

3.032 Problem Set 2 Solutions

Fall 2007

Due: Start of Lecture, 09.21.07

1. In the beam considered in PS1, a steel beams carried the distributed weight of the rooms above. To reduce stress on the beam, it is common to add extra support to these long beams at the point of internal maximum bending moment. Where should this support be located, and what are the shear forces and bending moments at this location?

To answer this, construct the shear force $V(x)$ and internal bending moment $M(x)$ diagrams along the entire beam length, and note the magnitude of each at every point in the beam where the loading situation changes, as well as at the point of maximum M .

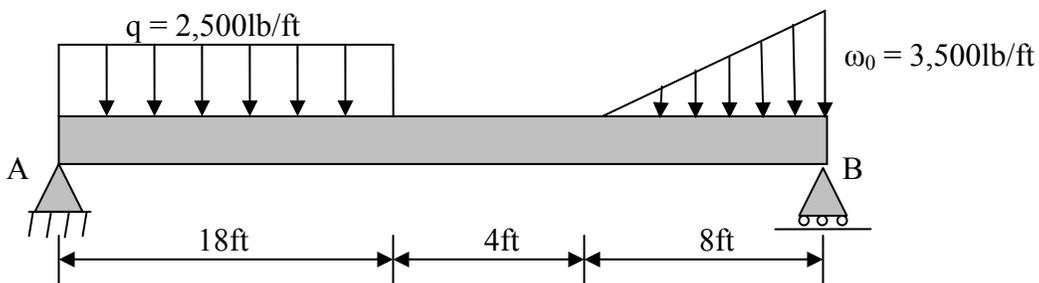


Figure 1. Floor joist beam of new MIT physics building.

2. At a seminar at MIT last week, Prof. Ilya Zharov (Univ. of Utah, Department of Chemistry) showed that his group could create self-assembled colloidal films, which are basically a few planes of close-packed silica (SiO_2) spheres with nanoscale pores between them (Fig. 2A). He proposed these as a potential new material for fuel cell proton-exchange membranes (PEMs), because he can coat the nanoscale pores between the silica spheres with sulfonated molecules that selectively allow proton exchange through the nanopores.

Figure 2. SEM images of the chemically-modified colloidal film prepared from 440 nm diameter silica spheres (A) top view; the geometric projection of a pore observed from the (111) plane is outlined in the inset. (B) Side view. (C) A scheme of its permselective behavior. *Langmuir* (2007, in revision).

This sounds exciting, but someone quickly pointed out this is basically a very thin, porous sheet of glass: as a free-standing film, it might fail mechanically under its own weight or under the loads due to proton flux. Prof. Zharov said he had no idea how strong these colloidal films were, but thought engineers would figure it out for him. He thought that laying strips of the films onto a metal grid (like a window screen) would keep the stresses low enough by supporting the film at several points along its length.

Let us see if he is correct in this guess, treating a strip of the film as a beam and assuming it deforms elastically up to the point of fracture.

- (a) We will first test the force required to break these films, using two different loading configurations: three-point bending and uniaxial tension (Fig. 2b). In both tests, we use identical samples which are 100 μm in length and have rectangular cross-sections of 20 μm width and 2 μm thickness. Draw the expected tensile stress along the beam length $\sigma(x)$ in each case. Assume all stresses remain within the elastic regime. (*It may be useful to construct shear and bending moment diagrams.*)

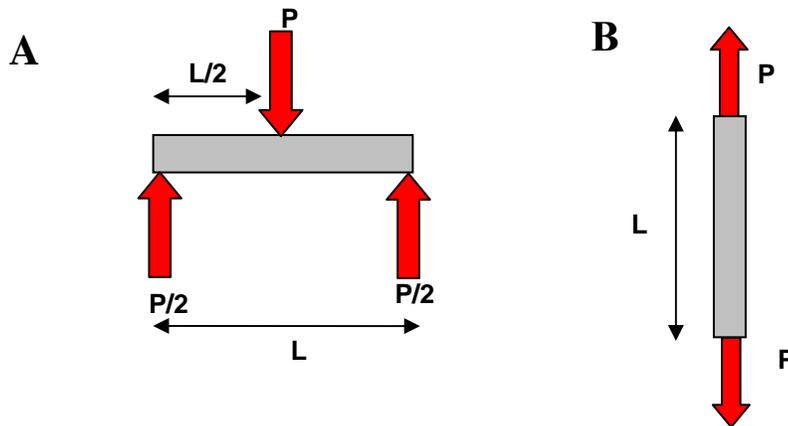


Figure 3. Experimental and schematic loading conditions for (A) Three-point bending; and (B) uniaxial tension.

