Lecture 25

Hysteresis in Ferromagnetic Materials

(Majority of illustrations in this lecture were generously provided by Prof. Geoffrey Beach)

Today

1. Magnetic anisotropy.
2. Transition metals: crystal structure and anisotropy.
3. Hard and easy axis.
4. Derivation of hysteresis loop for a single domain ferromagnet.
5. Coercive field vs. saturation magnetization.

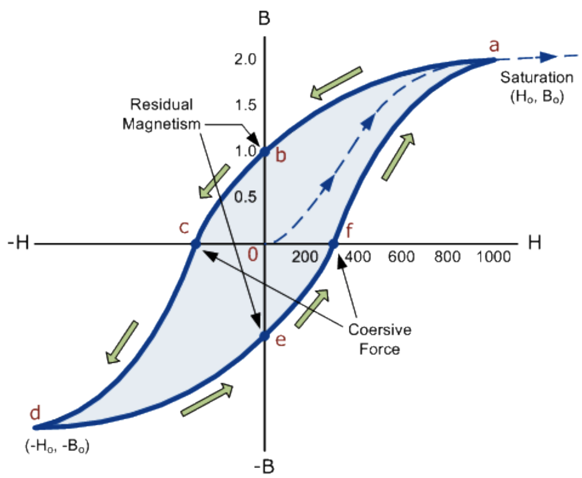
Questions you should be able to answer by the end of today's lecture

1. What is the origin of magnetic anisotropy?
2. What are hard and easy axis in transition metals?
3. Why does cobalt have very high anisotropy constant?
4. What is the origin of hysteresis?
5. How to plot magnetization energy density vs. angle between the magnetization and the easy axis?
6. What is coercive field?

From now on we will focus on ferromagnetic materials as they are used most frequently in applications ranging from magnetic data storage to power generation.

Last lecture we have shown how the fermionic nature of electrons leads to the spontaneous ordering of the magnetic dipoles inside the ferromagnetic material. So how does the magnetization of the material as a whole depend on the applied magnetic field?

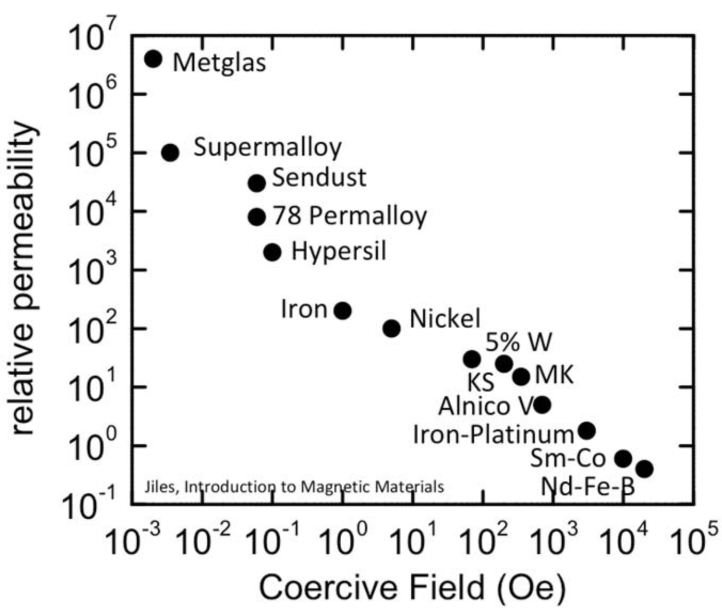
It turns out that ferromagnetic materials exhibit memory effects in their $M(H)$ dependence, this property is measured as a ***hysteresis loop***:



Courtesy of Wayne Storr. Used with permission.

If we apply sufficient magnetic field to produce complete saturation inside the ferromagnet and then start reducing the field back to zero, we will find that at zero applied field some residual magnetization = "***remnant induction***" will remain and it will take a significant field "***coercive field***" to completely demagnetize the material.

