

**Department of Materials Science and Engineering  
Massachusetts Institute of Technology  
3.044 Materials Processing – Spring 2012**

**Exam I**

**Wednesday, March 14, 2012**

**The Rules:**

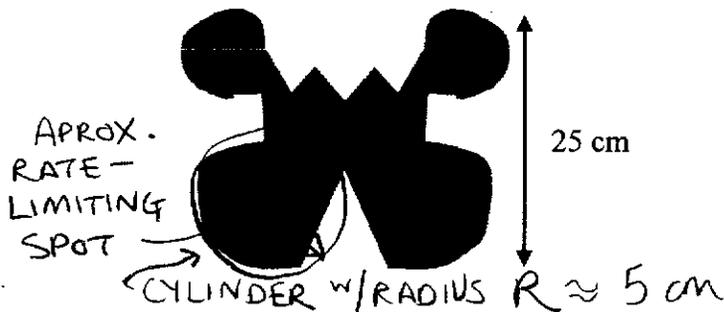
- 1) No books allowed; no computers allowed; etc.
- 2) A simple calculator is allowed
- 3) One hand written 3x5 index card may be used as a crutch
- 4) Complete 5 out of the 6 problems. If you do more than 5 problems, I will grade the first 5 that are not crossed out.
- 5) Make sure that you READ THE QUESTIONS CAREFULLY
- 6) Reasonable assumptions are permitted, but please state them clearly.
- 7) Supplementary materials are attached to the end of the test (eqns., etc.)
- 8) WRITE YOUR NAME HERE:

SOLUTIONS

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### Problem #1: In Which A Curious Shape is Heated

A complex shape of steel is shown below; it is long into the page (this is a cross-section).



In the heat treatment of this shape, we need the whole piece to achieve a temperature between 600° and 625° C for 2 hours.

#### Part A:

Our engineers claim that we have a unique process that can instantaneously fix the surface temperature at a desired value. What value should we choose? Write 1 sentence explaining your choice.

625° C — WE ARE TOLD THAT THE WHOLE PIECE MUST BE BETWEEN 600° & 625° C, SO CHOOSING 625° WOULD RESULT IN THE FASTEST

#### Part B: HEATING POSSIBLE WITHOUT EXCEEDING THE DESIRED BOUNDS.

Obviously, this shape is complicated, but we still need to know how long it takes to heat the whole thing up once we apply our process from Part A. In 4 sentences or less, explain how you would make a good estimate of the heating time for this specific part.

→ THE RATE-LIMITING SPOT IS THE POINT FARTHEST FROM ANY SURFACE.

→ ONE WAY TO MODEL HEATING: APPROXIMATE THE THICKEST LOBE OF THE PIECE AS AN INFINITE CYLINDER. THEN, USE P&CT: FIND TABLES FOR ∞ CYLINDER, USE RADIUS = R ≈ 5 cm, AND FIND THE TIME TO HEAT THE CENTER (r/R = 0) TO  $\theta = \frac{625-600}{625-25} = 0.042$

#### Part C:

Well, it turns out that our engineers were wrong—although our process heats the surface fast, it is not instantaneous. It actually has a convective surface coefficient that is high—about 10<sup>5</sup> W/m<sup>2</sup>s. Explain briefly how this changes your answers from Parts A and B.

$$\rightarrow Bi = \frac{hL}{k} = \frac{(10^5 \text{ W/m}^2\text{K})(0.25 \text{ m})}{\sim 37 \text{ W/mK}} \geq 676 \gg 10$$

(UPPER BOUND ON L)

→ SINCE Bi >> 10, CONVECTION IS EXTREMELY FAST, SO OUR ANSWERS TO A & B ARE UNCHANGED.

## Problem #2: Newtonian or Non-Newtonian

We always enjoy a problem when conduction is fast relative to the competing processes, don't we? For each of the following situations, figure out whether or not the cooling will be Newtonian.

- A) A slab of copper, 20 cm thick, is cooled differently on its two sides: one side has fans blasting air at it, and the other is in water.

