

Department of Materials Science and Engineering  
Massachusetts Institute of Technology  
3.044 Materials Processing – Spring 2011

Exam I

Monday, March 14, 2011

The Rules:

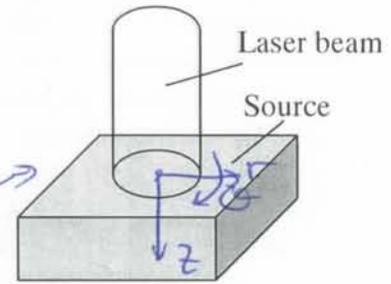
- 1) No books allowed; no computers allowed; etc.
- 2) A simple calculator is allowed
- 3) One hand written 3x5 index card may be used as a crutch
- 4) Complete 5 out of the 6 problems. If you do more than 5 problems, I will grade the first 5 that are not crossed out.
- 5) Make sure that you READ THE QUESTIONS CAREFULLY
- 6) Reasonable assumptions are permitted, but please state them clearly.
- 7) Supplementary materials are attached to the end of the test (eqns., etc.)
- 8) WRITE YOUR NAME HERE:

SOLUTIONS

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**Problem #1: In Which a Basic Problem of Setting up Boundary Conditions is Put Forth**

One way to induce local heating with extreme speed and precision is to use a laser pulse. (welding, evaporative coating, freeform fabrication, etc., can all be conducted by laser heating.) Consider the heat transfer problem for laser heating of a substrate in a vacuum. Take the laser beam to be cylindrical and at normal incidence to the surface, as shown in the figure.



You may assume the source is very thick, and also very large in the plane.

**Part A:**

Define a coordinate system for the heat transfer problem, and write down the proper equation to describe heat conduction in the source material in that coordinate frame.

Use radial (cylindrical coordinates), but w/o  $\theta$  dependence, since system is  $\theta$ -symmetric

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \quad \left\} \quad \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2}$$

1.5

1

0.5

**Part B:**

Write down a reasonable assumption regarding the heat transfer conditions where the laser hits the surface. Express this assumption as a mathematical boundary condition.

for  $r < R$ , assume a constant flux condition  
 $z=0$

$$q|_z = q_{laser}$$

$$\text{So } -k \frac{\partial T}{\partial z} = q_{laser}$$

1.5

0.5

**Part C:**

Write down the remaining boundary conditions for this problem.

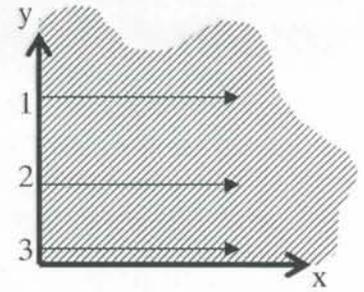
$$\left\{ \begin{array}{l} z = \infty \quad T = T_0 \\ r = \infty \quad T = T_0 \\ t = 0 \quad T = T_0 \\ \text{all } x, z > 0 \end{array} \right.$$

$z=0 \quad r > R$ , convection is negligible due to vacuum  
 $-k \frac{\partial T}{\partial z} = 0$   
 zero flux:

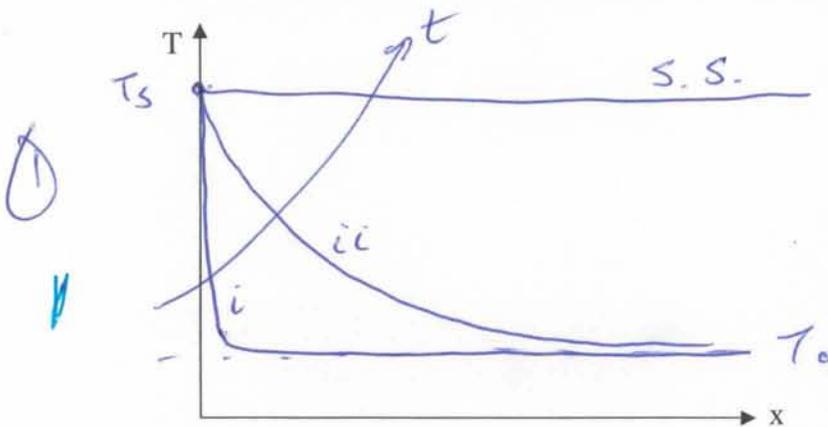
2

**Problem #2: Take a Position on Superposition**

A "semi-infinite corner" is shown here. It is the corner of a large object (there is a large bulk of material to the upper-right not shown). The object also extends into the page for a long distance. This object is at  $T_0$  when it is placed into a hot oil bath that is vigorously stirred, providing a constant surface temperature  $T_s$ . (The bold surfaces are exposed to  $T_s$ ).

