

Recitation 13

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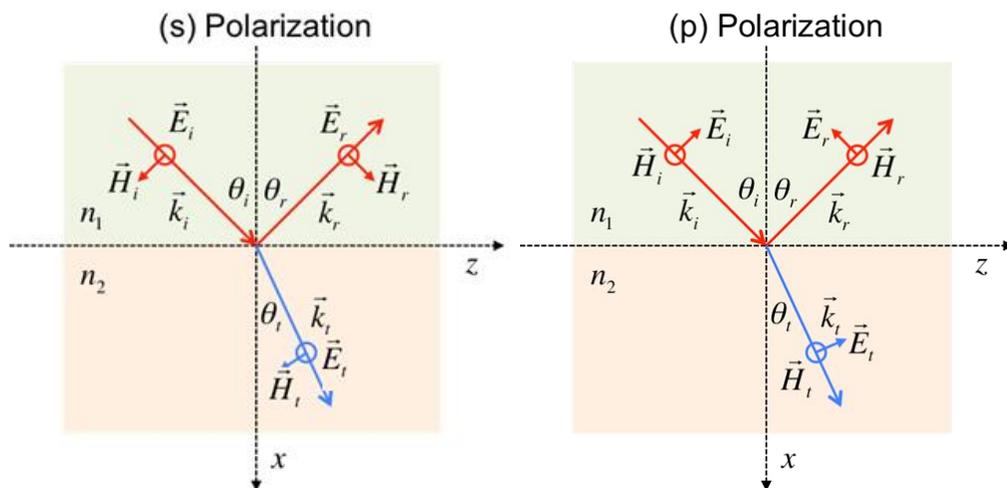
1. A “Light” Review
 - a) Interface Boundary Conditions

$$\begin{aligned}
 B_1 &= B_2 \\
 \sigma &= D_2 - D_1 \\
 \vec{E}_{1\parallel} &= \vec{E}_{2\parallel} \\
 \vec{K} &= \vec{H}_{2\parallel} - \vec{H}_{1\parallel}
 \end{aligned}$$

Here σ and \vec{K} are the surface charge density and surface current density respectively. For many cases, these densities are zero. By combining these boundary conditions with the constitutive equations for the materials, one can show how an incident light wave will reflect or refract at a given interface.

b) s- and p- Polarized Light

Since EMR waves are transverse waves, the field direction of the wave can be polarized at different angles relative to the interfaces with which the waves interact. By choosing two specific polarizations, one with the electric field parallel to the plane of incidence ((s)-Polarization) and one with the electric field perpendicular to the plane of incidence ((p)-Polarization), any general polarized light interactions can be viewed as a linear combination of these two specific polarizations. By considering the boundary conditions imposed on the fields (electric and magnetic) at the interface for both polarizations, one can derive reflection and transmission coefficients for the EMR waves interacting with the surface.



$$\begin{aligned}
 r_s &= \left(\frac{E_{ry}}{E_{iy}} \right) = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \\
 t_s &= \left(\frac{E_{ty}}{E_{iy}} \right) = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t}
 \end{aligned}$$

$$\begin{aligned}
 r_p &= \left(\frac{E_r}{E_i} \right) = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \\
 t_p &= \left(\frac{E_t}{E_i} \right) = \frac{2n_1 \cos \theta_i}{n_2 \cos \theta_i + n_1 \cos \theta_t}
 \end{aligned}$$

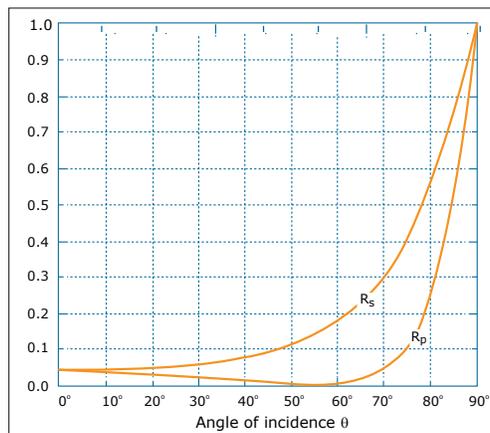


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c) Anti-Reflective Coatings

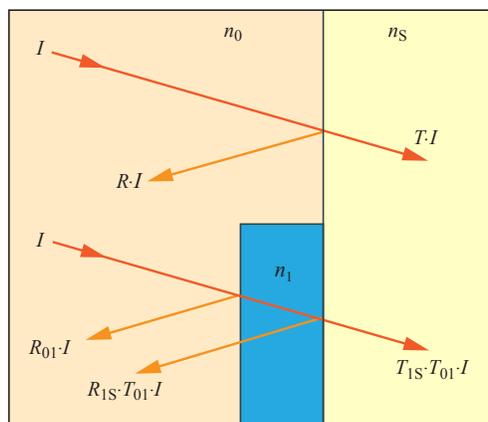


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Minimal Reflection Occurs When Refractive Indices Satisfy the Following:

$$n_1 = \sqrt{n_0 n_S}$$

Alternatively, for a specific wavelength, all light can be transmitted using a quarter wave stack:

$$d = \frac{\lambda_0}{4n_1}$$

2. Layered Optical Materials

a) Transfer Matrices

In general, we can consider the propagation of light through various media in terms of transfer matrices. By taking the input wave amplitude components of the incident and reflected light (forward and backward propagating amplitudes) as a 2X1 vector, for each interface encountered a 2X2 matrix is applied to the input amplitude vector, and for each medium of thickness d the light propagates through, a 2X2 propagation matrix is applied. Thus if the original medium the light is propagating through has index of refraction n_1 and the light is incident on a material with index of refraction n_2 , the transfer matrix for the interface is given by the matrix D_{12} , where:

$$D_{12} = D_2^{-1} D_1$$

$$D_1 = \begin{pmatrix} 1 & 1 \\ n_1 \cos \theta_1 & -n_1 \cos \theta_1 \end{pmatrix} \quad D_2 = \begin{pmatrix} 1 & 1 \\ n_2 \cos \theta_2 & -n_2 \cos \theta_2 \end{pmatrix}$$

