3.024 Electrical, Optical, and Magnetic Properties of Materials Spring 2012
Recitation 10

Outline:

p-n Junctions

p-type material (dopant is electron acceptor)

$$p_v \approx N_A$$

$$n_c \approx \frac{n_i^2}{N_A}$$

$${}_F = E_v + \frac{E_g}{2} + \frac{3}{4}k_B T \ln\left(\frac{m_v^*}{m_c^*}\right) - k_B T \ln\left(\frac{N_A}{n_i}\right)$$

n-type material (dopant is electron donor)

$$p_v \approx \frac{n_i^2}{N_D}$$

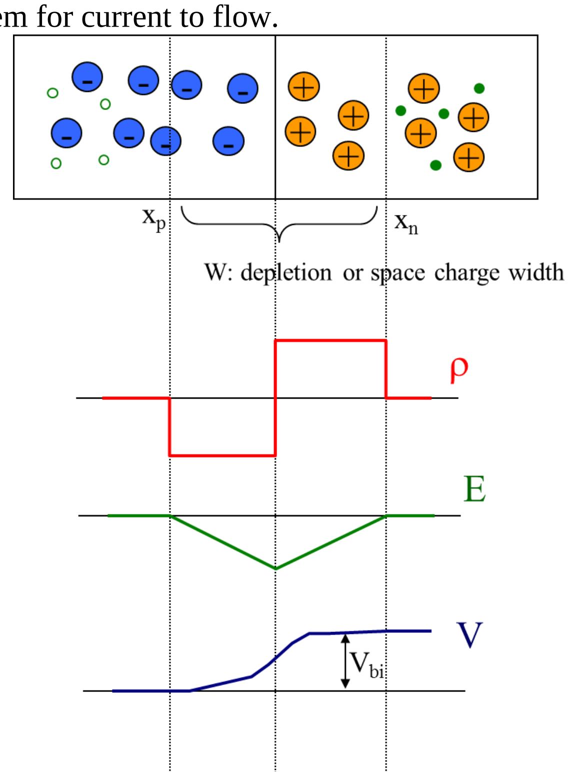
$$n_c \approx N_D$$

$${}_F = E_v + \frac{E_g}{2} + \frac{3}{4}k_B T \ln\left(\frac{m_v^*}{m_c^*}\right) + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

When p-type and n-type materials are brought together, they form a ***p-n*** junction. Immediately upon joining, holes will flow from p-type to n-type and electrons from n-type to p-type until an equilibrium is reached when the chemical potential/Fermi energies of each side are equal. This junction results in a redistribution of charge and thus the creation of an electric field being built into the system in a region called the depletion region.

Recall: $E(x) = \int \frac{\rho(x)}{\epsilon_r \epsilon_0} dx$ & $V(x) = -\int E(x) dx$

The built in potential results in no current flowing. To make current flow, an applied forward bias must be applied to the system for current to flow.



$$qV_{BI} = E_{Fn} - E_{Fp} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$
$$W = x_p + x_n = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{BI}}{q} \frac{N_A + N_D}{N_A N_D}}$$
$$x_p = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{BI}}{q} \frac{N_D}{N_A (N_A + N_D)}}$$
$$x_n = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{BI}}{q} \frac{N_A}{N_D (N_A + N_D)}}$$
$$N_A x_p = N_D x_n$$

e.g. Variable band gap semiconductor.
You are working on creating light absorbing ***p-n*** junction devices and have found a material semiconducting alloy whose band gap energy can be controlled with an external magnetic flux B. You have empirically determined that $E_g = 4.4\ \text{eV} - 0.3\frac{\text{eV}}{\text{T}^2}B^2$, where B is measured in Tesla T. The effective mass of the electrons in the system are ~0.5 m_e and the effective mass of the holes in the system are ~1.5 m_e. The system is at room temperature T = 300 K.

(a) What magnetic flux must you apply to get an intrinsic carrier concentration of 10^{15} cm^{-3}?

$$n_c(T) p_v(T) = N_c(T) P_v(T) e^{\frac{-E_g}{k_B T}}$$
$$n_c = p_v = n_i$$
$$N_c(T) \cong \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}}$$
$$P_v(T) \cong \frac{1}{4}\left(\frac{2 m_v^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}}$$
$$n_i^2 = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}} \frac{1}{4}\left(\frac{2 m_v^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}} e^{\frac{-E_g}{k_B T}}$$
$$\frac{16 n_i^2}{(m_c^* m_v^*)^{\frac{3}{2}} \left(\frac{2 k_B T}{\pi \hbar^2}\right)^3} = e^{\frac{-E_g}{k_B T}}$$
$$E_g = -k_B T \ln\left(\frac{16 n_i^2}{\left(\frac{m_c^* m_v^*}{m_e^2}\right)^{\frac{3}{2}} \left(\frac{2 m_e k_B T}{\pi \hbar^2}\right)^3}\right)$$
$$E_g = \frac{3}{2} k_B T \ln\left(\frac{m_c^* m_v^*}{m_e^2}\right) + k_B T \ln\left(\frac{\left(\frac{2 m_e k_B T}{\pi \hbar^2}\right)^3}{16 n_i^2}\right)$$

$$4.4\text{ eV} - 0.3\frac{\text{eV}}{\text{T}^2}B^2$$
$$= 8.62 \times 10^{-5}\frac{\text{eV}}{\text{K}}300\text{ K}\left(\frac{3}{2}\ln(1.5 * 0.5) + \ln\left(\frac{\left(2 * 9.11 \times 10^{-31}\text{kg } 1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}300\text{ K}\right)^3}{\pi^3 1.05 \times 10^{-34^6}\text{ J}^6\text{s}^6\ 16 * 10^{42}\text{m}^6}\right)\right)$$
$$B = 3.6\text{ T}$$

