

3.032 Problem Set 3
Fall 2006
Due: Start of lecture, 9/29/06

1. In PS2-4, you considered a femur under a torsional load that induced a spiral fracture, approximating the load as pure torsion.

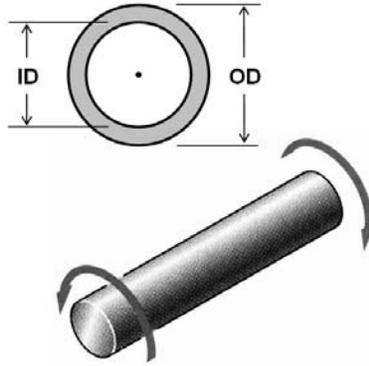


Figure 1: Femur under pure torsional loading T .

- (a) Draw a representative in-plane element from the outermost surface of the femur diaphysis. The femur is idealized as an annular cylinder of uniform diameter subjected to a pure torque T (Figure 1). Indicate all the normal and shear stresses σ_{ij} acting on that material volume element, expressed only as a function of T .
- (b) Construct Mohr's circle of stress to identify the average normal stress σ_{avg} in the bone.
- (c) Draw the orientation of the representative material element (indicate magnitude of θ_{ps}) and give the magnitudes of the principal normal stresses σ_1 and σ_2 , assuming a critical torsional load F of 67 N for a moment arm of 2 m.
- (d) Compare/contrast this finding with what is observed for the principal stress state and maximum shear state for the same bone subjected to uniaxial compression with a magnitude $\sigma_{11} = \sigma_1 = 60$ MPa.
2. Thin films used in microelectronics can develop considerable internal strain when they are deposited onto a substrate of a different material. The two materials may have different unit cell lattice parameters and/or may have different coefficients of thermal expansion, and the thin film is forced to take on the lateral dimensions of the substrate to which it is adhered. This strain can lead to substrate-film curvature (Figure 2). The midplane of such thin films is often approximated as being in a state of in-plane normal strain in cylindrical coordinates:

$$\epsilon_{r\theta} = 0; \epsilon_{rr} = \epsilon_{\theta\theta} = \epsilon_o - \kappa z.$$

where ϵ_o is an isotropic extension of the substrate (and the film), κ is the curvature of the film/substrate, and z is the distance from the film neutral axis (its midplane).

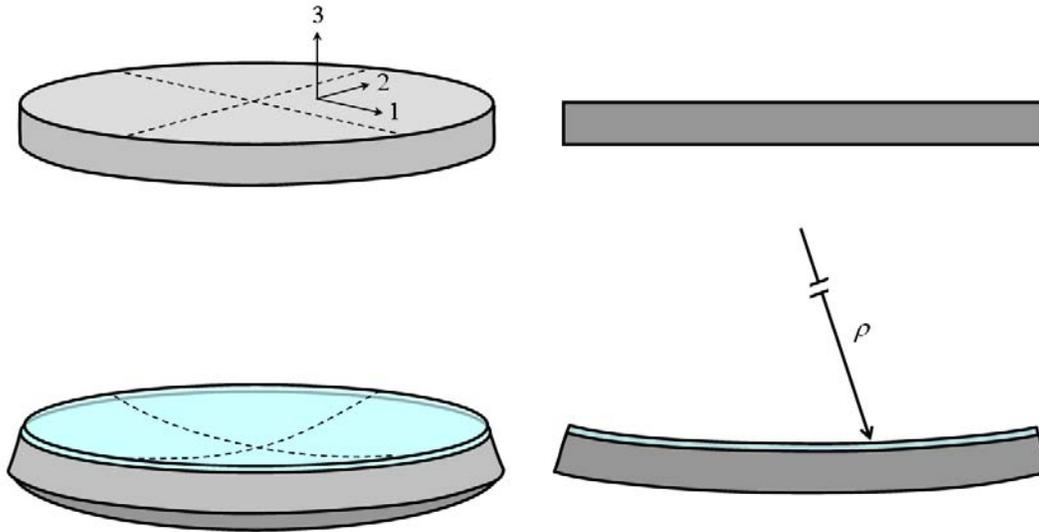


Figure 2: Top: Substrate before deposition; Bottom: Substrate and thin film after deposition (the actual curvature may be in the opposite direction depending on the mismatch between lattice parameters and thermal expansion coefficients).