Curiously, materials that have large magnetic permeability (which corresponds to the large saturation magnetization) exhibit small coercive field and vice versa.



Last lecture we have discussed the energies (Hamiltonians) contributing to the magnetization process:

$$\hat{H}_{ex} = -\sum_{i,j} J_{ij}\hat{S}_i\hat{S}_j$$

$$\hat{H}_{field} = \frac{\mu_B}{\hbar}\sum_i \vec{B}\hat{S}_i$$

But neither of the Hamiltonians above can explain why there exists a ***coercive field*** or why $\chi_m \uparrow \Rightarrow H_c \downarrow$ & $\chi_m \downarrow \Rightarrow H_c \uparrow$. (Here χ_m is magnetic susceptibility and H_c is coercive field).

The key to these question lies in the "***magnetic anisotropy***" **–** the dependence of the magnetic properties on the direction of the applied field with respect to the crystal lattice. It turns out that depending on the orientation of the field with respect to the crystal lattice one would need a lower or higher magnetic field to reach the saturation magnetization.

Easy axis is the direction inside a crystal, along which small applied magnetic field is sufficient to reach the saturation magnetization.

Hard axis is the direction inside a crystal, along which large applied magnetic field is needed to reach the saturation magnetization.

Consider the following three examples: Fe (bcc), Ni (fcc), Co (hcp):

Figure removed due to copyright restrictions. See Fig. 6.1: O'Handley, Robert C. *Modern Magnetic Materials*. Wiley, 1999.

- For **bcc Fe** the highest density of atoms is in the <111> direction, and consequently <111> is the hard axis. In contrast, the atom density is lowest in <100> directions and consequently <100> is the easy axis. Magnetization curves above show that the saturation magnetization in <100> direction requires significantly lower field than in the <111> direction.
- For **fcc Ni** the <111> is lowest packed direction and it is the easy axis. <100> is the hard axis.
- For **hcp Co** the <0001> is the lowest packed direction (perpendicular to the close-packed plane) and is the easy axis. The <1000> is the close-packed direction and it corresponds to the hard axis. Note, that hcp structure of Co makes it the one of the most anisotropic materials (see the different units on the figure above).

For a material with a single easy axis perpendicular to the hard axes (e.g. Co) the energy associated with the magnetic anisotropy can be written as:

$$E_a = \sum_{n=1}^{\infty} K_{un} \sin^{2n}\theta = K_{u1}\sin^2\theta + K_{u2}\sin^4\theta + ... \approx K_{u1}\sin^2\theta$$

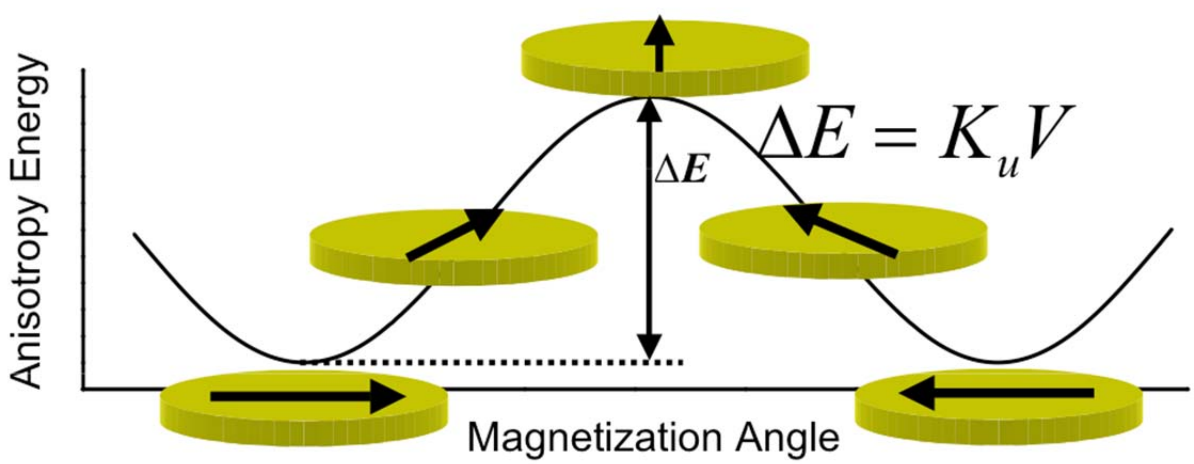
Where θ is the angle between the magnetization and the easy axis, and K_{un} are the anisotropy constants.