AIR  $h \approx 100 \text{ W/m}^2\text{K}$

↑ 20 cm Cu

WATER  $h \approx 1400 \text{ W/m}^2\text{K}$

Top:  $Bi_{\text{AIR}} = \frac{hL}{k} \leq \frac{(100 \text{ W/m}^2\text{K})(0.2 \text{ m})}{(\sim 400 \text{ W/mK})} = 0.05$  ← UPPER BOUND

Bottom:  $Bi_{\text{H}_2\text{O}} = \frac{hL}{k} \leq \frac{(1400 \text{ W/m}^2\text{K})(0.2 \text{ m})}{(\sim 400 \text{ W/mK})} = 0.7$  ← UPPER BOUND

$Bi_{\text{H}_2\text{O}} > 0.1$ , SO THERE WILL BE NON-NEGLIGIBLE THERMAL GRADIENTS DUE TO THE WATER. ⇒ **NON NEWTONIAN**

- B) A 2 mm thick plate of alumina ceramic is cooling in still air at a temperature of 1500° C.

$$Bi = \frac{hL}{k} = \frac{(10 \text{ W/m}^2\text{K})(0.002 \text{ m})}{(\sim 10 \text{ W/mK})} = 0.002$$

$$M = \frac{\epsilon \sigma L}{k} T_{\text{SURF}}^3 = \frac{(\sim 0.45)(5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4)(0.002 \text{ m})}{(\sim 10 \text{ W/mK})} (1800 \text{ K})^3 = 0.03$$

IF  $T > 1000^\circ\text{C}$   
⇒ SHOULD WORRY ABOUT RADIATION!

BOTH  $M$  &  $Bi < 0.1$  ⇒ NO GRADIENTS ⇒ **NEWTONIAN**

- C) A thin film of aluminum just 10 μm thick is sitting in still argon gas on a silicon substrate; the whole stack is cooled from 600° C.

Ar (COLD)  
↑ 10 μm Al

$h \approx 10 \text{ W/m}^2\text{K}$   
 $k \approx 235 \text{ W/mK}$

$$Bi = \frac{hL}{k} = \frac{(10 \text{ W/m}^2\text{K})(10^{-5} \text{ m})}{235 \text{ W/mK}} = 4 \times 10^{-7}$$

Si (HOT)

$L = 10 \mu\text{m}$   
(NOT 5 μm, BECAUSE HEAT IS ONLY LOST ON ONE SIDE)

$$Bi < 0.1$$

⇒ NO GRADIENTS

⇒ **NEWTONIAN**

### Problem #3: Serial or Parallel Casting

We are going to do some sand casting, of metal objects that are all the same size. We will do it two different ways, named process I and II. Here are schematics of the molds:



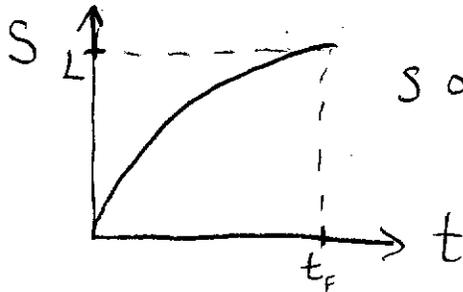
I



II

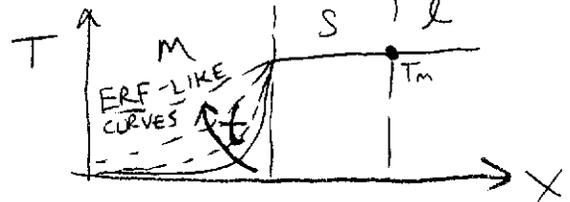
#### Part A:

Draw a schematic of how the solidification process occurs in process I, by charting the location of the solidification front vs. time. Make sure you show the point where the solidification is complete. Explain the shape of the curve in one sentence.



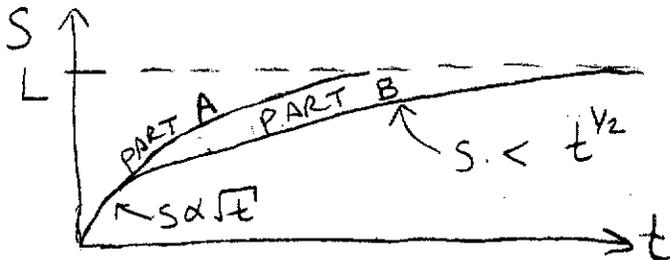
$$S \propto \sqrt{t}$$

BECAUSE WHEN CONDUCTION IS LIMITING, HEAT ACCUMULATES IN THE MOLD WITH TIME.



