

3.032 Problem Set 6

Fall 2007

Due: **Start** of Lecture, Wednesday 11.28.07

(NOTE THAT THIS DATE IS LATER THAN IN YOUR SYLLABUS.

PS7 WILL GO OUT ON 11.28 AND BE DUE ON 12.07.)

1. There is very little information available on wikipedia.org (<http://en.wikipedia.org/wiki/>) about the strengthening mechanisms available to increase the yield strength of crystalline and amorphous materials. Those wiki stubs that do exist are fragmented discussions of specific materials, and there is very little discussion of the physical mechanisms that govern this strengthening so that one could know how to choose a base material and/or alter the microstructure of that material to achieve a target strength, or to know the fundamental limits of the maximum attainable strength for a chosen mechanism.

Your task, individually and as a class, is to modify existing articles and create new articles, graphics, examples, and links that describe the concepts covered in lecture (and beyond) as clearly, accurately, and interestingly as possible. This is supposed to be interactive and the article(s) will get better as everyone contributes, and should help you really understand and integrate this material in prep for Quiz 3. The credit for this problem will be as follows:

- 67% from the material you personally edit/create. To do this, you will have to create one new wiki account on any computer (please use your MIT user name as your login “name”), and sign on using this account every time you edit any wiki pages from any computer. That way, we can also compare versions as the articles evolve. When you’re done, print your personal wiki “history” for this login name and turn that printout in with your pset, as you will have likely contributed to several different changes or made changes that are no longer visible. You don’t have to print out the pages you’ve created (please!) so we need this history to go find what wiki content you’ve edited.

- 33% for the total quality of the contributions to wikipedia that you as the 2007 3.032 class have made through your additions of material related to strengthening mechanisms in materials. It does not matter if you modify these pages or decide to create a new article that you all work on. You can add graphs, images, text, and equations, and link any new/existing pages created by classmates.

If you have not contributed much to wikis yet, there is a helpful FAQ at http://en.wikipedia.org/wiki/Wikipedia:Editing_FAQ. If you contribute early on, you’ll have the most chance to shape the site but the least to work with; if you wait too long to get started, you will have to work harder to think of creative and accurate ways to add to the site. *Remember that citations in your own academic work should NOT come from wikipedia, because it is not rigorously peer-reviewed and is only as accurate as the last person who edited the page.* Have fun – this is something you can do and enjoy while traveling on your Thanksgiving break!

2. We've discussed that hardness expressed in units of [Pa] is generally proportional to yield strength, and that engineers often assume $H = P/A = 3\sigma_y$. Here, you will derive this relation to understand the basis of this gross approximation.

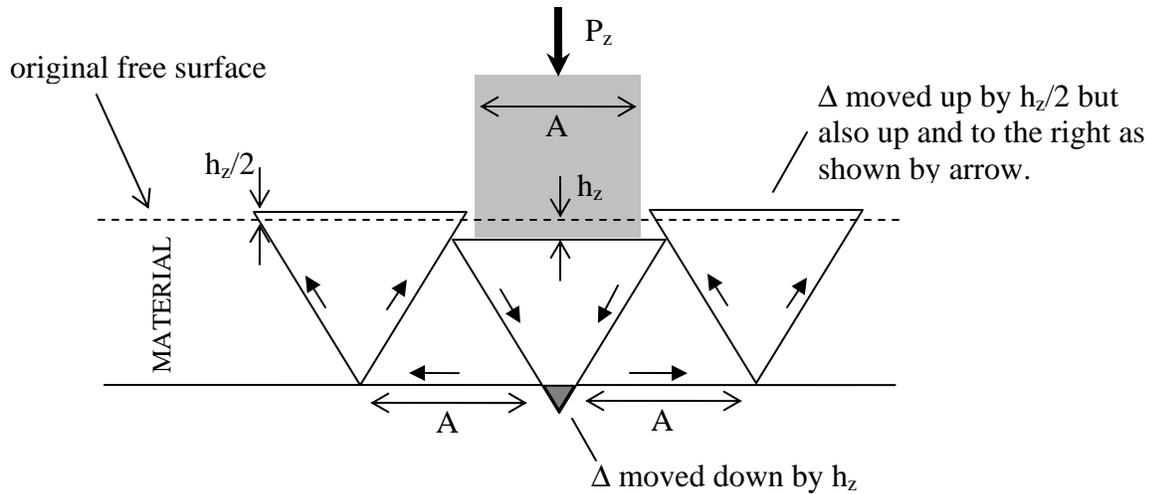


Fig. Q2: Two-dimensional approximation of indentation with a flat punch indenter, where material yields along planes of maximum shear stress.