For cubic material with three easy axes, the anisotropy energy is written as:

$$E_a = K_1\left(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2\right) + K_2\alpha_1^2\alpha_2^2\alpha_3^2 + ...$$

Where $\alpha_i = \cos\theta_i$ and θ_i are the angles between the magnetization and the easy axes.

The physical origin of the magnetic anisotropy energy is the interaction of the mean exchange field and the orbital angular momenta of the atoms (ions) in the lattice. This interaction is referred to as ***spin-orbit coupling***.

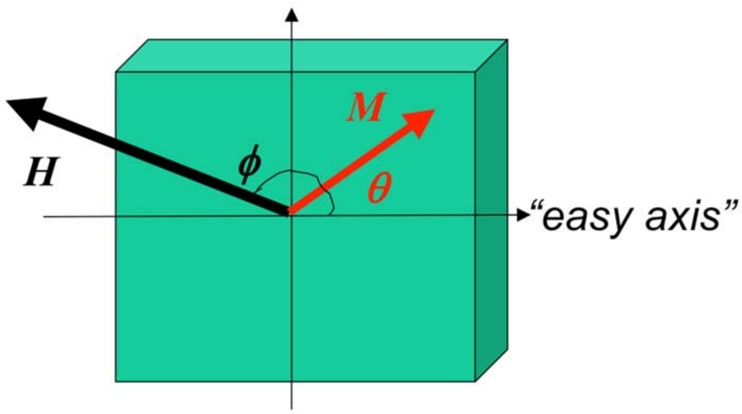


Derivation of the hysteresis loop for a single domain ferromagnet.

Let's start with an anisotropic (with single easy axis) ferromagnet sufficiently small so it consists of a single magnetic domain, i.e. all magnetic dipoles are aligned in the same direction maximizing the total magnetization $\vec{M}$. Assume that the angle between the magnetization and the easy axis is θ.

In the absence of an external field the energy of the ferromagnet as a whole is dominated by the anisotropy energy:

$$E_a \approx K_u \sin^2 \theta$$



When external field is applied the ferromagnet in addition to the anisotropy energy we will also have the magnetostatic or Zeeman energy:

$$E_{magnetostatic} = HM_s \cos(\phi - \theta)$$

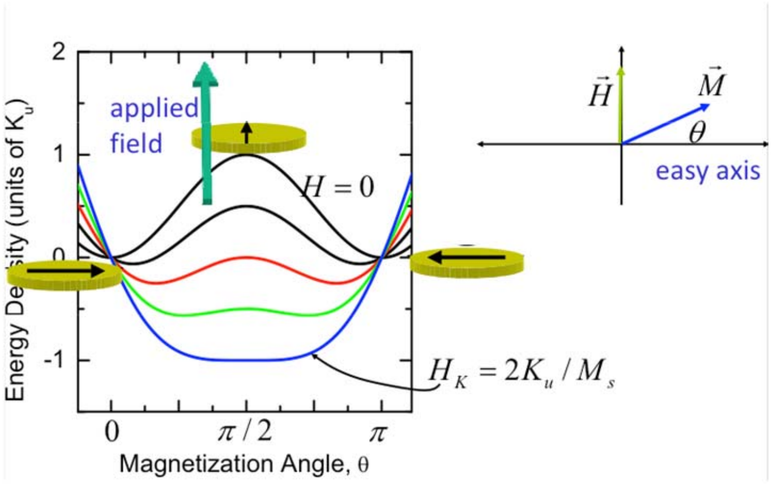
Where H is the applied magnetic field, M_s is the saturation magnetization and ϕ is the angle between the applied field and the easy axis.