(b) What wavelength of photons does this magnetic flux correspond to as the maximum wavelength the system can absorb?

$$E_g = 4.4\text{ eV} - 0.3\frac{\text{eV}}{\text{T}^2}B^2 = 0.514\text{ eV}$$
$$E_g = E = h\nu = \frac{hc}{\lambda}$$
$$0.514\text{ eV} = 1240\text{ eV}\frac{\text{nm}}{\lambda}$$
$$\lambda = 2414\text{ nm}$$

This wavelength is in the infrared.

(c) What is the location of the Fermi energy for this band gap at this applied magnetic flux?

$$\mu = \ {}_F = E_v + \frac{E_g}{2} + \frac{3}{4}k_B T\ln\left(\frac{m_v^*}{m_c^*}\right)$$
$${}_F = E_v + \frac{0.514\text{ eV}}{2} + \frac{3}{4}8.62 \times 10^{-5}\frac{\text{eV}}{\text{K}}300\text{ K}\ln\left(\frac{1.5}{0.5}\right)$$
$${}_F = E_v + 0.278\text{ eV}$$

This is very close to the middle of the band gap.

(d) If we dope the material with an n-type dopant, what donor concentration would we have to dope to move the Fermi level to within 0.1 eV under the conduction band?

$$E_c - \ {}_F = 0.1\text{ eV} = E_c - \left(E_v + \frac{E_g}{2} + \frac{3}{4}k_B T\ln\left(\frac{m_v^*}{m_c^*}\right) + k_B T\ln\left(\frac{N_D}{n_i}\right)\right)$$
$$0.1\text{ eV} = E_c - \left(E_v + 0.278\text{ eV} + k_B T\ln\left(\frac{N_D}{n_i}\right)\right)$$
$$0.1\text{ eV} = E_g - 0.278\text{ eV} - k_B T\ln\left(\frac{N_D}{n_i}\right)$$
$$0.1\text{ eV} = E_g - 0.278\text{ eV} - k_B T\ln\left(\frac{N_D}{n_i}\right)$$
$$0.1\text{ eV} - 0.514\text{ eV} + 0.278\text{ eV} = -8.62 \times 10^{-5}\frac{\text{eV}}{\text{K}}300\text{ K}\ln\left(\frac{N_D}{10^{15}\text{cm}^{-3}}\right)$$
$$N_D = 1.9 \times 10^{17}\text{cm}^{-3}$$

(e) If we dope the material with a p-type dopant, what acceptor concentration would we have to dope to move the Fermi level to within 0.1 eV over the valence band?

$$_F - E_v = 0.1\text{ eV} = \left(E_v + \frac{E_g}{2} + \frac{3}{4}k_BT\ln\left(\frac{m_v^*}{m_c^*}\right) - k_BT\ln\left(\frac{N_A}{n_i}\right)\right) - E_v$$

$$0.1\text{ eV} = \left(0.278\text{ eV} - k_BT\ln\left(\frac{N_A}{n_i}\right)\right)$$

$$0.1\text{ eV} - 0.278\text{ eV} = -k_BT\ln\left(\frac{N_A}{n_i}\right)$$

$$0.1\text{ eV} - 0.278\text{ eV} = -8.62\times10^{-5}\,\frac{\text{eV}}{\text{K}}\,300\text{ K}\ln\left(\frac{N_A}{10^{15}\text{cm}^{-3}}\right)$$

$$N_A = 9.8\times10^{17}\text{cm}^{-3}$$

(f) If we combine the p-type and n-type dopant from (f) and (g) together, what is the dielectric constant of the material if the depletion region width W = 750 nm?

$$W = \sqrt{\frac{2\epsilon_r\epsilon_0 V_{BI}}{q}\frac{N_A+N_D}{N_AN_D}}$$

$$qV_{BI} = E_{Fn} - E_{Fp} = 0.514 - .1 - .1\text{ eV} = 0.314\text{ eV}$$

$$V_{BI} = 0.314\text{ V}$$

$$W = \sqrt{\frac{2\epsilon_r\epsilon_0 V_{BI}}{q}\frac{N_A+N_D}{N_AN_D}}$$

$$750\text{ nm} = \sqrt{\frac{2\epsilon_r 8.85\times10^{-12}\,\frac{\text{F}}{\text{m}}\,0.314\text{ V}}{1.6\times10^{-19}\text{C}}\frac{9.8+1.9}{9.8*1.9*10^{17}\text{cm}^{-3}}}$$

$$\epsilon_r = 2577$$