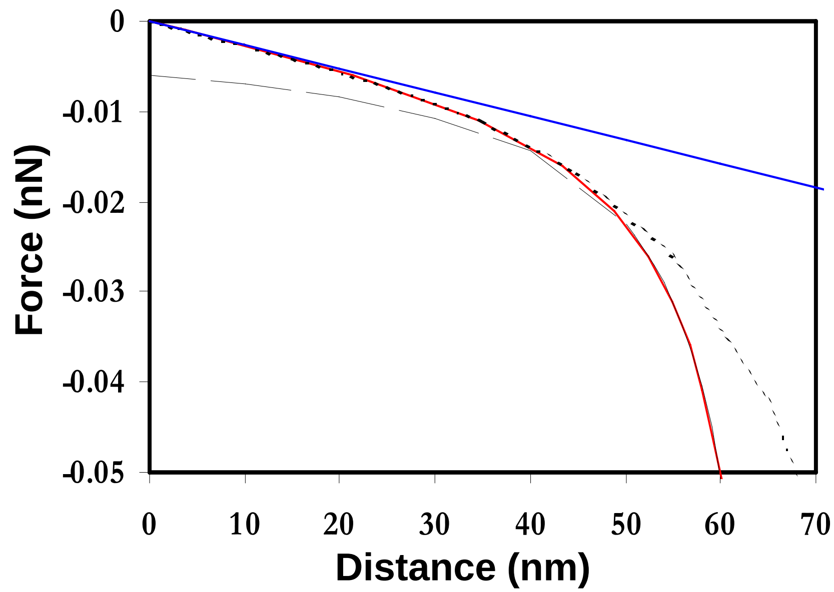


Two molecular level parameters (can be used as fitting parameters) :
a= statistical segment length (local chain stiffness)
n= number of statistical segments
$\mathbf{L_{contour}}$ = na= fully extended length of polymer chain

COMPARISON OF VARIOUS MATHEMATICAL FORMS FOR THE INEXTENSIBLE FREELY JOINTED CHAIN (FJC) MODEL







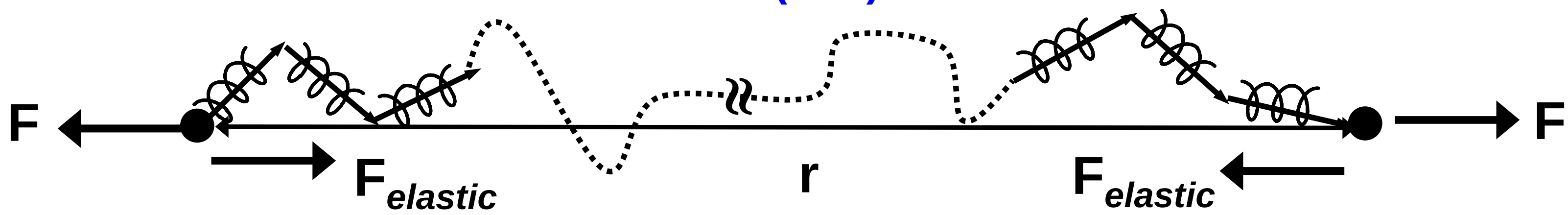
Surface separation distance, D= r, chain end-to-end distance; sign convention (-) for attractive back force, however some scientists plot as (+); e.g. Zauscher (podcast)

(1) Gaussian physically unrealistic; force continues to increase forever beyond $L_{contour,}$ valid for r<1/3 $L_{contour}$

(3) Langevin Series Expansion; finite force beyond $L_{contour}$ (physically unrealistic); valid for r<3/4 $L_{contour}$

(4) High stretch approximation underestimates force for r<3/4$L_{contour}$, valid for r>3/4$L_{contour}$

EXTENSIBLE FREELY JOINTED CHAIN (FJC) MODEL



- Take into account a small amount of longitudinal (along chain axis) enthalpic deformability (monomer/bond stretching) of each statistical segment, approximate each statistical segment as a linear elastic entropic spring (valid for small deformations) with stiffness, $k_{segment}$ →springs is series, forces are equal, strain additive;

