

Lecture 19-20
Light Emitting Diodes.

Today:

1. Carrier recombination in semiconductors.
2. *p-n* junctions with carrier injection. Light-emitting diodes (LEDs).

Questions you should be able to answer by the end of today's lecture:

1. What is the mechanism of operation of light emitting devices?
2. What are important LED design characteristics?
3. What is the joint density of states and the emission condition?
4. What determines electroluminescence spectrum of an LED?

Carrier recombination. Light-emitting diodes.

In any semiconductor electrons and holes can meet spontaneously and recombine, i.e. conduction electrons can fall down and fill the holes in the valence band, the excess energy is then released as a photon. At low temperatures (e.g. room temperature) the concentration of carriers even in

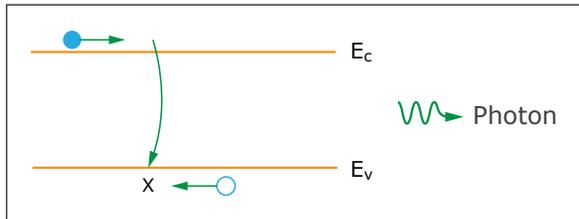


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doped semiconductors is too low for the carriers to meet and recombine spontaneously. However if we electrically inject carriers into the semiconductor we can raise their concentration dramatically so that the probability of recombination becomes significant and we can observe emitted

photons. Generally we would use a ***p-n junction diode in a forward bias regime*** to inject high concentrations of **electrons into the *n*-side** and **holes into the *p*-side**. These carriers can then meet at the junction and recombine releasing photons.

This process is called “***electroluminescence***”.

There are three general types of light-emitting devices:

(a) Light-emitting diodes (LEDs):

Electrically injected electrons and holes recombine at the p-n junction leading to spontaneous emission.

(b) Semiconductor optical amplifiers (SOAs):

In addition to injected carriers p-n junction is under illumination, which increases the overall population of carriers and for each incoming photon two photons are emitted, hence the device works as an amplifier.

(c) Laser diodes (LDs):

Extremely high carrier injection levels can lead combined with a positive feedback (a pair of mirrors – an optical cavity) can lead to stimulated emission or lasing.

Figure removed due to copyright restrictions. Fig. 16.0-1: Saleh, Bahaa E. A., and Malvin Carl Teich. *Optical Communications* 2nd ed. Wiley, 2007.

We will focus on light-emitting diodes (LEDs).

The efficiency of LEDs will be determined by the radiative recombination rate. As we have seen earlier there are many other ways that electron-hole pair can recombine, all these processes (R–G center recombination, Auger recombination etc.) will contribute negatively to the LED efficiency, so we need to maximize the radiative recombination rate and minimize all other recombination rates.

If we inject $\Delta n = \Delta p$ excess carriers (current has to be the same on n and p sides due to continuity and charge neutrality) into the p-n diode, the carrier concentrations at the junction will be: $p = p_0 + \Delta n$, $n = n_0 + \Delta n$ where n_0, p_0 are the equilibrium carrier concentrations without the current injection.

Let's assume that due to all recombination processes electron-hole pairs recombine within a time τ – electron-hole pair lifetime. We can also assume that in the steady state the excess of injected carriers is balanced by the recombination, then the carrier injection rate is: $R = \frac{\Delta n}{\tau}$

In low injection regime (no stimulated emission or amplification): $\tau \approx \frac{1}{k(n_0 + p_0)}$, where k is a recombination constant (generally experimentally determined number).

Then the injection rate R : $R = \frac{\Delta n}{\tau} \approx k\Delta n(n_0 + p_0)$

The internal quantum efficiency (IQE) of an LED is determined by the ration of the radiative recombination rate to the recombination rate due to all processes:

$IQE = \eta_i = \frac{k_R}{k} = \frac{k_R}{k_R + k_{NR}}$, where k_R, k_{NR} are radiative and no-radiative recombination constants.