Then the total energy is: $E = E_a + E_{magnetostatic} = K_u \sin^2 \theta + HM_s \cos(\phi - \theta)$

Let's consider two important special cases for the direction of the magnetic field: along easy or along hard axis.

Hard axis magnetization.

$$\phi = \frac{\pi}{2} \Rightarrow E = K_u \sin^2 \theta + HM_s \cos(\phi - \theta) = K_u \sin^2 \theta + HM_s \sin \theta$$



Let's find the angle θ that minimizes the energy in this case:

$$\frac{dE}{d\theta} = \left(2K_u \sin\theta + HM_s\right)\cos\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2} + \pi n,\ \ \vec{M}_s \uparrow\uparrow \vec{H} \ or\ \vec{M}_s \uparrow\downarrow \vec{H}$$

$$\frac{d^2E}{d\theta^2} = -2K_u \sin^2\theta - HM_s \sin\theta + 2K_u \cos^2\theta > 0 \Rightarrow \begin{cases} \theta = \dfrac{\pi}{2} \Rightarrow H < -\dfrac{2K_u}{M_s} \\ \theta = -\dfrac{\pi}{2} \Rightarrow H > \dfrac{2K_u}{M_s} \end{cases}$$

The only way the conditions above can be satisfied if: $H_a = \dfrac{2K_u}{M_s} \Rightarrow K_u = \dfrac{H_a M_s}{2}$.

Where H_a is referred to as ***anisotropy field*** (field at which magnetization reaches saturation).

For fields bellow the anisotropy field from the equation: $\dfrac{dE}{d\theta} = \left(2K_u \sin\theta + HM_s\right)\cos\theta = 0$

we also find: $2K_u \sin\theta + HM_s = 0$. For the component of magnetization that is parallel to the applied magnetic field we find: $\sin\theta = -\dfrac{M}{M_s}$. Substituting this into the previous equation we find:

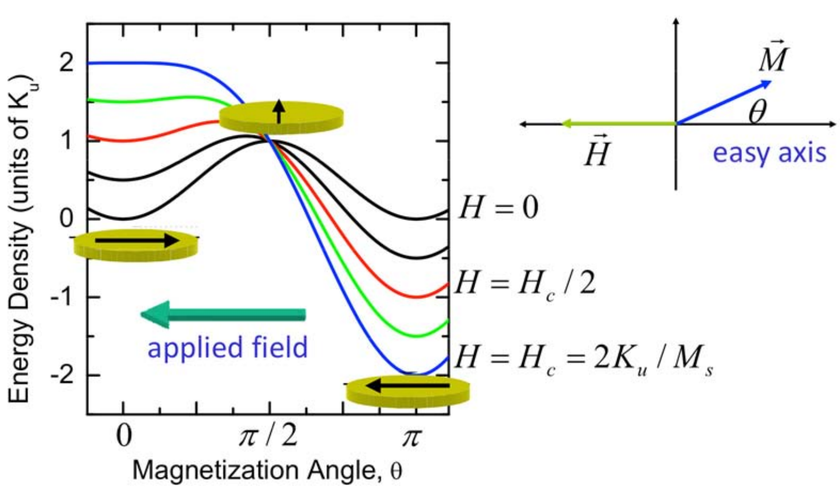
$$2K_u \sin\theta + HM_s = 0 \Rightarrow -2K_u \frac{M}{M_s} + HM_s = 0 \Rightarrow -2\frac{H_a M_s}{2}\frac{M}{M_s} + HM_s = 0$$

$$\Rightarrow M = M_s \frac{H}{H_a}$$

We find that for the hard axis magnetization changes linearly with applied field until it reaches saturation.

