

acting over the punch width (or cross-sectional area in 3D) A to exert a vertical displacement h_z onto the material surface.

Solution: $W = P_z h_z$

- (ii) Now assume the material beneath the indenter displaces only at locations of maximum shear stress, as shown. Determine the relative displacement of the material triangle edges (filled arrows) in terms of h_z , and the “area” of those isosceles triangle edges over which this displacement occurs.

Solution: As shown for $\theta = 45^\circ$,

Displacement u in shear diagonal direction = $h_z/2^{1/2}$;

Displacement u in horizontal direction = h_z

Area in shear diagonal direction = $A/2^{1/2}$

Area in horizontal direction = A

- (iii) The triangles will only start to displace when the work done by the punch = work required to attain shear stress in the material that is high enough to yield the material. Equate the work in (i) to the sum of the work done on all of these internal triangle edges (8 in total, shown in the Fig. Q2) that move under the shear stress τ . As the displacement u_z appears on both sides of this equality, you will obtain an equation of the form $P \propto A\tau$.

Solution: Each work term is the product of force \times distance, or shear stress \times area \times distance. There are 2 arrows (and thus 2 work terms) associated with the triangle just beneath the indenter, 2 horizontal arrows (and thus 2 work terms) associated with the two triangles next to that middle one, and 4 arrows from the triangles to the extreme left and right. Summing these work terms and equating with the input work from (i), we find:

$$P_z h_z = (2 \times A \tau^{1/2} h_z/2^{1/2}) + (2 \times A \tau \times h_z) + (4 \times A/2^{1/2} \times \tau \times h_z/2^{1/2}) \rightarrow P_z = 6A\tau$$

- (iv) Since we have implicitly assumed the maximum shear stress or Tresca criterion in this analysis, now express τ in terms of the yield strength according to this criterion, and note that H is defined as P/A , to obtain an equation that relates H to σ_y .

Solution: $H = P_z/A = 6\tau = 6(\sigma_y/2)$ according to Tresca yield criterion $= 3\sigma_y = H$

Here, the key is to note that Tresca claims the critical shear stress required to plastically deform a material to be equal to half the stress required to yield the material under uniaxial stress. Mohr's circle makes this easy to see.

- (v) Explain at least two cases where this approximation would be inaccurate, and whether you think this proportionality factor would go up (so H is more than thrice the yield strength) or down. To think about this, consider the simplifications that were made in order to derive this relation. Note especially that hardness is a value you measure

after considerable yielding over a material volume has occurred, not just at the instant that the yield stress is attained.

Solution: If the material is elastic-perfectly plastic (no strain hardening), then the stress generated at any stress AFTER the yield stress is the same (equal to the yield stress), but this is rarely observed in real materials.

Also, this model assumes the material undergoes yielding well predicted by the Tresca yield criterion. Many materials do not deform under high pressure according to Tresca – they may fail according to maximum normal stress or to von Mises yield criterion, or the materials may fracture, phase transform, or dissipate energy in other ways including viscoelasticity.

3. Materials engineers have become increasingly interested in whether the yield strength of crystalline materials changes as the physical size of the material decreases toward the nanoscale. Until very recently, conducting uniaxial tension/compression tests on tiny material volumes < 100 microns in diameter was technically infeasible, but Uchic et al. have shown that they can make and measure the compressive stress-strain response of single crystal Ni and other materials for pillar diameters as small as 5 microns (see Fig. 2). The full paper is available on MIT Server, and is a very cool read.

Images removed due to copyright restrictions. Please see Fig. 1 in Uchic, Michael D., et al. "Sample Dimensions Influence Strength and Crystal Plasticity." *Science* 305 (August 2004): 986-989.

Fig. Q3: (From Uchic et al., *Science* 305: 986, 2004). Mechanical behavior at room temperature for pure Ni microsamples having a $\{134\}$ orientation. (A) Stress-strain curves for microsamples ranging in size from 40 to 5 μm in diameter, as well as the stress-strain curve for a bulk single crystal having approximate dimensions $2.6 \times 2.6 \times 7.4 \text{ mm}$. (B) A scanning electron micrograph (SEM) image of a 20- μm -diameter microsample tested to $\sim 4\%$ strain. The circle milled into the top surface of the microsample is a fiducial mark used during sample machining. (C) A SEM image of 5- μm -diameter microsample after testing, where the sample achieved $\sim 19\%$ strain during a rapid burst of deformation that occurred in less than 0.2 s.