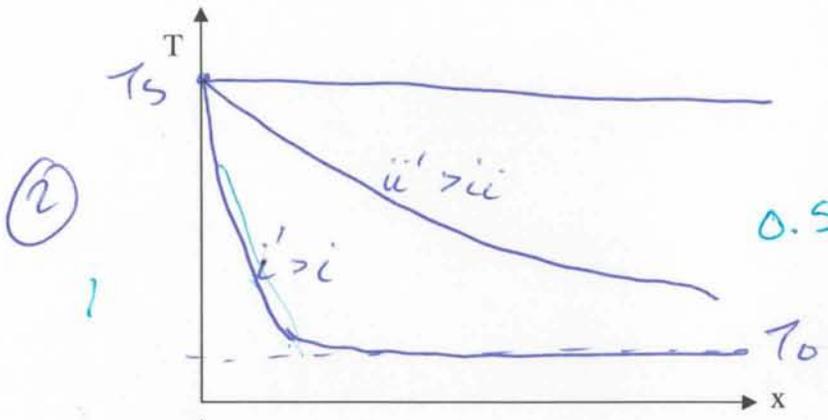


Three lines are shown on the object: 1, 2, and 3. Draw the temperature profile along each of these three lines, using one of the graphs below for each. Please draw profiles for an early time, at least one later time, and also for the steady-state. Also, next to each graph, write one brief sentence that explains your thoughts in making the graph.

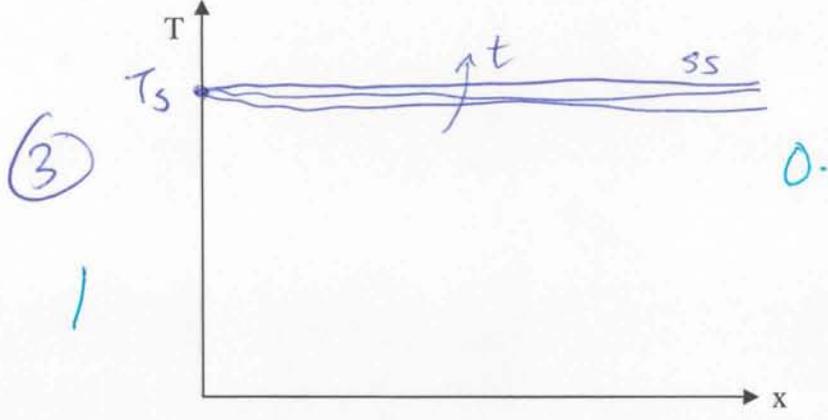


1 is far from the x-edge, so this is just like a semi-infinite situation: erf!

(1/1)



2 is between: it is erf from heat flux in the x-direction, but also receives heat from y-direction

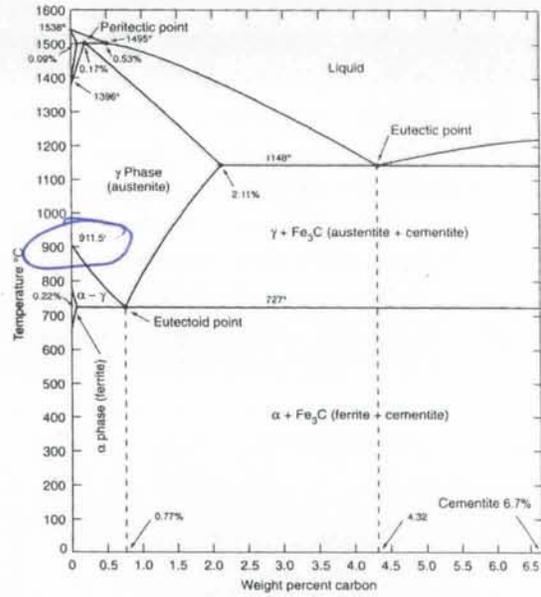


3 is on the edge (practically) where  $T$  is fixed...