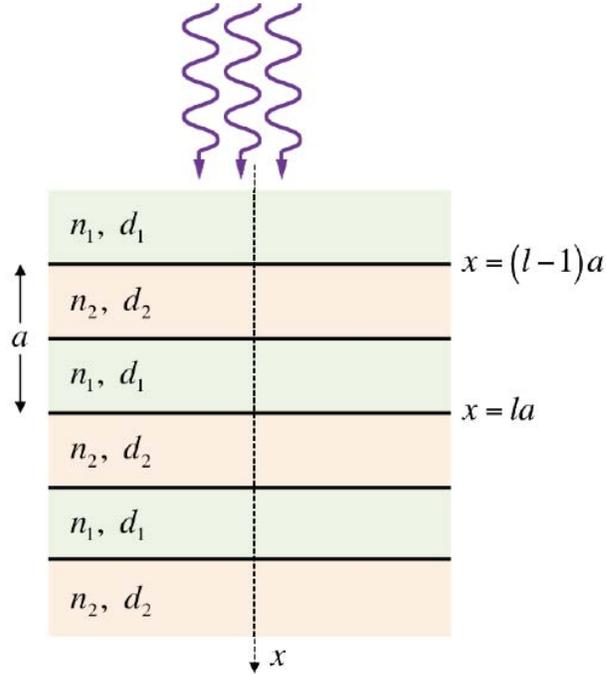
The propagation matrix through a material of thickness d with index of refraction n is given as:

$$P = \begin{bmatrix} e^{i\frac{\omega n}{c_0} \cos \theta d} & 0 \\ 0 & e^{-i\frac{\omega n}{c_0} \cos \theta d} \end{bmatrix}$$

This is just phase accumulation for a plane wave going through the d thick medium.

b) Photonic Crystals

Using the method of transfer matrices and applying it to the following periodic system:



A relation between any two layers can be found using transfer matrices:

$$\begin{pmatrix} E_{1(l-1)} \\ E'_{1(l-1)} \end{pmatrix} = D_{21}P_2D_{12}P_1 \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix} = D_1^{-1}D_2P_2D_2^{-1}D_1P_1 \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix} = M \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix}$$

$$P_1 = \begin{pmatrix} e^{ik_1x d_1} & 0 \\ 0 & e^{-ik_1x d_1} \end{pmatrix} \quad P_2 = \begin{pmatrix} e^{ik_2x d_2} & 0 \\ 0 & e^{-ik_2x d_2} \end{pmatrix}$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

Since the system is periodic, it should have a Bloch type solution as well.

$$\begin{pmatrix} E_{1(l-1)} \\ E'_{1(l-1)} \end{pmatrix} = e^{\mp iKa} \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix}$$

Thus we have an eigenvalue problem where:

$$M \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix} = e^{\mp iKa} \begin{pmatrix} E_{1l} \\ E'_{1l} \end{pmatrix}$$

This yields the following:

$$e^{\mp iKa} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\frac{1}{4}(M_{11} + M_{22})^2 + M_{12}M_{21}}$$

The coefficients of M are all functions of frequency ω and phase β , thus this relationship serves as the dispersion relation between the Bloch wavenumber K and ω and β .

Combining the two possible eigenvalues of $e^{\mp iKa}$:

$$\frac{e^{iKa} + e^{-iKa}}{2} = \cos Ka = \frac{1}{2}(M_{11} + M_{22}) \rightarrow K(\omega, \beta) = \frac{1}{a} \cos^{-1} \left(\frac{1}{2}(M_{11} + M_{22}) \right)$$

Upon inspection, if $\left| \frac{1}{2}(M_{11} + M_{22}) \right|$ is less than unity, the light will be propagation, if the value is unity, this corresponds to the photonic band edge, and if it is greater than 1, the waves are decaying or evanescent (K is complex).

Figures removed due to copyright restrictions. Bloch waves corresponding to the A and B solutions for frequency at the edge of the Brillouin zone: Unknown source.

Dispersion relationship for $\beta = 0$ (incidence angle is 0).

Figure removed due to copyright restrictions. TM and TE polarization: Unknown source.

Dispersion relationship for $\beta \neq 0$ (incidence angle is 0).

c) Dielectric Mirrors

By concatenating many periods, one can make a so called distributed Bragg reflector (DBR) that has almost 100% reflection.

$$\begin{pmatrix} E_0 \\ E'_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^N \begin{pmatrix} E_N \\ E'_N \end{pmatrix}$$

$$R = |r_N|^2 = \left| \frac{E'_0}{E_0} \right|^2 = \frac{|M_{21}|^2}{|M_{21}|^2 + \left(\frac{\sin Ka}{\sin NKa} \right)^2}$$

In the limit of long wavelengths or small periods:

$$Ka \rightarrow 0 \Rightarrow R = \frac{|M_{21}|^2}{|M_{21}|^2 + \left(\frac{1}{N} \right)^2}$$

In the limit of large N :

$$N \rightarrow \infty \Rightarrow R = \frac{|M_{21}|^2}{|M_{21}|^2} = 1$$

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3.024 Electronic, Optical and Magnetic Properties of Materials
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