(a) The authors show an engineering stress vs. engineering strain response, but I don't observe the ultimate stress beyond yielding where the stress decreases before fracture. Why not?

Solution: No UTS in compression because compression generally does not create a localized neck of deformation such that the cross-sectional area is changing in only one place prior to fracture.

- (b) For the Ni sample of 10 μm diameter and 2nd highest engineering fracture stress (~100 MPa), replot these data as true stress vs. true strain (yes, you will need to pull some points off the curve).

Solution: To convert:

$$\sigma_t = \sigma_e(1 + \epsilon_e) = F/A_{\text{instantaneous}}$$

$$\epsilon_t = \ln(1 + \epsilon_e) - \ln(L/L_{\text{initial}}) \text{ before necking, and } = \ln(A/A_{\text{initial}}) \text{ both before and after necking.}$$

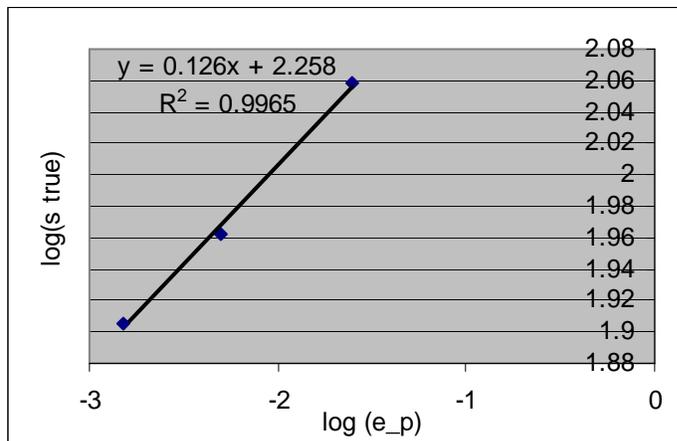
You can graph this as negative stress and negative strain if you like, or graph the absolute value so that stress-strain looks similar to tensile graph, but note that it is in compression. Let ϵ_e be a negative number when converting to true strain, because physically, the length is decreasing with respect to the initial length of the material sample.

- (c) Determine the strain hardening coefficient of this sample from these data, and compare that to literature values of n for Ni.

Solution: The simplest way to do this is to recognize that at stresses > yield stress, $s = K\epsilon_p^n$, where ϵ_p is the plastic strain and is equal to $e_{\text{total}} - e_{\text{elastic}}$. Then, we have

$\log \sigma = \log K + n \log(\epsilon_p)$ such that n is the slope of this $\log \sigma$ vs. $\log \epsilon_p$ graph.

s eng (MPa)	s true (MPa)	e_total (m/m)	e_plastic	log(e_p)	log(s)
80	80.4	0.005	0.0015	2.82391	1.905256
90	91.8	0.02	0.005	2.30103	1.962843
110	114.4	0.04	0.025	1.60206	2.058426



This value of $n = 0.13$ is much lower than typical literature values for Ni ($n \sim 0.2$ to 0.3). This may be because the capacity of a material with so much free surface to strain harden is reduced, as dislocations easily create slip steps on the free surface rather than entangle inside the material.

- (d) The authors wanted to consider whether the yield strength as a function of pillar diameter followed a Hall-Petch type relation. Why would they make this analogy between polycrystals of decreasing size and single crystal pillars of decreasing size?