- (i) Assume the indenter is a flat punch and that we consider indentation only in a plane (2D, for simplicity). Define the work done by the indenter as it applies a force P_z acting over the punch width (or cross-sectional area in 3D) A to exert a vertical displacement h_z onto the material surface.
- (ii) Now assume the material beneath the indenter displaces only at locations of maximum shear stress, as shown. Determine the relative displacement of the material triangle edges (filled arrows) in terms of h_z , and the “area” of those isosceles triangle edges over which this displacement occurs.
- (iii) The triangles will only start to displace when the work done by the punch = work required to attain shear stress in the material that is high enough to yield the material. Equate the work in (i) to the sum of the work done on all of these internal triangle edges (8 in total, shown in the Fig. Q2) that move under the shear stress τ . As the displacement u_z appears on both sides of this equality, you will obtain an equation of the form $P \propto A\tau$.
- (iv) Since we have implicitly assumed the maximum shear stress or Tresca criterion in this analysis, now express τ in terms of the yield strength according to this criterion, and note that H is defined as P/A , to obtain an equation that relates H to σ_y .
- (v) Explain at least two cases where this approximation would be inaccurate, and whether you think this proportionality factor would go up (so H is more than thrice the yield strength) or down. To think about this, consider the simplifications that were made in order to derive this relation. Note especially that hardness is a value you measure after considerable yielding over a material volume has occurred, not just at the instant that the yield stress is attained.

3. Materials engineers have become increasingly interested in whether the yield strength of crystalline materials changes as the physical size of the material decreases toward the nanoscale. Until very recently, conducting uniaxial tension/compression tests on tiny material volumes < 100 microns in diameter was technically infeasible, but Uchic et al. have shown that they can make and measure the compressive stress-strain response of single crystal Ni and other materials for pillar diameters as small as 5 microns (see Fig. 2). The full paper is available on MIT Server, and is a very cool read.

Images removed due to copyright restrictions. Please see:
 Fig. 1 in Uchic, Michael D., et al. "Sample Dimensions Influence
 Strength and Crystal Plasticity." *Science* 305 (August 2004): 986-989.

Fig. Q3: (From Uchic et al., *Science* 305: 986, 2004). Mechanical behavior at room temperature for pure Ni microsamples having a $\{134\}$ orientation. **(A)** Stress-strain curves for microsamples ranging in size from 40 to 5 μm in diameter, as well as the stress-strain curve for a bulk single crystal having approximate dimensions $2.6 \times 2.6 \times 7.4 \text{ mm}$. **(B)** A scanning electron micrograph (SEM) image of a 20- μm -diameter microsample tested to $\approx 4\%$ strain. The circle milled into the top surface of the microsample is a fiducial mark used during sample machining. **(C)** A SEM image of 5- μm -diameter microsample after testing, where the sample achieved $\approx 19\%$ strain during a rapid burst of deformation that occurred in less than 0.2 s.

- The authors show an engineering stress vs. engineering strain response, but I don't observe the ultimate stress beyond yielding where the stress decreases before fracture. Why not?
- For the Ni sample of 10 μm diameter and 2nd highest engineering fracture stress ($\sim 100 \text{ MPa}$), replot these data as true stress vs. true strain (yes, you will need to pull some points off the curve).
- Determine the strain hardening coefficient of this sample from these data, and compare that to literature values of n for Ni.
- The authors wanted to consider whether the yield strength as a function of pillar diameter followed a Hall-Petch type relation. Why would they make this analogy between polycrystals of decreasing size and single crystal pillars of decreasing size?
- Compute the average yield strength for each pillar diameter and graph σ_y as a function of d to determine (i) whether Hall-Petch type strengthening is observed; (ii) What the "power" x of such strengthening would be for a Hall-Petch type equation.

4. Creep can occur via several different mechanisms in amorphous and crystalline materials, each with their own activation energy Q . Typically, there is an Arrhenius behavior to this creep response, with an exponential prefactor that is particular to the material considered and on the operating mechanism.

(a) You are running two separate creep experiments on polycrystalline Ni, one at 527°C (=800 K) and the other at 532°C (=805 K). You observe that strain rate $d\epsilon/dt$ increases by a factor of 1.5 for the sample tested at the higher temperature, as compared to the lower temperature. Assuming there is a single dominant mechanism (and therefore a single activation energy Q) for creep over this temperature range, determine Q .