#### Part B:

Draw a similar chart for the center of the three objects in process II. Explain how it is different from your answer in part A, or if it is the same; why it is the same.

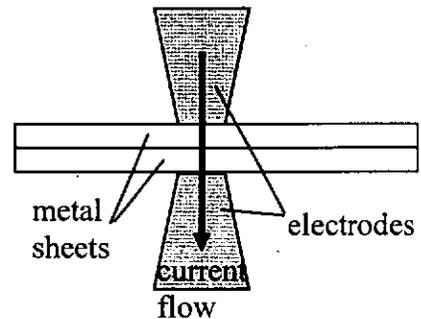


AT EARLY TIMES, THE SOLIDIFICATION FRONT GOES AS  $t^{1/2}$ , JUST AS BEFORE. BUT ONCE HEAT ACCUMULATES IN THE MOLD

FROM BOTH SIDES, ITS ABILITY TO REMOVE HEAT IS FURTHER DIMINISHED, SO THE SOLIDIFICATION FRONT SLOWS DOWN AS COMPARED WITH PART A.

**Problem #4: See Spot Weld. Weld, Spot, Weld.**

Spot welding involves a pair of electrodes, which are used to apply a high current through two laid-up sheets, as shown here. Take the sheets to be 0.5 mm thick each.



Heat is generated evenly within the two sheets as a consequence of the current passing through them (resistive heating). Melting eventually occurs, and solidification on cooling yields a joint between the sheets.

In this problem, you will consider the heating portion of this process.

**Part A:**

Write down the full form of the heat conduction equation that we need to understand heat flow in the sheets. Be sure to use the proper coordinate system—the system is radially symmetric.

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$$

FOR A RADIALLY-SYMMETRIC CYLINDRICAL COORDINATE SYSTEM:

**Part B:**

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{q}}{\rho c_p}$$

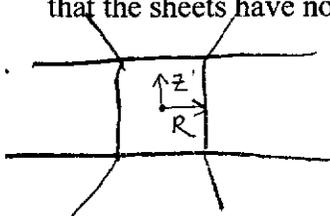
If the sheets are steel, and we are welding in air, indicate whether or not we need to consider convection at the free surfaces.

$$Bi = \frac{hL}{k} = \frac{(10 \text{ W/m}^2\text{K})(0.005 \text{ m})}{(37 \text{ W/mK})} = 0.0014 \ll 0.1$$

NO GRADIENTS IN  $z \Rightarrow$  NEGLIGIBLE HEAT LOSS VIA CONVECTION  $\Rightarrow$  DO NOT CONSIDER CONVECTION

**Part C:**

Write down all the boundary conditions and initial conditions that we need to solve this problem. Please make sure that these are consistent with your answer to part B. Also—you may assume that the sheets have no interface resistance between them.



I.C.: AT  $t=0$ ,  $T=T_0$  EVERYWHERE.

B.C. # 1: FOR  $t > 0$ ,  $r \leq R$ ,  $\dot{q} = \text{CONSTANT}$   
 $r > R$ ,  $\dot{q} = 0$

B.C. # 2: AT  $r = \infty$ ,  $T = T_0$

B.C. # 3: AT  $r = 0$ ,  $\frac{\partial T}{\partial r} = 0$

B.C. # 4: FOR ALL  $z$ ,  $\frac{\partial T}{\partial z} = 0$ .

**Problem #5: A Heat Balance Problem—How Can You Even Think About Skipping This?**

Here is a small (differential) element from the cylindrical system, through which heat is conducting. In general, there need not be any symmetry around  $\theta$ , so there can be a heat imbalance on  $\theta$  as well as on  $r$ . For this problem let's stick to 2D, so you can neglect the  $z$  axis.

