3.032 Mechanical Behavior of Materials

Fall 2007

STRESS AND STRAIN TRANSFORMATIONS:

Finding stress on a material plane that differs from the one on which stress is known...

or "Why it's easier to remember Mohr's circle"

Note: Derived in class on Wednesday 09.19.07.

Force balance for stress over a face inclined an angle θ with respect to the original (x, y) axes give:

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2} cos2\theta + \tau_{xy} sin2\theta \tag{1}$$

$$\sigma_{y'y'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2} cos2\theta - \tau_{xy} sin2\theta \tag{2}$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2} sin2\theta + \tau_{xy} cos2\theta \tag{3}$$

Taking the derivative of Eq. (1) with respect to θ to obtain the orientation of maximum normal stress gives:

$$tan2\theta_{normalstress,max} = \frac{\tau_{xy}}{\frac{\sigma_{xx}-\sigma_{yy}}{2}} \tag{4}$$

and substituting the corresponding sin2θ and cos2θ expressions into Eqs. (1 - 2) to obtain the maximum normal stresses in this 1-2 plane gives:

$$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{(\frac{\sigma_{xx}-\sigma_{yy}}{2})^2 + \tau_{xy}^2} \tag{5}$$

where, by convention, $\sigma_1 \geq \sigma_2$.

Taking the derivative of Eq. (2) and going through the same process to obtain the orientation and magnitude of the maximum shear stresses gives:

$$tan2\theta_{shearstress,max} = \frac{-\frac{(\sigma_{xx}-\sigma_{yy})}{2}}{\tau_{xy}} \tag{6}$$

and

$$\tau_{max,in-plane} = \sqrt{(\frac{\sigma_{xx}-\sigma_{yy}}{2})^2 + \tau_{xy}^2} \tag{7}$$

Note that the equations for coordinate transformations of strain (strain transformation equations) are completely analogous. For example,

$$\epsilon_{x'x'} = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} + \frac{\epsilon_{xx}-\epsilon_{yy}}{2} cos2\theta + \epsilon_{xy} sin2\theta \tag{8}$$

but the only thing to note is that this ϵ_{xy} is equal to half the engineering shear strain, $\gamma_{xy}/2$. In other words, if you are given a state of engineering strain for a material body, you have to multiply the engineering shear strain components by 2 before using these equations to find the full strain state on some other plane inclined an angle θ.

As you will see in the next class, a very smart engineer named Otto Mohr figured out how to represent these equations in the shape of a circle, so that one can quickly and graphically locate the orientations and magnitudes of maximum normal and shear stresses / strains!