Easy axis magnetization.

$$\phi = 0 \Rightarrow E = K_u \sin^2\theta + HM_s \cos(\phi - \theta) = K_u \sin^2\theta + HM_s \cos\theta$$



If we find the angle θ that minimizes the energy in this case:

$$\frac{dE}{d\theta} = \left(2K_u \cos\theta - HM_s\right)\sin\theta = 0 \Rightarrow \theta = \pi n,\ \ \vec{M}_s \uparrow\downarrow \vec{H} \ or\ \vec{M}_s \uparrow\downarrow \vec{H}$$

$$\frac{d^2E}{d\theta^2} = 2K_u \cos^2\theta - HM_s \cos\theta - 2K_u \sin^2\theta > 0 \Rightarrow \begin{cases} \theta = 0 \Rightarrow H < \dfrac{2K_u}{M_s} \\ \theta = -\pi \Rightarrow H > -\dfrac{2K_u}{M_s} \end{cases}$$

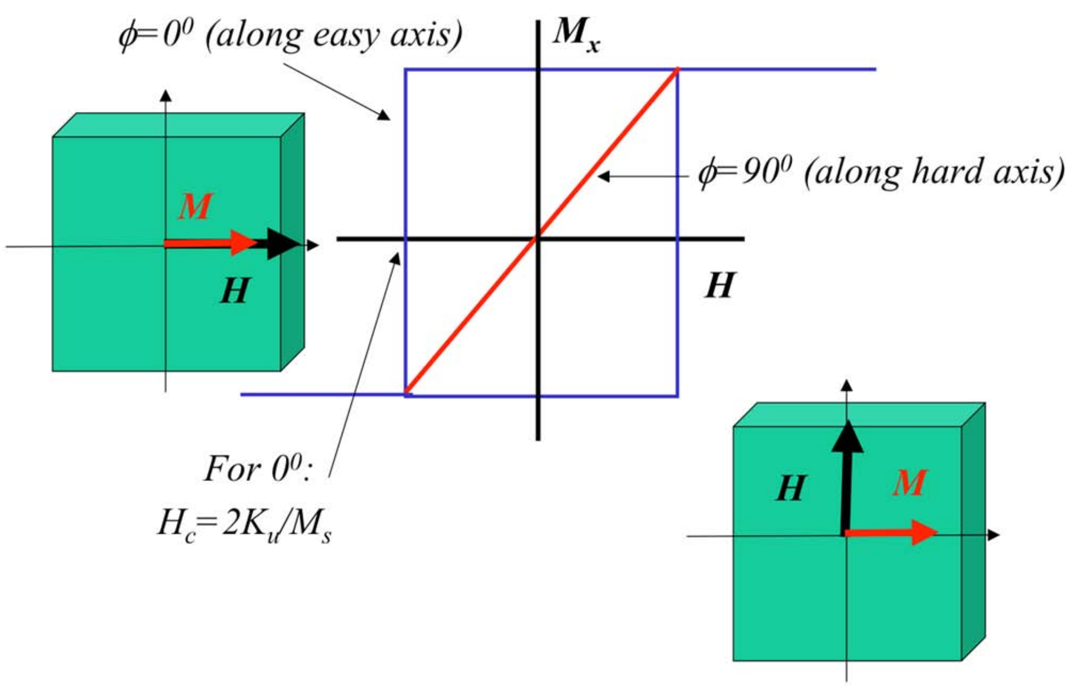
Energy will be minimized for all magnetic field values between:

$-\frac{2K_u}{M_s} < H < \frac{2K_u}{M_s}$. The coercive field is: $H_c = \frac{2K_u}{M_s}$

All of these fields will yield saturation magnetization. This results in the ideal rectangular shape of the hysteresis loop.

From the expression for the coercive field $H_c = \frac{2K_u}{M_s}$, it is obvious that materials with high saturation magnetization would have low coercive field unless they have really high anisotropy. But given a particular anisotropy constant: $M_{sat} \uparrow \Rightarrow H_c \downarrow$.

Ideal hysteresis loop:



3.024 Electronic, Optical and Magnetic Properties of Materials
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