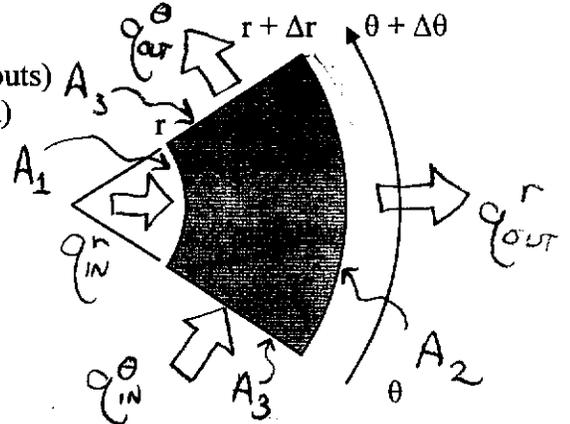
**Part A:** Draw all the relevant conductive heat fluxes (ins and outs) on the diagram. Label the area (since we are only in 2D, length) over which these fluxes act. (Hint: arclength is given by  $r \cdot \theta$ )

$$A_1 = r \Delta \theta$$

$$A_2 = (r + \Delta r) \Delta \theta$$

$$A_3 = \Delta r$$

ELEMENT VOLUME:  
 $V = r \Delta r \Delta \theta$



**Part B:** Write out the heat balance equation for this system, including one term for each of the balance elements in the picture.

$$Q_{IN} - Q_{OUT} = V \cdot q_{ACCUM} = A_1 q_{IN}^r + A_3 q_{IN}^\theta - A_2 q_{OUT}^r - A_3 q_{OUT}^\theta$$

$$r \Delta r \Delta \theta q_{ACCUM} = r \Delta \theta q_{IN}^r + \Delta r q_{IN}^\theta - (r + \Delta r) \Delta \theta q_{OUT}^r - \Delta r q_{OUT}^\theta$$

**Part C:** Simplify your expression from part B, and derive the heat conduction equation in radial coordinates. (Hint: you can check your answer)

$q_{ACCUM} = \rho c_p \frac{\partial T}{\partial t}$  & FOURIER'S LAW:  $\vec{q} = -k \nabla T$   
 $q^r = -k \frac{\partial T}{\partial r}$        $q^\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$

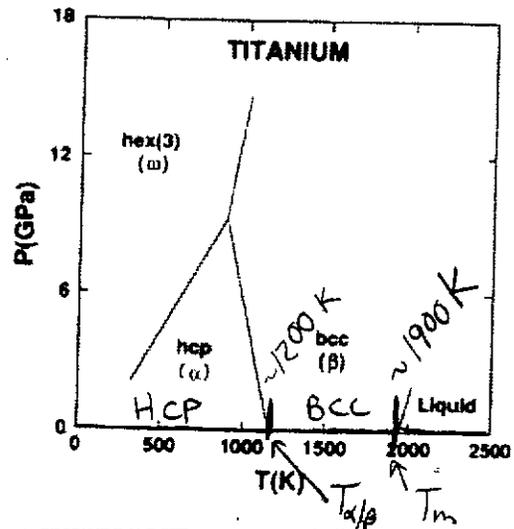
$$\rho c_p \frac{\partial T}{\partial t} = \frac{r}{r \Delta r} \left( -k \frac{\partial T}{\partial r} \Big|_r \right) + \frac{1}{r \Delta \theta} \left( -k \frac{\partial T}{\partial \theta} \Big|_\theta \right) - \frac{r + \Delta r}{r \Delta r} \left( -k \frac{\partial T}{\partial r} \Big|_{r + \Delta r} \right) - \frac{1}{r \Delta \theta} \left( -k \frac{\partial T}{\partial \theta} \Big|_{\theta + \Delta \theta} \right)$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{k}{r} \left( \frac{(r + \Delta r) \frac{\partial T}{\partial r} \Big|_{r + \Delta r} - r \frac{\partial T}{\partial r} \Big|_r}{\Delta r} \right) + k \frac{1}{r^2} \left( \frac{\frac{\partial T}{\partial \theta} \Big|_{\theta + \Delta \theta} - \frac{\partial T}{\partial \theta} \Big|_\theta}{\Delta \theta} \right)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right]$$