### Problem #3: Steel Anneal Revealed

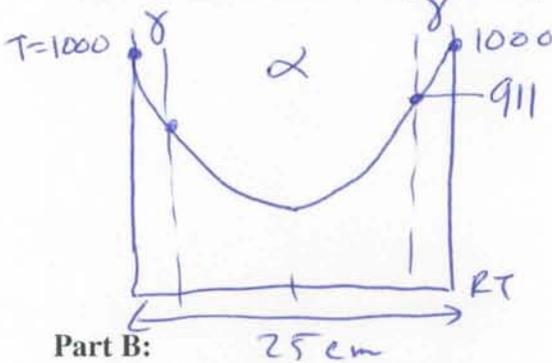
In class, we discussed the heat transfer during annealing of steel. Specifically, we talked about heating a 1-D block of steel, with thickness  $2L = 25$  cm, up to the austenite field at, say,  $1000^\circ\text{C}$ . For this problem, let's take our composition to be 0% carbon (just pure iron) for simplicity.

One thing we neglected in our class discussion was the phase transformation from  $\alpha$  to  $\gamma$  on heating.



#### Part A:

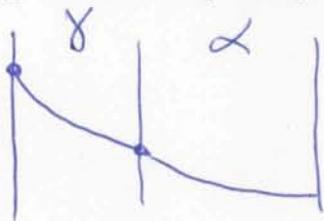
First, draw a schematic of the temperature profile in such a sheet of steel, at an arbitrary time after we have begun to heat it, but before it is uniformly heated. Indicate how the addition of the  $\alpha/\gamma$  phase transformation impacts this profile.



phase trans at  $911^\circ\text{C}$   
 conductivity is the same  
 in both phases ( $\approx 30$ )  
 so the profile looks similar,  
 but there are two phase  
 now!  $\alpha$  inside,  $\gamma$  outside

#### Part B:

How would you modify this heat conduction problem mathematically to account for the effect of having two different phases present ( $\alpha$  and  $\gamma$ ), and for having a phase transformation in the solid?



in  $\gamma$ , solve  $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$   
 in  $\alpha$ , solve  $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$

#### Part C:

at  $\alpha/\gamma$  interface, Stefan!  $\frac{\partial T}{\partial t} = \frac{k}{H_s \rho} \left( \frac{\partial T}{\partial x} \Big|_{\alpha} - \frac{\partial T}{\partial x} \Big|_{\gamma} \right)$

In class we neglected the transformation. Do you think that we overestimated or underestimated the time needed to heat the billet? Write one sentence to explain.

underestimated, b.c. trans. requires extra enthalpy to be delivered

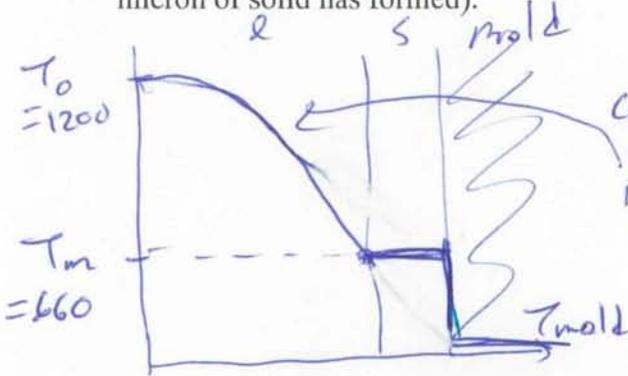
**Problem 4: Wherein Another Permutation of the Casting Problem is Encountered**

The procedure for understanding a casting process is the one we followed in class, and it always starts by identifying the limiting kinetic process. There are many other variations on solidification that we did not cover in class. If you ever encounter one, you should follow the same procedure. You are about to encounter one.

Liquid aluminum at  $1200^\circ\text{C}$  is cast into an actively cooled steel mold of characteristic dimension  $L = 25\text{ cm}$ .

Superheat!

**Part A:** If there is an interface resistance between the mold and casting of  $4000\text{ W/m}^2\text{K}$ , identify the rate-limiting heat transfer step for early times just after pouring (say, when only a micron of solid has formed).



Compare:

mold	solid	liquid
4000	$2 \times 10^8$	400
$\uparrow$	$\uparrow$	$\uparrow$
$h$	$K/L_s$	$K/L_l$
	$\uparrow$	$\uparrow$
	$200(10^{-6})$	100 $\approx 0.25$
	$\uparrow$	$\uparrow$
	fast! no gradient here	slow: largest gradient here!

**Part B:** As the casting solidifies, the limiting transfer step can change. In the present case, identify if there is such a transition, and if so, where/when it happens.