Alternatively we can write it as: $\eta_i = \frac{\tau}{\tau_R} = \frac{\tau_R + \tau_{NR}}{\tau_R}$, where τ_R is the radiative lifetime.

Then the number of emitted photons per unit volume per second will be proportional to the radiative recombination constant and the concentration of injected carriers Δn :

$$\Phi_i = \eta_i R = k_R \Delta n (n_0 + p_0)$$

$$\Phi_i = \eta_i R = \eta_i \frac{\Delta n}{\tau} = \frac{\Delta n}{\tau_R}$$

The flux of photons emitted by a volume V is: $\phi_i = V\Phi_i = V \frac{\Delta n}{\tau_R}$

As you noticed the internal quantum efficiency does not take into account photon extraction efficiency but is solely determined by the radiative recombination time. The external quantum efficiency of the LED takes into account the device design parameters and is simply defined as:

$$EQE = \eta_{ex} = \frac{\text{photons out}}{\text{electrons in}} = \frac{\text{photons/s}}{\text{electrons/s}} = \frac{\phi}{I}$$

Where ϕ is the measured photon flux and I is injected current.

During the experiment photodetector is used to measure the total optical power P [W] emitted from an LED. Then for a given wavelength λ we can find the photon flux:

$$\phi = \frac{P}{\text{photon energy}} = \frac{P}{ch/\lambda}$$

$$\text{Then: } EQE = \eta_{ex} = \frac{\phi}{I} = \frac{P\lambda}{Ich}$$

However in order to find total efficiency we need to integrate over the entire electroluminescence spectrum.

Note that expression above is exactly opposite to the EQE for the solar cell, this means that device optimized to be a good LED will generally be a poor solar cell and vice versa.

LED electroluminescence spectra.

The spontaneous emission rate depends on the photon energy:

$$k_{sp}(\omega) = \frac{1}{\tau_R} g_j(\omega) f_e(\hbar\omega)$$

Where $g_j(E)$ is the joint density of states and $f_e(E)$ is the joint occupation probability for the valence and conduction bands also known as the emission condition.

The joint density of states is analogous to the density of states for the conduction or valence band but in this case we are considering the number of states available to electrons and holes, which upon recombination would yield to photons of particular frequency. In order to determine the joint density of states we use the parabolic approximation for the bands at the band edge. Recall that:

Figure removed due to copyright restrictions. Fig. 16.1-2: Saleh, Bahaa E. A., and Malvin Carl Teich.) XQGDP HQVWV RI 3KRVRQIFV. 2nd ed. Wiley, 2007.

$$E_2 = \frac{\hbar^2 k^2}{2m_c^*} + E_g \quad \text{for electrons in the conduction band}$$

$$E_1 = -\frac{\hbar^2 k^2}{2m_v^*} \quad \text{for holes in the valence band}$$

Then the energy of the photon emitted when the electron with energy E_2 recombines with the hole with energy E_1 is:

$$\hbar\omega = E_2 - E_1 = \frac{\hbar^2 k^2}{2m_c^*} + E_g + \frac{\hbar^2 k^2}{2m_c^*} = \frac{\hbar^2 k^2}{2m_r} + E_g$$

Where $m_r = \frac{m_c^* m_v^*}{m_c^* + m_v^*}$ is the reduced mass and $E = \hbar\omega$ is the energy of the emitted photon.