$k_{segment}$; $f_{segment} = k_{segment}\ \delta_{segment}$

solve for: $\delta_{segment} = f_{segment}/k_{segment}$

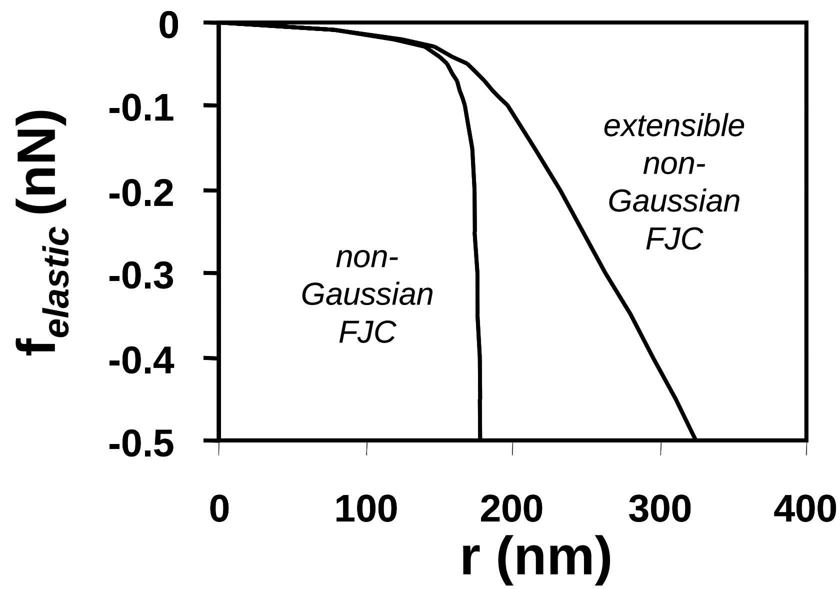
Add displacement term to $L_{contour}$:

$$L_{total} = \underbrace{L_{contour}}_{=na} + \underbrace{n\left(\frac{f}{k_{segment}}\right)}_{\text{extension beyond } L_{contour} \text{ due to enthalpic stretching of chain segments}}$$

n= number of statistical segments

$$f(r) = \left(\frac{k_B T}{a}\right)\mathscr{L}^{-1}\left(\frac{\mathbf{r}}{L_{total}}\right)$$

Now we have three physical (fitting) parameters; a, n, $k_{segment}$



Schematic of the stretching of an extensible freely jointed chain and (b) the elastic force versus displacement for the extensible compared to non-extensible non-Gaussian FJC (a = 0.6 nm, n = 100, $k_{segment}$ = 1 N/m)

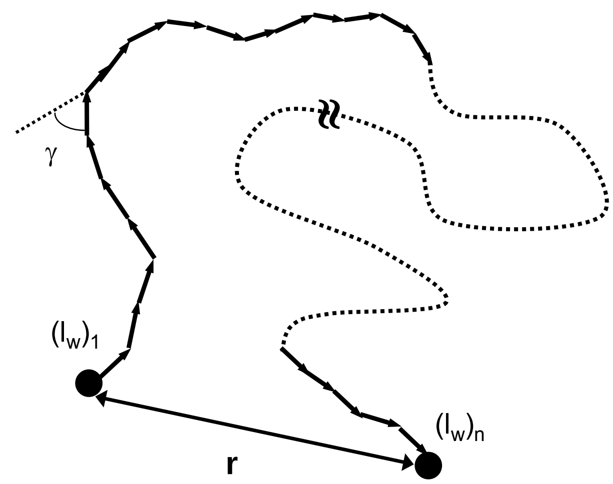
WORM LIKE CHAIN (WLC) MODEL

*(*Kratky-Porod Model)*

"Directed random walk"- segments are correlated, polymer chains intermediate between a rigid rod and a flexible coil (e.g. DNA)