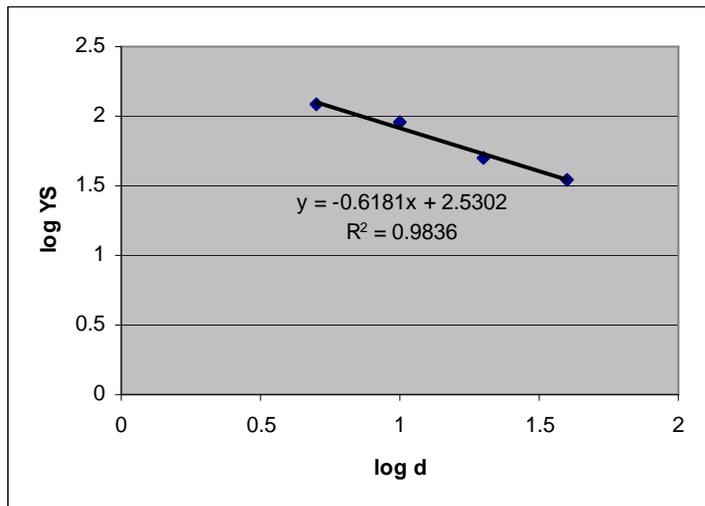
Solution: One pillar = one single crystal, so one pillar = one grain.

- (e) Compute the average yield strength for each pillar diameter and graph σ_y as a function of d to determine (i) whether Hall-Petch type strengthening is observed; (ii) What the “power” x of such strengthening would be for a Hall-Petch type equation.

Solution: Hall-Petch says $\sigma_y = \sigma_o + kd^{-x}$, so $\rightarrow \log \sigma_y = \log \sigma_o + \log k - x \log d$

Pulling off the approximate yield strengths of the four different pillar diameters, we find the slope (best fit least squares linear regression) of the log-log graph to be $x = 0.62$. Fairly decent example of Hall-Petch expected value of $x = -1/2$.

s (MPa)	d (um)	log(s)	log(d)	log(s)
120	5	2.079181246	0.69897	2.079181
90	10	1.954242509	1	1.954243
50	20	1.698970004	1.30103	1.69897
35	40	1.544068044	1.60206	1.544068



4. Creep can occur via several different mechanisms in amorphous and crystalline materials, each with their own activation energy Q . Typically, there is an Arrhenius behavior to this creep response, with an exponential prefactor that is particular to the material considered and on the operating mechanism.

(a) You are running two separate creep experiments on polycrystalline Ni, one at 527°C (=800 K) and the other at 532°C (=805 K). You observe that strain rate $d\epsilon/dt$ increases by a factor of 1.5 for the sample tested at the higher temperature, as compared to the lower temperature. Assuming there is a single dominant mechanism (and therefore a single activation energy Q) for creep over this temperature range, determine Q .

Solution: Here, we consider a ratio of strain rates to solve for Q , because we assume that the same mechanism and thus the same Q dominates over this small temperature range:

$$\begin{aligned} 1 &= \text{Strain rate 1} \propto e^{-Q/RT_1} \rightarrow Q = \ln(1/1.5) / (1/RT_2 - 1/RT_1) = 434 \text{ kJ/mol} \\ 1.5 &= \text{Strain rate 2} \propto e^{-Q/RT_2} \end{aligned}$$

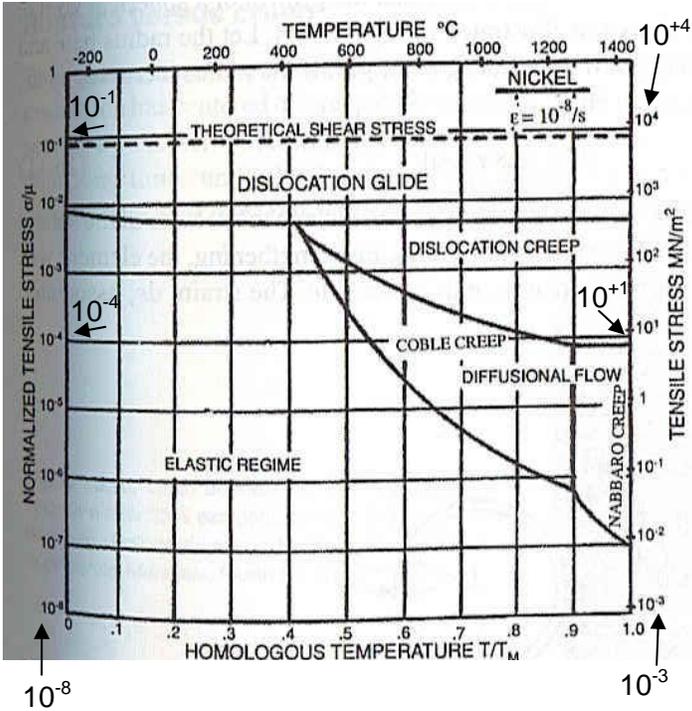
(b) What do you think this deformation mechanism could be, and why? Note that the actual grain size d and stress magnitude need not be stated to figure this out, but you can use deformation mechanism maps (Fig. Q4) to narrow down your options, with key data for Ni indicated in figure caption.