Then we can write the energies of the electron and the hole as:

$$E_2 = E_c + \frac{m_r}{m_c^*} (\hbar\omega - E_g) = E_c + \frac{m_r}{m_c^*} (\hbar\omega - E_g)$$

$$E_1 = E_v - \frac{m_r}{m_v^*} (\hbar\omega - E_g) = E_2 - \hbar\omega$$

As the number of recombining electron hole pairs corresponds directly to the number of emitted photons we can write that the total number of electrons dropping into the valence band and filling holes is exact same as number of photons in the ideal case, then:

$$\left. \begin{aligned} g_c(E_2) dE_2 &= g_j(\omega) d\omega \\ g_c(E) &= \frac{(m_c^*)^{\frac{3}{2}}}{\pi^2 \hbar^3} \sqrt{2(E - E_c)} \end{aligned} \right\} \Rightarrow g_j(\omega) = g_c(E_2) \frac{dE_2}{d\omega} = \frac{(m_c^*)^{\frac{3}{2}}}{\pi^2 \hbar^3} \sqrt{2(E_2 - E_c)} \frac{d(E_2)}{d\omega}$$

$$\Rightarrow g_j(\omega) = \frac{(m_c^*)^{\frac{3}{2}}}{\pi^2 \hbar^3} \sqrt{2 \left(E_c + \frac{m_r}{m_c^*} (\hbar\omega - E_g) - E_c \right)} \frac{d \left(E_c + \frac{m_r}{m_c^*} (\hbar\omega - E_g) \right)}{d\omega}$$

$$\Rightarrow g_j(\omega) = \frac{(m_r)^{\frac{3}{2}}}{\pi^2 \hbar^2} \sqrt{2(\hbar\omega - E_g)}$$

The emission condition is then expressed in terms of the Fermi distributions of the electrons and holes:

$$f_e(\hbar\omega) = f_c(E_2)(1 - f_v(E_1))$$

Here $f_c(E)$, $f_v(E)$ are the Fermi distribution functions for electrons and holes in the non-equilibrium injection conditions:

$$f_c(E) = \frac{1}{1 + e^{E - E_{Fc}/k_B T}}$$

$$1 - f_v(E) = \frac{1}{1 + e^{E_{Fv} - E/k_B T}}$$

Figure removed due to copyright restrictions. Fig. 16.1-3:
Saleh, Bahaa E. A., and Malvin Carl Teich.) *XQGDH HQWUW*
RI 3KRVRQLFV. 2nd ed. Wiley, 2007.

Where E_{Fc} , E_{Fv} are the non-equilibrium Fermi levels or quasi-Fermi levels (due to injection) for electrons and holes. During injection the concentration of carries can be so high that the conduction band would have much higher concentration of electrons and the valence band would have much higher concentration of holes than they would have in thermal equilibrium, which can be expressed in terms of quasi-Fermi levels moving into the bands making the SC behaving almost like a metal.

We can determine the quasi-Fermi levels based on the injection concentrations. Remember how we calculated carrier densities in equilibrium using D.O.S function and the Fermi distribution, so the same logic applies here:

$$n = n_0 + \Delta n = \int_{E_c}^{\infty} g_c(E) f_c(E) dE, \quad g_c(E) = \frac{m_c^*}{\pi^2 \hbar^2} \sqrt{\frac{2m_c^*(E - E_c)}{\hbar^2}}$$

$$p = p_0 + \Delta n = \int_{-\infty}^{E_v} g_v(E) f_v(E) dE, \quad g_v(E) = \frac{m_v^*}{\pi^2 \hbar^2} \sqrt{\frac{2m_v^*(E_v - E)}{\hbar^2}}$$

In the conditions when $E_2 \gg E_{Fc}$, $E_1 \ll E_{Fv}$ we can approximate the Fermi functions with exponentials:

$$f_c(E) = \frac{1}{1 + e^{E - E_{Fc}/k_B T}} \approx e^{-(E - E_{Fc})/k_B T}$$

$$1 - f_v(E) = \frac{1}{1 + e^{E_{Fv} - E/k_B T}} \approx e^{-(E_{Fv} - E)/k_B T}$$