- (a) In a scientific programming language of your choice, write a program that can be used to determine the normal strain $\epsilon_{x'x'}$ and engineering shear strain $\epsilon_{x'y'}$ on the (thin film material) plane oriented at an angle θ with respect to your original coordinate system (x, y) . Your program must report:
- $\epsilon_{x'x'}$ and engineering shear strain $\epsilon_{x'y'}$ for a given θ
 - principal strains ϵ_1 and ϵ_2 and the material volume element orientation θ_{pe}
 - maximum in-plane shear strain $\epsilon_{x'y',max}$ and the corresponding material volume element orientation $\theta_{pe,max}$
 - average normal strain ϵ_{avg}

Note: Mathematica, Matlab, Maple, C++, and Fortran90 are excellent choices to build this program. Practice with Mathematica will benefit you in future lab experiment analyses, so here's a good excuse to brush up (available on MIT Server). However, a brute-force execution in Excel is acceptable as a last resort. In all cases, you must provide the program and indicate the embedded equations with your problem set solution.

- (b) Assume all the strain at the midplane of a Ge film deposited on a Si substrate comes from a mismatch of the unit cell lattice parameters a of those two materials. Compute that strain.
- (c) Is the film in a state of tensile or compressive strain, and will the film/substrate system show positive (smile) or negative (frown) curvature?
- (d) Assume the film is 500 nm in thickness and that you have measured the radius of curvature ρ of the film/substrate system as 10 km, the resolution of laser-based instruments used to measure wafer curvature. What is the in-plane normal strain $\epsilon_{rr} = \epsilon_{\theta\theta}$?
- (e) This strain state is the principal strain state. How do you know this?

- (f) Assume a Cartesian coordinate system is a fair approximation in the plane strain state of the film (i.e., $\epsilon_{rr} = \epsilon_{\theta\theta} = \epsilon_1 = \epsilon_2$). Use your program to determine the orientation and magnitude of maximum in-plane shear strain in the Ge film.
- (g) If the principal strain state of a thicker piece of Ge were not in an equibiaxial strain state as described above, but instead a random biaxial strain state (e.g., $2\epsilon_1 = \epsilon_2$; $\epsilon_3 = 0$), use Mohr's circle to determine the magnitude of maximum shear strain and the plane of the film on which that shear strain would be (1–2, 1–3, or 2–3). You can assume the 3-direction to be through the film thickness.
3. A beam of 2000 mm^2 cross-sectional area is composed of two pieces of wood glued together along a plane at an angle θ with the beam axis (Figure 3). It has been found from previous experiments that the joint will fail at 20 MPa and 10 MPa normal and shear stresses, respectively, and the wood at 56 MPa and 28 MPa normal and shear stresses, respectively. Determine the maximum allowable axial load that the bar can carry with a safety factor of two and the corresponding value of the angle θ .

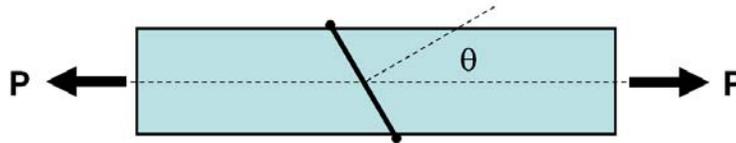


Figure 3: Wood pieces connected by glued joint.

4. In 1654, the German scientist Otto von Guericke performed an experiment for Emperor Ferdinand III to demonstrate the nature of atmospheric pressure. Guernicke connected two copper bowls with an outer diameter D of 14 in, pumped out the air between them, and tried to pull them apart with two teams of eight horses each (Figure 4). The horses were unable to separate the bowls.

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http://chem.ch.huji.ac.il/history/guericke_magdenburg_experiment.jpg

Figure 4: Otto von Guericke's experiment with two connected copper bowls, a vacuum pump, and two teams of horses.

- (a) Assuming that Guernicke's pump was capable of removing 75% of the air from the assembly and idealizing the connected bowls as a sphere with a wall thickness of 0.4 cm, what was the maximum normal stress in the copper bowls before the horses started pulling?
- (b) Using a failure stress σ_F for copper in compression of 370 MPa, how thin in theory could the copper bowls be made without failure? If this value seems impractical, explain why using the terminology and concepts of 3.032.
- (c) Assume that the bowls are not hemispheres but rather two identical shells of revolution of $y = (7 \text{ in}) \sin [x\pi/(14 \text{ in})]$ for x between 0 and 14 inches with axis of revolution $x = 7$ inches. The bowls are sealed at the joint with epoxy (Figure 5). The wall thickness is 0.4 cm and the interior is 75% evacuated as before. What is the actual shape of the stress distribution $\sigma(x)$, and what is its average value?

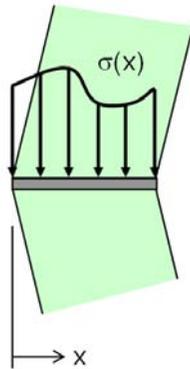


Figure 5: Epoxy joint between two shells of revolution.