Solution: Must be a diffusive mechanism (not elasticity or plasticity or fracture) because the above analysis indicates an Arrhenius relationship with a diffusion barrier Q .

At $T/T_m = 0.36$ (530/1455C), the deformation mechanism map indicates this could be dislocation creep (QL) or Coble creep (Q_{gb}) if the strain rate is $10^{-8}/s$ (map on the left) or dislocation creep (QL) if the grain size is 0.1 mm (map on the right). To check this, look up energy barrier for self diffusion in Ni. If it's on the order of 434 kJ/mol, reasonable to assume dislocation creep is the dominant mechanism. If it's smaller than that, you know that Q_{gb} is less than QL , and it's reasonable to assume that Coble creep is the dominant mechanism and that the grain size is NOT 0.1 mm.

(c) You are running these tests because you would like to develop a new polycrystalline Ni alloy for Boeing aircraft engines. The Ni parts will be under a constant stress of about 1.3 GPa due to centrifugal forces, and a temperature of about 1100°C. Which creep mechanism are you most concerned about in this application, and *how do you propose to modify the polycrystalline Ni alloy to minimize creep rates for this application?*

Solution: Now, $T/T_m = 0.8$ and $s/G = 2 \times 10^{-2}$, so dislocation glide is the dominant mechanism of deformation. This is NOT creep, so I'm not worried about designing a microstructure that is particularly resistant to lattice or gb diffusion, and I'd want to include in my microstructure any strengthening mechanisms that would increase the resistance to dislocation glide (e.g., solute atoms, grain boundaries via grain size reduction, work hardening via prior plastic deformation to increase dislocation density). However, since I know the next-fastest mechanism around this temp/stress is dislocation creep, I would also design the microstructure to reduce lattice diffusion (increase QL), potentially by adding solute atoms.



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Fig. Q4. Deformation mechanism map for Ni of grain size $d = 32 \mu\text{m}$ (left) and $d = 100 \mu\text{m}$ (bottom).

Ni shear modulus $G = 76 \text{ GPa}$; tensile strength $\text{UTS} = 320 \text{ MPa}$; melting temp $T = 1455^\circ\text{C}$; fcc with atomic radius $r = 0.125 \text{ nm}$. $R = 8.31 \text{ J/mol}\cdot\text{K}$.

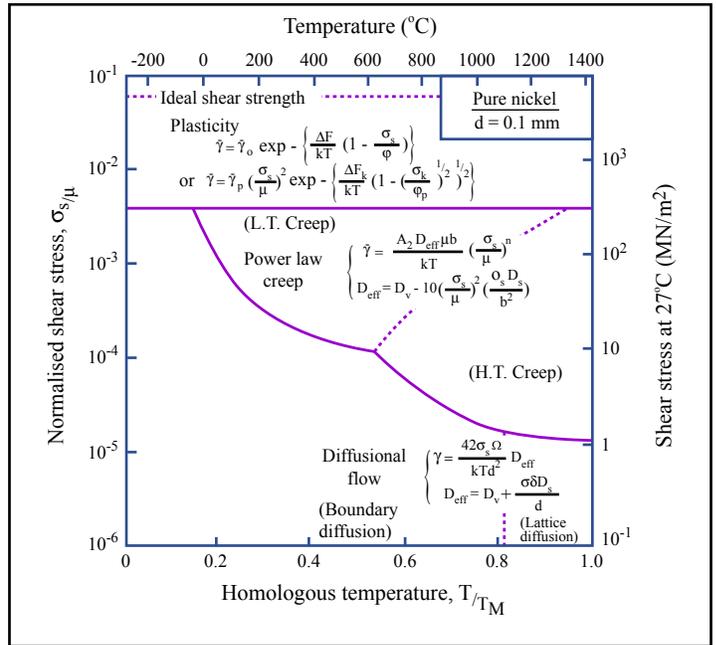


Figure by MIT OpenCourseWare.