Compare liquid + solid

$$\frac{k_s}{L_s} = \frac{k_l}{L_l}$$

$$200 \rightarrow \frac{k_s}{L_s} = \frac{k_l}{(0.25 - L_s)}$$

$$L_{s,crit} \approx 0.17$$

← as  $l$  shrinks +  $s$  grows, when solid is 17cm thick, the gradient in the solid will be bigger

compare liquid + mold

$$100 \rightarrow \frac{k_l}{L_l} = 4000$$

$$L_{l,crit} = 0.025$$

$$L_{s,crit} = 0.225$$

and when the solid is 22.5 cm, the mold resistance will be large!

**Part C:** Draw a graph in which you estimate the shape of the curve describing the solidification front position as a function of time. Explain your reasoning briefly.



conduction in liquid dominates for some part of the time. As heat is removed from the liquid, it can freeze faster.  $\therefore$  the process is accelerating!

**Problem #5: True or False or Sometimes True or Sometimes False (Bearing in Mind that Sometimes True is the same as Sometimes False, Now, Isn't It?)**

For each of the five statements below, indicate whether the statement is true, false, or sometimes false. Explain in one sentence or less.

- A) Upon heating a material, at a sufficiently high temperature the rate-limiting heat transfer step will become radiation.

Mostly ~~True~~ <sup>False</sup>: radiation  $\sim T^4$ , at higher temperatures will always outscale the  $\sim T$  dependence of conductivity or convection, and not be limiting (only exception is if  $\Sigma = 0$  ... rare/never)?

- B) Thanks to the symmetry of dimensionless numbers, an object, whether it is being heated or cooled by radiation, follows the same kinetics.

False:  $T^4$  scaling poses a problem to the usual inversion for heating/cooling.  
(See the Newtonian charts for an example)

- C) We are going to heat a rod of copper (1 cm diameter) in vacuum by shining a lamp at  $1000^\circ\text{C}$  on it. We can heat one end of the rod, and the whole thing will heat up uniformly.

Usually <sup>False</sup>: relevant dimension is the length of the rod (not given, so can be very large)

- D) The superposition principle does not apply to objects heated or cooled by radiation.

False: it does, provided only one reduced temperature for the problem

- E) If there is a body at a constant very high temperature, and we would like to know whether radiation or convection is dominant, we should calculate the dimensionless constant:

$$\epsilon\sigma T^3 h^{-1}$$

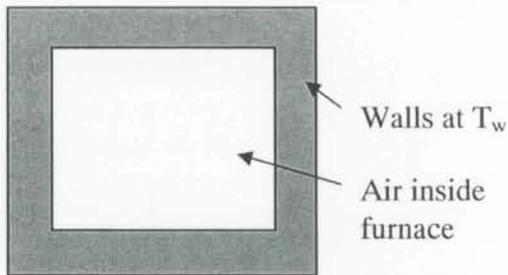
True: ratio of resistances  $1/\epsilon\sigma T^3$  and  $1/h$

### Problem #6: Outside-In Heat Transfer

We have frequently talked about processes in which a solid was heated or cooled by virtue of its contact with a fluid (or vapor) that exchanged heat at the interface by convection. What if the problem went the other way around? How long does it take to heat a vapor with a hot solid? We have never thought through this problem, but I think you should be able to do it on the fly.

Consider a furnace where the heating elements are in the walls; the walls may be considered as instantaneously being raised to a temperature  $T_w$  at  $t = 0$ . Let's write out an equation that tells how the temperature rises in the furnace as a function of time.

How will you do this? Hmm. How have we derived the solution to every heat transfer problem in this class? Something about balancing something or other...



You may need to introduce some quantities/variables (such as the volume of air in the furnace,  $V$ , among other things). Please explain when you do and label the symbols so I know what you are thinking.

heat balance:

$$\text{heat in: } h(T_w - T_{air}) A_w = \text{heat accumulated } \rho C_p V_{air} \frac{dT_{air}}{dt}$$

↑ ↑  
density + heat cap of air

$$\frac{dT_{air}}{dt} = \frac{h A_w}{\rho C_p V_{air}} (T_w - T_{air})$$

$$\frac{dT_{air}}{T_w - T_{air}} = \frac{h A_w}{\rho C_p V_{air}} dt$$

$$\ln(T_w - T_{air}) = \frac{h A_w t}{\rho C_p V_{air}} + A \quad \text{apply B.C., etc.}$$

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