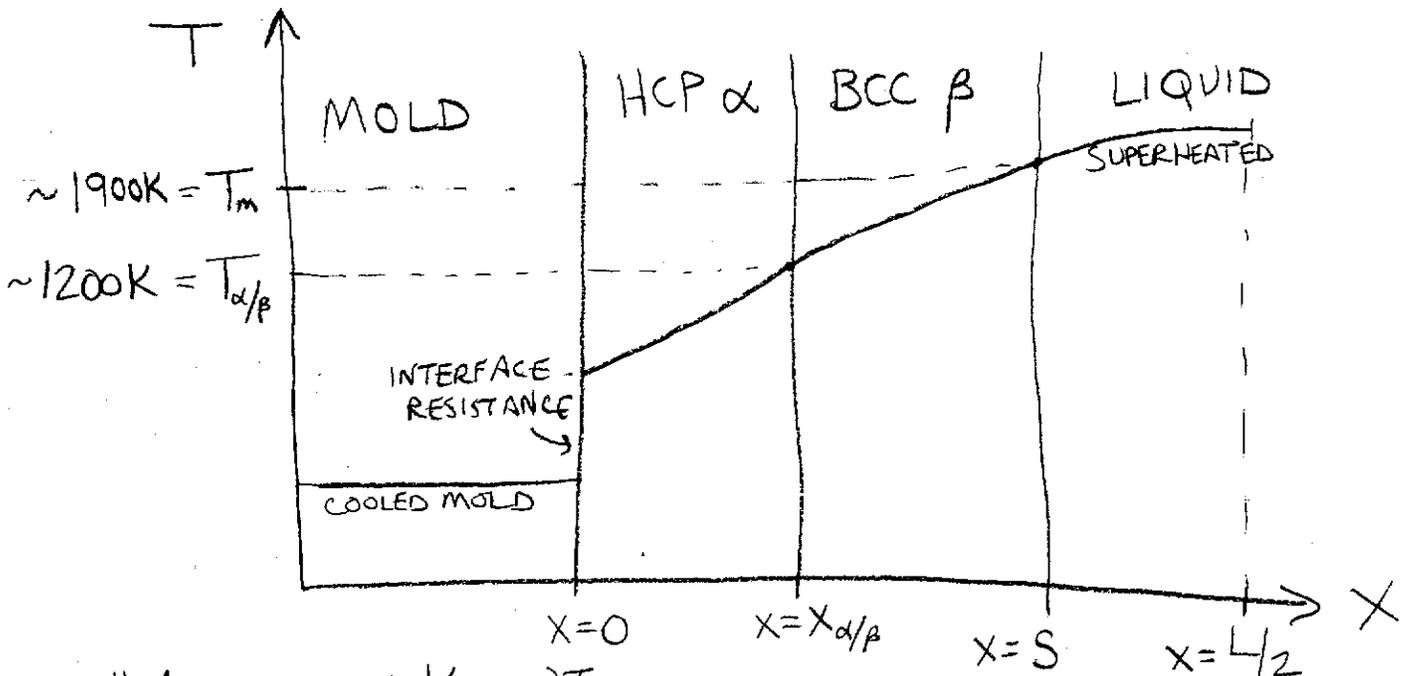
### Problem #6: Casting Plus Bells and Whistles

On the right is the phase diagram of pure titanium, which you intend to melt, superheat, and cast into a mold. You will conduct this operation at 1 atm of pressure (~0 GPa in the diagram at the right).



Draw a picture that captures the essential features of the temperature profile in a titanium casting. Your drawing should assume a 1-D heat flow problem, but should include all of the following elements: A casting which is inside a mold which is actively cooled.

Draw your schematic of the temperature profile for an arbitrary time after solidification has begun, and WITHOUT any simplifying assumptions. Label every boundary condition that should be applied.



B.C. #1: AT  $x = L/2$ ,  $\frac{\partial T}{\partial x} = 0$

B.C. #2: AT  $x = S$ ,  $T = T_m = \sim 1900 \text{ K}$  &  $\frac{ds}{dt} = \frac{k}{H_f \rho} \left( \frac{\partial T}{\partial x} \Big|_s - \frac{\partial T}{\partial x} \Big|_l \right)$

B.C. #3: AT  $x = x_{\alpha/\beta}$ ,  $T = T_{\alpha/\beta} = \sim 1200 \text{ K}$

B.C. #4: AT  $x = 0$ ,  $q_{\text{INTO MOLD}} = h_{\text{INTERFACE}} (T_{\text{CASTING, } x=0} - T_{\text{MOLD}})$

★ IF  $H_f^{\alpha \rightarrow \beta} \neq 0$ , THEN WE WILL ALSO HAVE A STEFAN CONDITION AT  $x = x_{\alpha/\beta}$ .

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