Then the spontaneous emission rate can be found as:

$$k_{sp}(\omega) = \frac{1}{\tau_R} g_j(\omega) f_e(\hbar\omega) \approx \frac{(m_r)^{\frac{3}{2}}}{\pi^2 \hbar^2 \tau_R} \sqrt{2(\hbar\omega - E_g)} e^{-(E_2 - E_{Fc})/k_B T} e^{-(E_{Fv} - E_1)/k_B T}$$

$$k_{sp}(\omega) = \frac{(m_r)^{\frac{3}{2}}}{\pi^2 \hbar^2 \tau_R} e^{(E_{Fc} - E_{Fv})/k_B T} \sqrt{2(\hbar\omega - E_g)} e^{-(E_2 - E_1)/k_B T} = \frac{(m_r)^{\frac{3}{2}}}{\pi^2 \hbar^2 \tau_R} e^{(E_{Fc} - E_{Fv} - E_g)/k_B T} \sqrt{2(\hbar\omega - E_g)} e^{-(E_2 - E_1 - E_g)/k_B T}$$

Recalling that: $E_2 - E_1 = \hbar\omega$

$$k_{sp}(\omega) = \frac{(m_r)^{\frac{3}{2}}}{\pi^2 \hbar^2 \tau_R} e^{(E_{Fc} - E_{Fv} - E_g)/k_B T} \sqrt{2(\hbar\omega - E_g)} e^{-(\hbar\omega - E_g)/k_B T}$$

$$k_{sp}(\omega) = D \sqrt{\hbar\omega - E_g} e^{-(\hbar\omega - E_g)/k_B T}, \text{ where } D = \frac{(2m_r)^{\frac{3}{2}}}{2\pi^2 \hbar^2 \tau_R} e^{(E_{Fc} - E_{Fv} - E_g)/k_B T}$$

Note that D increases exponentially as the spacing between the quasi-Fermi levels increases with respect to the bandgap, i.e. the emission rate increases exponentially as we inject more carriers.

On the other hand the dependence of the rate on photon frequency is independent of injection, i.e. electroluminescence spectrum does not change with increasing injection.

Figure removed due to copyright restrictions. Fig. 16.1-4: Saleh, Bahaa E. A., and Malvin Carl Teich.) *XQGDH HOWDQ RI 3KRVROLFV* 2nd ed. Wiley, 2007.

The electroluminescence peak can be found approximately by taking a derivative of the spontaneous rate with respect to photon frequency:

$$\begin{aligned} \frac{d}{d\omega}(k_{sp}(\omega)) &= 0 \\ \frac{d}{d\omega}\left(D\sqrt{\hbar\omega - E_g} e^{-(\hbar\omega - E_g)/k_B T}\right) &= D\left(\frac{\hbar}{2\sqrt{\hbar\omega - E_g}} e^{-(\hbar\omega - E_g)/k_B T} - \frac{\hbar}{k_B T} \sqrt{\hbar\omega - E_g} e^{-(\hbar\omega - E_g)/k_B T}\right) = 0 \\ \frac{1}{2} - \frac{1}{k_B T}(\hbar\omega_{peak} - E_g) &= 0 \Rightarrow \hbar\omega_{peak} = E_g + \frac{1}{2}k_B T \end{aligned}$$

The bandgap of the material and the ambient temperature determine the emission wavelengths of the LED.

Material	Wavelength (nm)
GaAs	838 (4.2 K)
	843 (77 K)
InP	910 (77 K)
InAs	3,100 (77 K)
InSb	526 (10 K)
PbSe	8,500 (4.2 K)
PbTe	6,500 (12 K)
Ga(As _x P _{1-x})	650-840
(Ga _x In _{1-x})As	840-3,500
In(As _x P _{1-x})	910-3,500
GaSb	1,600 (77 K)
Pb _{1-x} Sn _x Te	(9.5-28 μm) (12 K)
Ga _{1-x} Al _x As	690-850
InGaAsP	1,200-1,600
InGaP	500-700
ZnSe	490-500 (4.2 K)
In _x Ga _{1-x} N	450-650

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Figure removed due to copyright restrictions. Graph of semiconductor [band gap vs. lattice constants](#).

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