- (b) We measure that these test samples catastrophically fail (i.e., transition from elastic deformation to fracture, without any prior plastic deformation or yielding) at an average load of **75.0 μN** for the 3-pt. bend tests and at a load of **3.4 mN** for the tensile tests. Determine the stress at which the colloidal films failed in both cases; this is called the flexural strength and tensile strength, respectively.
- (c) In this case, the colloidal film is brittle: when the film fails, it fails at the nanopore defects as the spheres break free from one another. The mechanical strength of brittle materials depends strongly on the existence and density of defects in highly stressed regions of the material. Considering the stress distributions determined in (a), explain any differences in the flexural and tensile strength properties of the colloidal films calculated in (b).
- (d) For this new proton-exchange membrane application, Zharov wants to “simply support” these beams of colloidal film by laying them on a copper transmission electron microscopy (TEM) grid. the standard grid holes are 3 mm x 3 mm. He estimates the distributed load on the colloidal film due to proton flux to be 10 mN/mm. Considering this new loading condition and the flexural/tensile strengths we determined experimentally, will the colloidal film fail under this application? If so, how would you redesign the film and/or support to enable this fuel cell application?

3. Solid polymer pressure-sensitive adhesives (PSA) are used for electronic component-to-heat sink attach to minimize interfacial thermal resistance, typically comprising polyacrylates filled with conductive ceramic particles. These thermally conductive adhesives are applied between the interfaces of the electronic component and the heat sink (Fig. 4). The polymer becomes highly adhesive under elastic deformation, due to application of this uniform pressure between the electronic component and heat sink.

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Fig. 1 in Eveloy, Valerie, et al. "Reliability of Pressure-Sensitive Adhesive Tapes for Heat Sink Attachment in Air-Cooled Electronic Assemblies." *IEEE Transactions on Device and Materials Reliability* 4 (December 2004): 650-657.

Figure 4: Illustration of PSA application to component-heat sink attachment.

Let us treat this PSA tape as an elastic continuum of Poisson's ratio $\nu = 0.45$. A press applies 1000 N to the electronic package to compress the adhesive tape (Figure 5). The initial gap distance between the package and the heat sink is 0.35 mm.

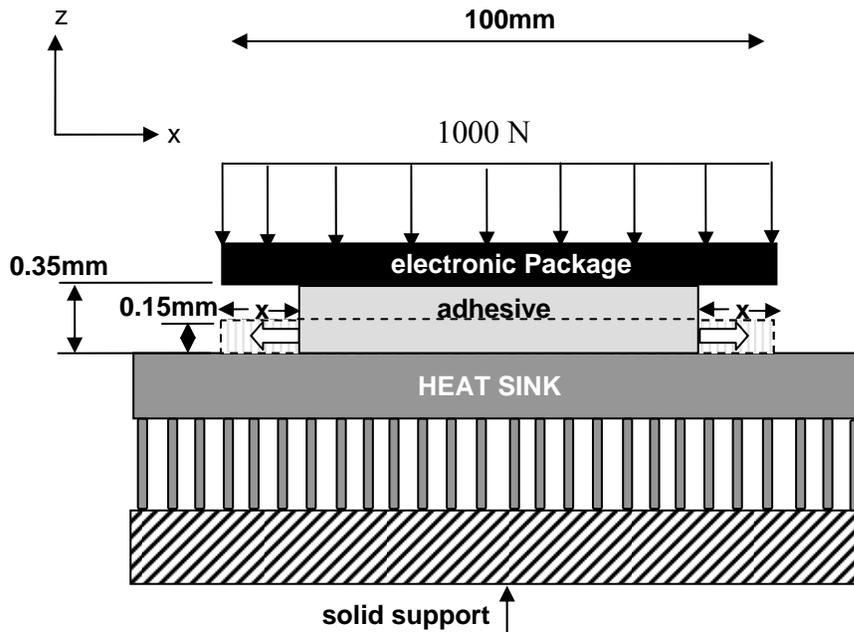


Figure 5: Schematic of component-to-heat sink joining procedure using pressure-sensitive adhesive tape.

- If the electronic package is 100 mm wide x 20 mm deep (into page), what is the applied stress?
- Derive the relationship between true strain and engineering strain: $\epsilon_t = \ln(\epsilon_e + 1)$. Is there a significant difference in these strain values for the normal strain resulting from compression of the PSA from a thickness of 0.35 mm to 0.15 mm?
- To fit this PSA-joined device into the larger circuit, a final joint thickness of 0.15 mm is required (Figure 5). To ensure a reliable joint, the polymeric adhesive must deform laterally under the applied pressure to completely fill the joint (a change in length equal to $2x$ in Fig. 5). Given these two requirements, what is the minimum initial length of PSA tape that we need to ensure complete interfacial contact between the adhesive and electronic package after pressing? Use engineering strain calculations and idealize the PSA tape as an elastic continuum.

4. Strain gages are designed only to measure normal (tensile or compressive) strains, but the engineering state of strain of the material plane is defined by ϵ_{xx} , ϵ_{yy} and γ_{xy} . Strain gage rosettes as you used in Lab 1 are one solution to this problem, because γ_{xy} can be calculated from experimentally measured ϵ_{xx} , ϵ_{yy} and $\epsilon(45^\circ)$. Considering the axes transformation equations of strain, determine this relation.