(b) What do you think this deformation mechanism could be, and why? Note that the actual grain size d and stress magnitude need not be stated to figure this out, but you can use deformation mechanism maps (Fig. Q4) to narrow down your options, with key data for Ni indicated in figure caption.

(c) You are running these tests because you would like to develop a new polycrystalline Ni alloy for Boeing aircraft engines. The Ni parts will be under a constant stress of about 1.3 GPa due to centrifugal forces, and a temperature of about 1100°C. Which creep mechanism are you most concerned about in this application, and *how do you propose to modify the polycrystalline Ni alloy to minimize creep rates for this application?*

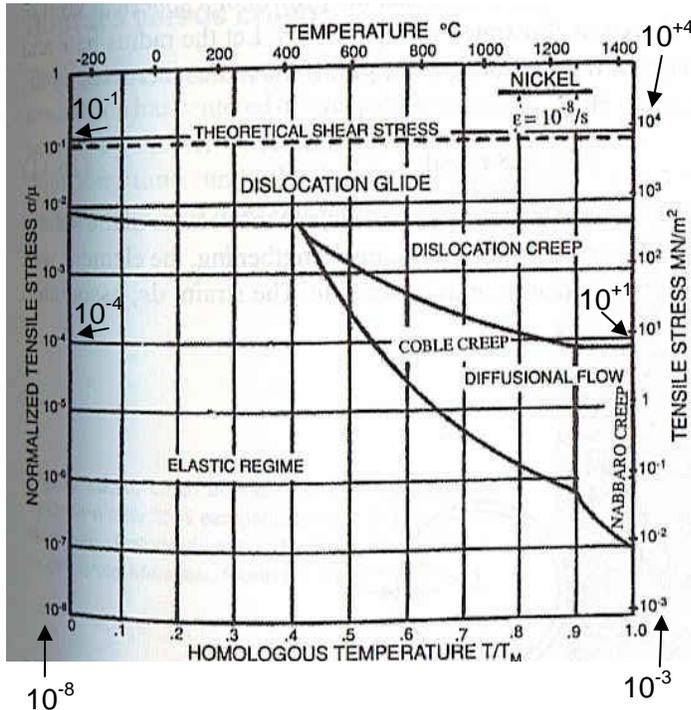
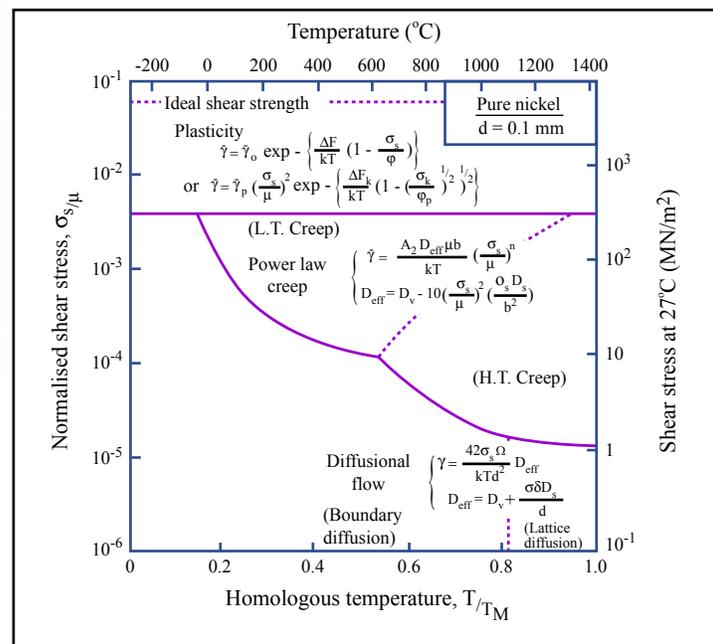


Fig. Q4. Deformation mechanism map for Ni of grain size $d = 32 \mu\text{m}$ (left) and $d = 100 \mu\text{m}$ (bottom).

Ni shear modulus $G = 76 \text{ GPa}$; tensile strength $\text{UTS} = 320 \text{ MPa}$; melting temp $T = 1455^\circ\text{C}$; fcc with atomic radius $r = 0.125 \text{ nm}$. $R = 8.31 \text{ J/mol}\cdot\text{K}$.



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