- takes into account both local stiffness and long range flexibility

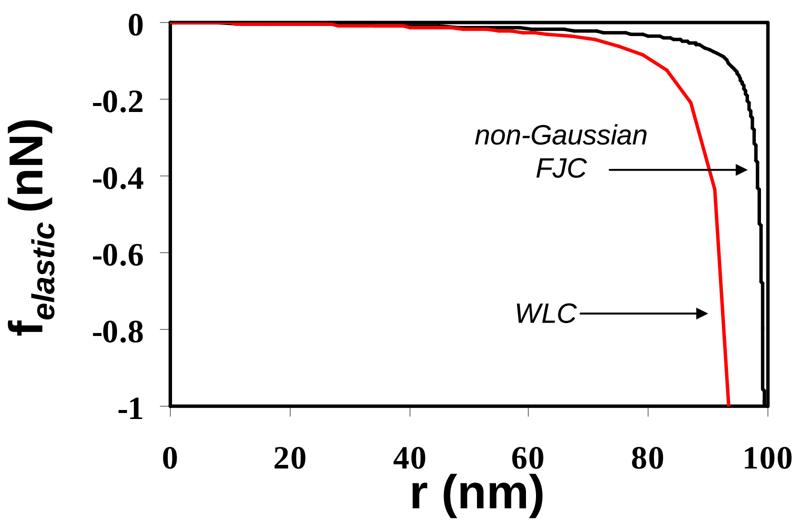
-chain is treated as an isotropic, homogeneous elastic rod whose trajectory varies continuously and smoothly through space as opposed to the jagged contours of the FJC



p= persistence length, length over which statistical segments remain directionally correlated in space

Exact : Numerical Solution

Interpolation Formula : $f(r) = \left(\frac{k_B T}{p}\right)\left(\frac{r}{L_{contour}} + \frac{1}{4\left(1-\frac{r}{L_{contour}}\right)^2} - \frac{1}{4}\right)$



-WLC stiffer at higher extensions, force rises sooner than FJC since statistical segments are constrained, can also make an extensible form of WLC→ replace $L_{contour}$ by L_{total} as before for FJC

APPENDIX : THEORETICAL MODELS FOR THE ELASTICITY OF SINGLE POLYMER CHAINS : FULL CITATIONS OF ORIGINAL REFERENCES

FJC

1. Kuhn, W., "Röntgenoskopie and elektronoskopie von dispersen systemen, fäden, filmen und grenzschichten." *Kolloid-Zeitschrift* 68 (**1934**) 2-11.
2. Guth, E.; Mark, H. "Zur innermolekularen statistik, insbesondere bei kettenmolekülen I." *Monatshefte für Chemie* 65 (**1934**) 93-121.

Extensible FJC

3. Smith, S. B.; Finzi, L.; Bustamante "Direct Mechanical Measurements of the Elasticity of Single DNA Molecules Using Magnetic Beads," *Science* 258 5085 **1992** 1122-1126.

WLC

4. Kratky, O. and Porod, G., “X-ray investigation of dissolved chain molecules.” Recueil des Travaux Chimiques des Pays-Bas et de la Belgique (*Recl. Trav. Chim. Pas-Bas*) 68 (**1949**) 1106-1122.
5. Fixman, M. and Kovac, J., “Polymer conformational statistics. III. Modified Gaussian models of stiff chains.” *J. Chem. Phys.* 58 (**1973**) 1574-1568.
6. Bouchiat, C.; Wang, M.D.; Allemand., J.F.; Strick, T.: Block, S.M.; and Croquette, V., "Estimating the persistence length of a worm-like chain molecule from force-extension measurements." *Biophysical Journal* 76 (**1999**) 409–413.

Extensible WLC

6. Odijk, T., “Stiff chains and filaments under tension.” *Macromolecules* 28 (**1995**) 7016-7018.
7. Wang, M.D.; Yin, R.; Landick, J.; Gelles, J.; Block, S. **1997** "Stretching DNA with Optical tweezers." *Biophysical Journal* 72 : 1335-1346.